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A CLASSROOM QUASI-EXPERIMENTAL INTERVENTION TO EXPLORE PERFORMANCE IN CIRCLE THEOREMS AMONG 11th GRADERS IN A RURAL PLACE IN ZAMBIA

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Abstract

Purpose: This study was designed to compare classroom performance in circle theorems among 11th graders with a view to determine the causal difference in performance in circle theorems when concrete-representational-abstract instructional approach as a method of instruction was employed on the experimental group compared to the conventional method of instruction which was employed on the control group.

Materials and Methods: This was a post-test only two group quasi experimental design of 11th graders. A control group was exposed to the conventional approach and an experimental group was exposed to the concrete representational approach. Post-test and delayed Post –test of the two groups were gathered and analysed using mean, paired sample t-test, independent sample t-test, and Mann Whitney U test.

Findings: Findings revealed Post-test and delayed Post –test changes in performance. There were higher rates of performance in the experimental group. Beneficial effects were evidenced when concrete-representational-abstract instructional approach was employed than conventional. Mann-Whitney U test showed that the medians of the post test scores were independent – and statistically significantly higher with values being 83.8 for the experimental group when concrete-representational-abstract instructional approach was employed compared to the value 52.13 for the control group and U was 110 and p was .024. In addition, Mann-Whitney U test for the delayed post test scores were independent – and were statistically significantly higher that was 85.5 for the experimental group compared to 52.47 and U was 221 and p was 0.001.

Conclusion: Concrete-representational-abstract instructional approach is better than the conventional in enhancing learners' mathematics performance in circle theorem.

Key words: *Concrete representational abstract, traditional approach, circle theorem, performance*
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Introduction

Mathematics has been accepted as an important component of formal education from ancient period to the present day. Mathematics is one of the most important subjects in our human life. Without the knowledge of Mathematics, we can say nothing is possible in the world. Mathematics learning in each educational unit is expected to equip learners with the skills and abilities to face various problems of math and daily life. This ability is referred to as mathematical power. The application of Mathematics is wide ranging including problem - solving ability, reasoning ability, communication ability to make connection, and render representation ability. One of the important roles of learning Mathematics is to understand the abstract object of Mathematics directly (NCTM, 2000; Santrock, 2002). To achieve student abstraction ability, it takes something bridging from concrete to the abstract. In its application, learners can present ideas in the form of mathematical representations, in the form of concrete models, in the form of images or other forms. Most of the learners today might have difficulties in understanding the lesson in Mathematics especially in Geometry because of many reasons. These reasons might include the ways on how the teachers teach the lessons in class. Given this, Mathematics teaching does not merely convey information such as rules, definitions, and procedures to be memorized by learners but teachers must actively involve learners in the learning process. Active participation of learners will strengthen their understanding of mathematical concepts. This is in accordance with the principles of constructivism which is; knowledge built by learners themselves, both personally and socially, knowledge cannot be transferred from teacher to student. Each student has a different way to construct his knowledge. Problem solving approach is one part of mathematical curriculum. This approach allows learners to gain experience to use the knowledge and skills they have gained in solving non-routine problems. In problem solving, teachers present problems that are not to be solved routinely by the learners. In this case, learners are required to have the ability to synthesize knowledge, skills, and understanding so that in the end they can solve the problems faced well. However, teachers face difficulties in teaching how to solve problems well, on the other hand learners face difficulties on how to solve problems given by teachers. These difficulties arise partly because searching for answers is seen as the only goal to be achieved.

The teaching of Mathematics does not merely convey information such as rules, definitions, and procedures to be memorised by learners but teachers ought to actively involve learners in the learning process. Active participation of learners tends to strengthen their understanding of mathematical concepts. This is in accordance with the principles of constructivism which is knowledge built by learners themselves, both personally and socially, knowledge cannot be transferred from teacher to student. Each student has a different way to construct his knowledge. One of the constructive approaches is concrete-representational- abstract sequence of instruction.

A plethora of literature has examined teaching Mathematics to learners using the CRA model and among a few to teach fractions (Bouck et al., 2017; Flores et al., 2020; Lemonidis et al., 2020; Morano et al., 2020), mathematical concepts and skills (Flores, 2010[1]; Mancl et al., 2012[2]; Sealander et al., 2012[3]; Kim, 2015[4]; Bouck et al., 2018[5]; Flores, & Hinton, 2019[6]), vocabulary problems and place value (Doabler and Fien, 2013[7]), perimeter and area (Satsangi and Bouck, 2015[8]; Indriani,2019[9]) and logical-mathematical thinking (Novaliyosi, 2020[10]; Rittle et al., 2021[11]; Root et al., 2021[12]). The outcomes from these studies have demonstrated that learners grasped concepts and skills and they tended to perform better than other methods of instruction.

The Research Problem

The performance of learners in Grade 12 especially in Geometry has been observed over the years not to be improving. There have been no interventions that have been designed to mitigate this poor performance notwithstanding the fact that Geometry is a critical key area of knowledge related to professions that are based on science, technology, engineering, and mathematics (STEM) (Carnevale et al., 2011[13]). It has been feared that if there was no modification in the teaching of Geometry, since Geometry is also interwoven with the individual's life and everything that surrounds a person, its application generally in life, science, technology, engineering, and mathematics (Crompton, 2013[14] AL-Salahat, 2022[15]). This situation was highlighted in the 2021 board of examiners and called for pedagogical interventions. It therefore follows that the aim of this study

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was to determine the causal difference in performance in circle theorems when Concrete-representational-abstract sequence of instruction as a conventional pedagogical method of instruction among Grade 11 pupils is compared to the traditional method of instruction.

The conceptual paradigm of the study is based on the assumption theorising that the Concrete-Representational-Abstract (CRA) is potent in creating an impact to improve learners ' performance in circle theorems within geometry. This study as such was designed to respond to the call to mitigate poor performance in Geometry among senior graders in secondary schools.

Research Setting

Katete District of Eastern part of Zambia is the focus of the study. The district is located Eighty (80) kilometres from the provincial capital (Chipata) of Eastern province of Zambia. The choice of this area is based on the fact that the researcher had observed that this is one of the districts in Zambia which has been performing poorly in Mathematics (ECZ, 2020- 2022) and especially geometry. Katete has fifteen secondary schools. All these are government funded schools.

Research Design and Methodology

For this study, we opted to employ a two phased sequential explanatory mixed methods approach as defined by Creswell et al. (2003). The phasing was appropriate to determine causation. The first phase was evaluation of the Post-test. This was done soon after completing the syllabus on circle theorems and the second one was a delayed post-test. This was administered after four weeks. At the heart of this design, is the researcher's ability to (a) manipulate the independent variable (i.e., treatment) that is hypothesized to affect the dependent variable (i.e., outcome being pupil performance in circle theorems), and (b) not to randomly assign participants to treatment (i.e., a group that experiences the manipulation) and control (i.e., a group that does not receive the treatment) conditions (Shadish et al., 2002;[16]White and Sabarwal , 2014[17]). The groups for observation following an intervention occur naturally (i.e., not by researcher). Because the field experiment was designed to examine phenomena in the two natural settings while also employing research design features that support causal inference, the researcher considers this also as one of the "gold standards" of quasi experimentation (Remler and Van Ryzin; 2014; [18] Bärnighausen et al., 2017[19]). It is methodologically prudent to render justification for using a quasi-experimental design that has been selected in this study and no other better designs. When designing a study to estimate the causal effect of an intervention, the experiment (particularly the randomised controlled trial (RCT) is generally considered to be the least susceptible to bias. A defining feature of the experiment is that the researcher controls the assignment of the treatment or exposure. If properly conducted, random assignment balances unmeasured confounders in expectation between the intervention and control groups. In many evaluations of education interventions, however, it was not possible to conduct randomised experiments in this study because of the nature of the problem and the respondents at hand. Instead, standard observational quasi experimental study design is traditionally used. Quasi experimental designs are known to be susceptible to unmeasured confounding.

Enlisting Procedure

Our focus was to enlist two Mathematics teachers who were handling the best Grade 11 Class. A Grade 11 Class was a cohort of learners who were purposefully selected by the school management following extemporary performance in Grade 9 examinations. Each school had the best Grade 11 class and it was then easy to undertake the study. By accommodating this baseline data (pre-intervention) which made sure that the comparison group was as similar as possible to the treatment. This allowed the researcher to conduct the experiment with greater internal validity (Aguinis et al., 2020; Grant & Wall, 2009; Podsakoff & Podsakoff, 2019).

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The procedure to enlist teachers to be part of this intervention was as follows; we requested the Head teachers of all the 15 schools to connect us to a Mathematics teacher who was handling the best Grade 11 Class. We held an unplanned discussion with each teacher who was handling a Grade 11 class with a view to assess the pedagogical inclination when teaching mathematics. A teacher with the preponderance for traditional approach was selected and assigned to an intervention befitting a control group (Pot 1) to employ the traditional approach whereas one who showed flexibility was assigned to the intervention group (Pot 2) to employ concrete-Representational- abstract sequence instructional approach.

In total there were five teachers from the fifteen secondary schools in Katete who demonstrated the use of concrete representational- abstract sequence instructional approach and eleven for the traditional approach. A number was written on a piece of paper linked to each teacher. Only one number was to be picked randomly from each pot. It happened that the teachers of the best classes from Katete Day Secondary School and Jersey Day Secondary School were selected. For ethical reasons, teaching preponderance by school type is withheld.

Procedure for the Application of the Traditional Method of Instruction

One of the teaching approaches teachers use when teaching the learners is the lecture method (Mancl & Miller, 2021). The approach is faster and appropriate more especially if the class size is big. It is a teacher centered approach where learners have less participation in the lesson. In this study, a teacher who showed an inclination to the lecture method of teaching was asked to use the method without telling him how to use it in order to avoid contaminating the results. The teacher who was assigned to teach circle theorem was requested to notify us to go and administer the test once the circle theorem syllabus was completed.

Procedure for the Application of the Concrete-Representational- Abstract Sequence of Instruction Approach

The teacher who was assigned to teach circle theorem using the intervention approach was requested to notify us to go and administer the test once the circle theorem syllabus was completed.

The intervention was designed to incorporate widely accepted practices of effective instruction, such as systematic and explicit instruction (Goeke, 2009; Fuchs et al., 2008; Miller & Hudson, 2007, 2006), the use of CRA (Witzel et al., 2003, 2009), consistent and immediate instructor feedback, and ample opportunities for learners (Stein et al., 2006) provided in scripted lessons. The intervention targeted specific skills necessary to develop mathematics knowledge and be successful in computations regarding circle geometry in order to prepare the learners for the future tertiary high school geometric courses (NMAP, 2008). Scripted lessons from the book *Computation of circle geometry: Math Intervention for Grade 11 Learners* were used for the intervention. Below is the procedural approach the researcher used to carry out this study.

Steps:

1) ***Introduced the lesson.*** Teachers briefly introduced the lesson topic and shared with the learners their expectations for the day. They told learners that they would be learning circle theorem (e.g., addition, subtraction) skill).
2) ***Step 2: Modelled the lesson***. Teachers modelled the correct procedures necessary to complete the desired task. As scripted, teachers practiced using metacognitive strategies out-loud to model the reasoning and thinking behind the behaviours. During this step, the teacher was the only individual using the concrete manipulatives, representational pictures, or numbers and symbols. Teacher demonstrated the behaviours necessary to successfully complete the computation of Geometry. Teaching the math circle theorem concept/skills by using manipulatives (concrete level). Learners were taught the circle geometry (proof of circle theorems) using a plank where a circle was drawn and placed with some pins along the circle. Using rubber bands, a teacher started proving same circle theorems.
3) ***Step 3: Guided learners through practice.*** During this step, the teacher scaffolded learners learning by guiding learners through the steps of the desired tasks. The structured practice allowed learners to correctly display the desired behaviours and successfully complete the task with the support of the teacher. During the third step, both the teacher and the learners practiced by using the concrete manipulatives, representational pictures, or numbers and symbols. Learners used hands-on manipulatives to develop a conceptual understanding of the circle geometric computations. The teacher modelled the learning objectives in the lesson by physically moving objects and concurrently describing

the process of solving the computation problems. Over the course of the lesson, learners were taught how to use the manipulatives to solve similar computations and eventually achieved independent practice. Wooden circle, Thumb Tacks pins, rubber bands, paper symbols (e.g. formation of the circle and angles in the circle) were used as physical manipulatives.

4) ***Step 4:*** This stage is called the 'work' stage where learners physically manipulated objects to solve a Mathematical problem through the use of 3 D artefacts to help them learn new concepts (Miller, & Leffar, 2011). With this in mind, learners would move and manipulate 3 D objects to represent their thinking. For example, the student used geometric tapes and a pencil to draw a circle on a plank and then placed some pins along the circle and manipulated the object to prove a lot of circle theorems as shown in the next diagram (see Table 4.1). This allowed for ample opportunities for learners to practice the concept using various manipulatives.
5) ***Step 5:*** Provided opportunities for independent practice and feedback. After learners had demonstrated that they were successful in step three (i.e., by showing approximately 90% accuracy), they were able to practice independently under the observation of their teacher. This step allowed learners the opportunity to become fluent with the mastered materials. The teacher circulated around the classroom and was available to work with learners who did not reach approximately 90% accuracy during the guided practice. This was done to make sure the learners understood the concepts at the concrete level before moving to the representational level.
6) ***Step 6:*** Introduced pictures to represent objects (representational level). Learners bridged their knowledge from concrete manipulatives to abstract numbers and symbols through the use of picture representations as a means to develop a more thorough understanding of the concepts. Similar to the first phase of the lesson, the teacher modelled how to solve the computation problems with the use of pictures that were visually similar to the manipulatives used in the first phase. Representational symbols (i.e., drawings of circles and different angles in the circle) were introduced during the second phase of the lessons.
7) ***Step 7:*** Provided plenty of time for learners to practice the concept using drawn or virtual images.
8) ***Step 8:*** Checked for student understanding. Do not move to the abstract phase if learners haven't mastered the representational level.
9) ***Step 9:*** Taught learners the maths circle theorem concepts using only numbers and symbols (abstract level). Modelled the concept.
10) ***Step 10***: Provided plenty of opportunities for learners to practice using only numbers and symbols.
11) ***Step 11***: Provided opportunities for practice and review. As learners learned the strategies, it was important that they had opportunities to practice discriminating between when to use and when not to use this strategy (i.e., conditional knowledge). Teachers had the opportunity to provide feedback and determine mastery of previously taught strategies. This was to check for student understanding. If learners are struggling, go back to the concrete and representational levels
12) ***Step 12:*** Once the concept is mastered at the abstract level, periodically bring back circle theorems concepts for learners to practice and keep their skills fresh.
13) ***Step 13:*** Monitor progress weekly with monitoring tool to probe.

Delivery format of intervention. The intervention was designed to last for approximately Three weeks (15 approximately 80-minutes lessons) and each mathematics concept was taught in at least three phases. It was noted by the researcher and the experimental teacher that the concrete lessons took more time to deliver than the representational and abstract lessons. This may be due to the fact that concrete lessons introduced new topics or that passing out the materials and physically manipulating the objects required more time to complete.

Materials

The study included multiple materials suitable for application during the CRA stages. In the concrete stage, geometric strips, rubber bands of different colors and lengths and pins were perforated around the circle and at the centre. Rubber bands were used to make some angles following the pins.

CRA Intervention

The CRA instruction strategy implemented in the study consisted of three stages: (1) The concrete stage where movable and tactile materials are used, (2) The representational stage where the lines in which the materials used are represented, and (3) The abstract stage where the numbers, symbols and mathematical expressions are used.

The sessions at all stages of the concrete-semi-concrete-abstract instruction strategy implemented with the direct instruction method were implemented by performing the steps of being a model, guided practices and independent practices.

The intervention sessions in this study were prepared by merging these three levels in one session. Learners were taught the circle theorems using the concrete level, drawing the geometric shape at the representational level and finally the strategy of remembering numbers at abstract level by converting the drawing into an equation to calculate the missing angles. A detailed presentation of the method of intervention through CRA was followed as outlined below.

Stage One: Concrete

This was the “work” stage where learners physically manipulated objects to solve mathematical problems through the use of 3D artifacts to help them learn new concepts. With this in mind, learners were able to move and manipulate 3D objects to represent their thinking. For example, the learners used rubber bands to form triangles with different sides, or equilaterals, and represented the length of each side using numbers. In addition, the use of manual methods increased the number of sensory inputs that the learners used while learning a new concept which improved the student’s chance of remembering the procedural steps necessary to solve a certain problem, for example, the colors and lengths of geometric rubber bands.

Stage Two: Representational

This was the “vision” stage and involves using pictures to solve mathematical problems. This stage required learners to do a simple drawing of the concrete objects they used in the first stage. So, learners’ mastery of the first stage is a prerequisite for moving to the second stage. At this stage, the teacher clarified the relationship between drawing and concrete objects and provided many training examples for learners to get them to work independently. For example, drawing triangle and straight segment. . Also retraining in visualizing the concrete stage with a simple drawing helped learners understand the mathematical skills/concepts.

Stage Three: Abstract

This stage also called “symbolic”, began after the student demonstrated a thorough understanding of the representational level, and involved using only numbers and symbols to solve arithmetic problems. Learners no longer relied on manual methods or graphics to solve problems. At this stage, students only used mathematical strategies to solve problems. For example, speak in their language how to find angles in a circle. The problem was also solved using abstract symbolic notation, which involved memorizing mathematical procedures and continued until the learners learnt the procedure and concept automatically. The three stages are shown in Table 1 below.

Table 1- CRA Stages

Step	Procedures
Concrete “Work stage”	▪ Modelled the desired shape using circle geometric bands and pins. ▪ Put the number that represented the angle in the circle. ▪ Keep tight the bands to make a straight segment. ▪ Measure the angles using a protractor.
Representational “Vision stage”	▪ Draw the desired shape. ▪ Write the length of each side of the desired shape on the drawing. ▪ Draw a line segment and made some angles. ▪ For a triangle, where straight lines meet, angles to be created ▪ Write all numbers that represented the angles on the triangles.
Abstract “Symbolic stage”	▪ Express using their own language how to find the desired angle. ▪ Represent the angles of the desired shape with an equation. ▪ Calculate the missing angle of the desired shape using numbers only.

Organisations of lessons. During each lesson, the teacher progressed through an explicit sequence of steps, which required to (a) introduce the material, (b) model the desired behaviour, (c) guide learners through practice, and (d) monitor learners ’ independent practice. The instructional steps within each lesson allowed the learners opportunities to practice the computation of operations involving circle theorems and receive corrective feedback as necessary. This was achieved based on a number of experimental assumptions.

Organization of lesson series. Lessons progressed in a sequential fashion, building up to more difficult concepts from the prerequisite concepts. During the first phase of the instruction the teachers and learners used concrete manipulatives to physically practice the mechanics of the construct. The second phase acted as a transition from concrete to abstract, offering instruction and practice at a semi-concrete, representational level. The pictures mirrored the concrete manipulatives used in the second phase of the lesson series, aiding learners understanding as a scaffold step between the concrete instruction and abstract instruction. The last instructional phase was the abstract phase. In the last phase of the intervention, learners were introduced to the computation of circle theorems at the abstract level, where numbers and symbols were used to represent computations involving circle theorems. Below in Table 2 is an example of CRA sequence.

Table 2: An example of CRA sequence

Steps	procedures	Theorem
Concrete 'Work stage'	▪ Model a desired shape using geometric tapes and clips. ▪ Put the pins along the circumference of the circle. ▪ Start making the angles inside the circle using the rubber bands. ▪ Measure each angle made in the circle by the rubber bang using a Protractor ▪ Measure angle X and then measure angle Y ▪ You discover that $< X = 2 < Y$	The angle subtended by an arc at the Centre is double the angle subtended by the same arc at the circumference. **PROOF** Given that X is the angle at the centre and Y is the angle at the Circumference of the circle from the same arc. ▪ You discover that $< X = 2 < Y$ ▪ So many angle theorems can be proved by using this method and pupils' knowledge and skills can be developed if such a sequence is used.
Representational 'Vision stage'	▪ Draw the desired shape on the chart or Manila paper. ▪ Write the angles in the circle as a representation from the concrete stage	
Abstract 'Symbolic stage'	▪ Represent the theorem with an equation ▪ Calculate the required missing angle using numbers only	

Making the Model for Concrete-Representational- Abstract Sequence of Instruction Approach for Teaching Circle Geometry.

Materials needed; various manipulatives could be used to teach skill/concept during the concrete phase. In this study, wood, cut card box, office pins, and rubber bands were selected.

Method of making;

- Place on a smooth plank of length 40cm by 35cm a card box.
- Draw and trace a circle out on the card box, can use a plate.
- Fix each of the pins on the circumference of the circle (use different colours of pins to make it look more attractive.
- Find a centre of a circle, use a bisection method by using a rubber bang.
-

Appearance of the instructional material for teaching circle geometry made will look as follows;

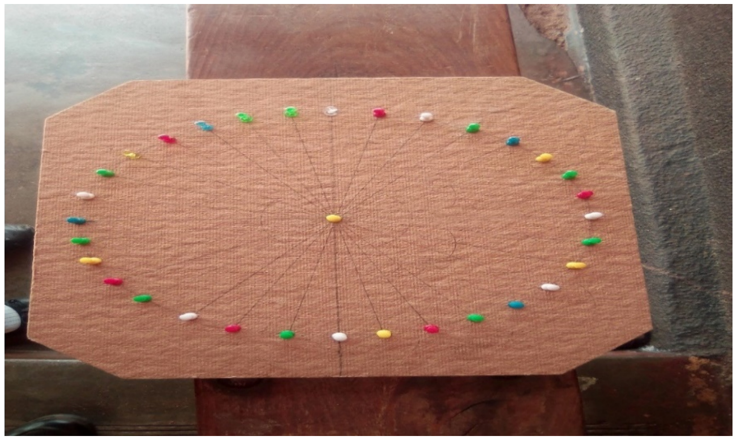


Figure 1: Constructing A circle

Proof; 1: concretely (work stage).

Theorem: *The angle at the centre of a circle is twice the angle on the circumference of the circle when the points all lie on the same arc;*

Method:

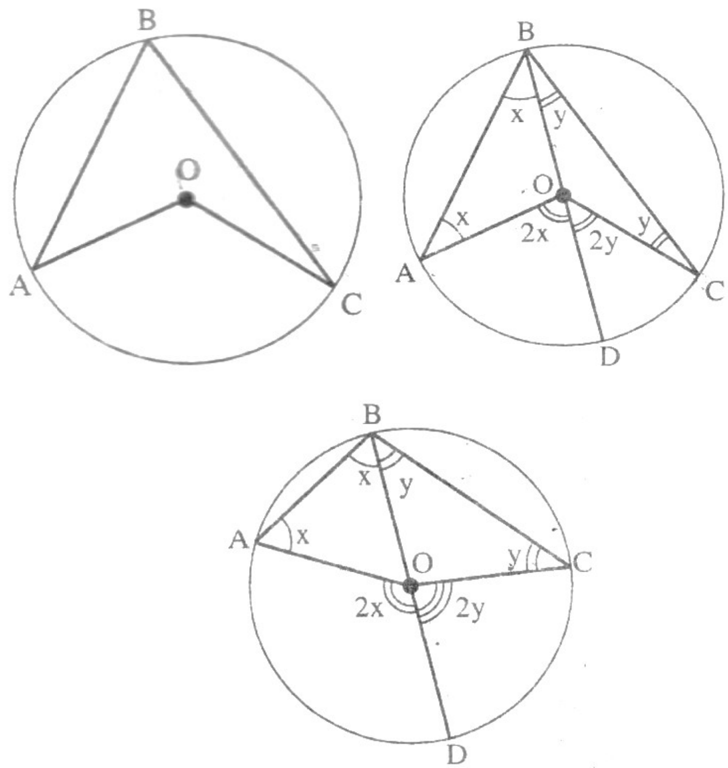
- **O** is the centre of the circle and angle AOC at the centre of the circle stands on the same arc as angle ABC at the circumstance.
- The relationship between $<$ AOC and $<$ ABC can be measured using a protractor.

-After measurement is done, the relationship is that $<$ AOC= 2 $<$ ABC

Representational (vision stage)

- Draw the desired shape of a circle
- Draw two lines from the different points on the circumference of a circle and meet them at the centre of a circle.
- From the same points on the same arc draw two lines again to meet on the circumference of the circle.

Figure 1 is outcome of the vision stage.



Conclusion; < AOC = 2 < ABC.

Figure 2: Construct from CRA application

Abstract (symbolic stage).

Proof;
Join B to O and extend the line to D
Let < ABO = x and < OBC = y, as shown in figure above.
∆ AOB is Isosceles, with < ABO = < BOA.
< AOD = < BAO + < ABO (The exterior angle of
∆ ABO = sum of all the opposite internal / angles)
: < COD = y +y
= 2 y
< AOC = 2 x +2 y
= 2 (x +y)
= 2 x < ABC
: < AOC = 2 < ABC proved abstractly.

Several such proofs can be made using this approach of concrete – Representation- Abstract

The researcher observed 80% of the lessons to ensure teachers followed scripted lessons for the CRA instruction as well as measures to document and operationally define traditional instruction. The researcher sat in the back of the classroom with a laptop and recorded observational notes during the lessons. However, it should be noted that the researcher conducted all fidelity checks for both the treatment and control groups.

Procedure for the Traditional Group

The researcher requested the second teacher to handle his class in circle theorems at a time he felt it was convenient and to notify the researcher when the teaching was done.

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Instrument to Measure Performance

When the teaching was all done, it was then followed by an end of teaching evaluation. The first evaluation the Post-test- was done soon after completing the syllabus on circle theorems and another test, delayed post-test was administered after four weeks. In order to measure the level of learners' performance in circle theorems, the researcher prepared a test. The test was based on mathematics curricular and the diagnostic tests approved by the Heads of Departments of Mathematics. The researcher and the teachers designed the two test questions in order to confer reliability. Test 1 Post-test and Test 2 delayed Post-test appear in the appendices.

The test consisted of a set of skills such as (calculating the Circumference of a circle, missing angles in the circle). To verify the validity of the test content, it was reviewed by four teachers of Mathematics and Heads of Departments in the two schools. Among the four teachers, two of them were part of the teaching and two were independent. Based on the observations, the items that were agreed upon by the team were retained, i.e. with an agreement rate set at above (84%). Thus, the final version of the test consisted of ten (10) questions: The reliability of the test was verified using Coder Richardson 21. The reliability coefficient on the overall test score was (89%).

Measurement of Outcomes

Performance was measured as the grades in percentage points obtained following an assessment. This performance was then categorised as shown in Table 3.

Table 3: ECZ Performance Table

Marks Interval	Division	Level
75-100	One	Distinction 1
70-74	Two	Distinction 2
65-69	Three	Merit 1
60-64	Four	Merit 2
55-59	Five	Credit 1
50-54	Six	Credit 2
40-49	Seven	Satisfactory 1
35-39	Eight	Satisfactory 2
0-34	Nine	Fail

This grading system was then categorised for purposes of easy description. The repetitious scores in terms of level were adopted and collapsed to have one representative level as follows:

Table 4: Modified Performance Table

Marks Interval	Level
75-100	Distinction
60-74	Distinction
50-59	Credit
35-49	Satisfactory
0-34	Fail

Outcome Evaluation of Performance

Primary and secondary outcomes were assessed on the individual participant level four weeks apart. The primary outcome was the performance after learning (post test score %) and remembrance (Delayed-Post-Test Score %) after 4 weeks.

Data Analysis

Descriptive statistics on the study participants was presented as proportions for categorical variables and as mean values with corresponding standard deviation (SD) for continuous variables. When comparing the two groups, differences between proportions were assessed by chi-square test and differences between means with t-test or

ANOVA as the case was. The conventional significance level of 0.05 was used in all analyses in order to reject the null hypothesis of no difference between groups. The researcher used the difference-in-differences methods to assess the impact of the interventions on pupil performance.

Findings
General Characteristics of Study Participants

The results of the findings of this chapter in which the researcher wants to present came from appendix one (1).

These findings are from a sample of Grade 11 pupils from two secondary schools whose ages were rather uniform ranging from16 to 18 years. Just over half n = 47 (52.2%) in the sample were males and less than half n = 43 (47.8%) were females (Figure 3).

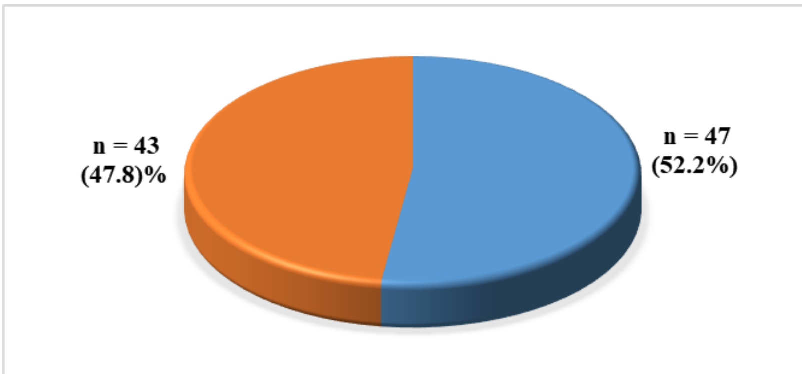

Figure 3: Distribution of respondents by gender

Performance Distribution of Control Group for Post Test and delayed Test

This section covers the performance outcome after using Concrete – Representational – Abstract (CRA) Instructional Approach in the teaching of circle theorems in Mathematics and shows the performances of both post-test and delayed post- test for a control Group. A performance distribution of the control group looking at the post-test and delayed post-test shows an unequal distribution of performance within the group. However, the distributions were non-normal as shown by the skewness values that are lower than 3 (See table 5). A statistical comparison of individuals within the control group was then performed. The within group assessment of differences was considered to be important in order to examine if the traditional method alone was effective. The researcher compares the test results after the intervention – traditional instruction and again four weeks after traditional approach, this traditional intervention compares the two test results. In this case, the researcher is not looking at the differences between two groups, but rather the differences between the same group taken at two time points' t_1 and t_2. The mean scores were rather close between the post-test (52.13%) and delayed post-tests (52.47%).

Table 5: Performance Distribution- Control Group for Post Test and delayed Test

Statistic	Post-test- Traditional Instructional Approach	Delayed Post Test- Traditional Instructional Approach
Mean	52.13	52.47
Median	44.00	52.00
Std. Deviation	17.30	18.30
Minimum	28.00	20.00
Maximum	96.00	88.00

While the mean scores were rather close, the median scores showed a marked variation between the post-test (44) and the delayed post test scores (52). A pictorial presentation as shown in the box plot figure 5.2 below. It is evident that the remembrance scores following four weeks of the delayed post assessment were rather poor in the traditional approach.

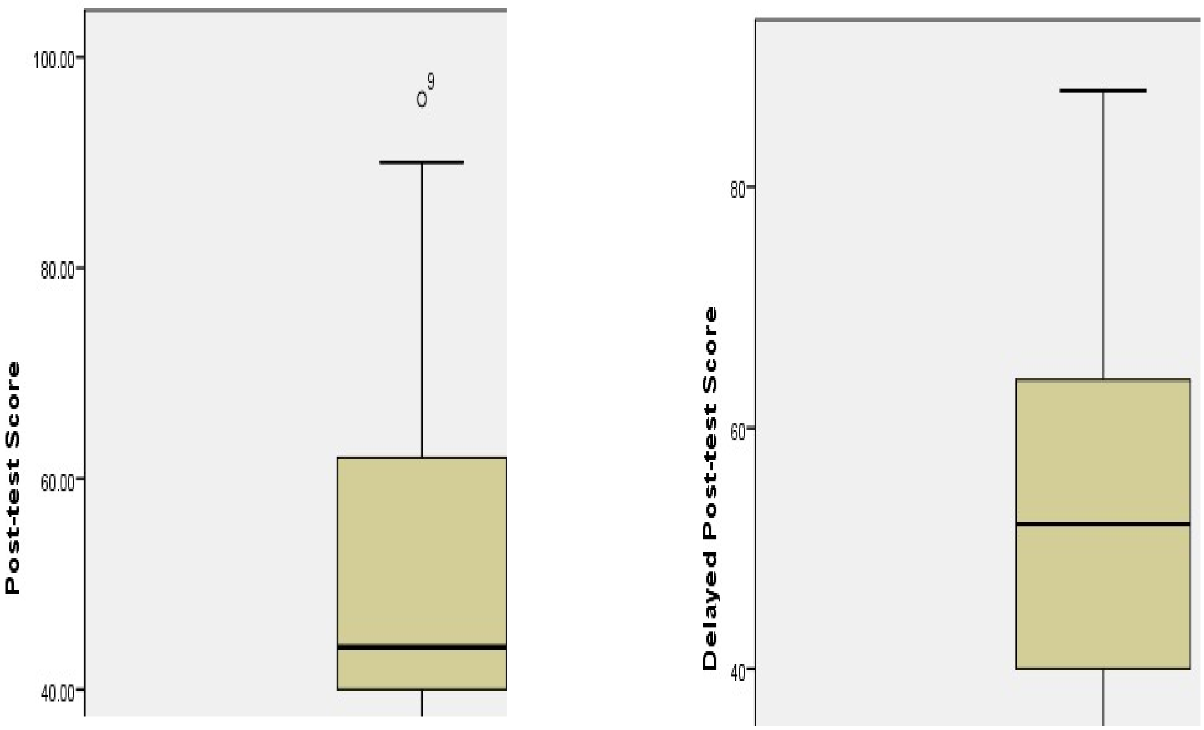

Figure 4 Median scores Performance Distribution- Control Group for Post Test and delayed post test

The researcher assessed the distribution of grades with the control group and the findings show a mixed response. Though there was a rise in those who failed, learners who were in the satisfactory group in the post-test moved into the merit and credit grades after four weeks as shown by the delayed post-test grades.

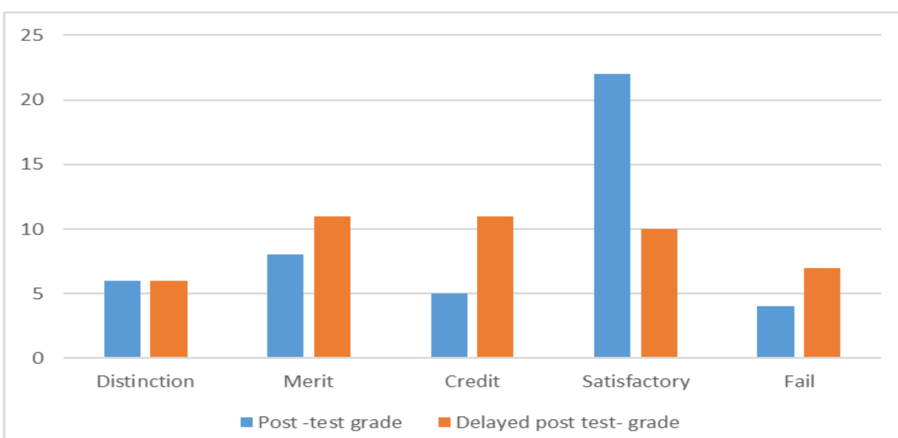

Figure 5 Grade Performance Distribution- Control Group for Post Test and Delayed Post-Test

The descriptive statistics in this section are suggestive of the existence of a difference between the post-test and delayed post-test in the group, a statistical comparison of individuals within this given group. This was one way of determining the effectiveness of the traditional intervention. Within group differences were considered by the researcher to be important. This called for the researcher to assess whether a difference existed just after the intervention of the program and again at the end of four weeks and compare the two at two time points t_1 and t_2. One sample t tests below provide the data that the means are different. The traditional Instructional Approach's mean for the Post-test Score was 52.13 and for the Delayed Post Test- Approach was 52.47 for the control group as shown in Table 6. Table 7 shows that the score was statistically significantly higher than a normal of 0 (the test level) t (44) = 20.20, p = .001.

Table 6: One-Sample Statistics -Control Group for Post Test and delayed Test

	N	Mean	Std. Deviation	Std. Error Mean
Group	45	2.00	.000[a]	.000
Post-test Score	45	52.13	17.3	2.57
Delayed Post-test Score	45	52.47	18.300	2.72

Table 7: One-Sample Test

	Test Value = 0					
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
Post-test Score	20.207	44	.000	52.13	46.93	57.33
Delayed Post-test Score	19.233	44	.000	52.47	46.97	57.96

Performance Distribution of the Experimental Group for Post Test and Delayed Test

This section covers the performance outcome after using Traditional approach (TA) in the teaching of circle theorems. The section shows the performances of both post-test and delayed post- test of an experimental Group. A performance distribution of the experimental group looking at the post test and delayed post-test shows an unequal distribution of performance within the group (see Table 8 and Figure 6).

Table 8: Performance Distribution Experimental Group for Post Test and Delayed Test

Statistic	Post-test- Abstract (CRA) Instructional Approach	Delayed Post Test- Abstract (CRA) Instructional Approach
Mean	83.87	85.58
Median	82.00	85.00
Std. Deviation	12.43	8.88
Minimum	53.00	58

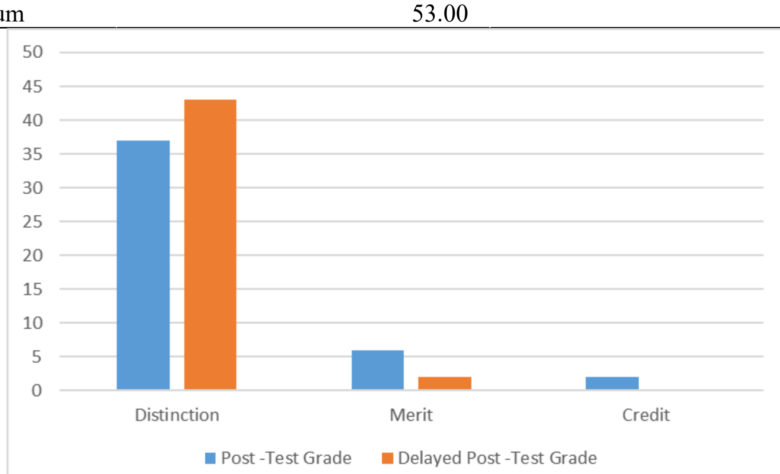

Figure 6 Grade Performance Distribution in the Experimental Group for Post Test and delayed Tests

The descriptive statistics in this section are suggestive of the existence of a difference between the post-test and delayed post-test in the group, a statistical comparison of individuals within this given group. This was one way of determining the effectiveness of Concrete- Representational- Abstract (CRA) Instructional Approach intervention. Within group differences were considered by the researcher to be important. This called for the researcher to assess whether a difference existed just after the intervention of the program and again at the end of four weeks and compare the two at two time points t_i and t_2. One sample t tests below provide the data that the means are different. The Concrete- Representational- Abstract (CRA) Instructional Approach's mean for the Post-test Score was 83.8 and for the Delayed Post Test- Approach was 85.58 for the experimental group as shown in Table 9.

Table 9: One-Sample Statistics Experimental Group for Post Test and delayed Test

	N	Mean	Std. Deviation	Std. Error Mean
Group	45	1.00	.000a	.000
Post-test Score	45	83.86	12.42	1.85
Delayed Post-test Score	45	85.58	8.88	1.324

Table 10 shows that the score was statistically significantly higher than a normal of 0 (the test level) t (44) = 45.26, p = .001.

Table 10: Experimental group One-Sample Test

	Test Value = 0					
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
Post-test Score	45.26	44	.000	83.86	80.13	87.60
Delayed Post-test Score	64.63	44	.000	85.57	82.91	88.25

Between Group Comparisons

This section looks at differences in performance in circle theorems of Grade 11 pupils when taught using Concrete- Representational- Abstract (CRA) Instructional approach and comparing it to its counterpart method of Traditional approach. The section covers the performances of both post-test and delayed post- test of a control and experimental Groups.

Between Groups differences examine how independent groups – groups that are not the same and in this case the control and experimental groups – may differ from each other on performance. Between Groups difference tests were done as they are useful for examining the efficacy of interventions or treatments. Each group is considered a single entity, and between-group comparisons were computed simultaneously. Since the data was not normally distributed, in order to determine the difference between the two groups, Mann-Whitney U test was employed.

Post-test Group Comparisons between Control and Experimental Groups

In Table 11, are Post test scores of the two groups and these show the actual significance value of the Mann-Whitney U test. The Test Statistics table provides the test statistic, U statistic, as well as the asymptotic significance (2-tailed) p-value. The table concludes that the medians of the post test score are independent – and are statistically significantly higher that is 83.8 (for the experimental Group-(CRA) Instructional Approach (see Table 5.3) compared to 52.13 (See Table 5.6) for verification and this is shown as $U = 110$ and $p = .024$. We thus reject the null hypothesis which claimed that there is no difference between the post test scores in the two groups arising from the two interventions. There are indeed marked differences with experimental Group-(CRA) results higher than those of control group - (TA) in the post test scores.

Table 11: Post test scores Test Statistics

	Post test scores
Mann-Whitney	110.000
Wilcoxon W	320.000
Z	-2.446
Asymp.Sig. (2-tailed)	.024.
Exact Sig. [2*(1-tailed Sig.)]	.024[a]

a. Not corrected for ties
b. Grouping Variable: Group - Experimental Group-(CRA)Instructional Approach and Control Group-(Traditional)Approach

Delayed Post-test Group Comparisons between Control and Experimental Group

In Table 12, displayed are Delayed Post test scores of the two groups and these show the actual significance value of the Mann-Whitney U test. The table concludes that the medians of the delayed post test score are independent – and are statistically significantly higher that is 85.5 (for the experimental Group-(CRA) Instructional Approach compared to 52.47 and for verification, this is shown as $U = 221$ and $p = .001$ in Table 12below. We thus reject the null hypothesis which claimed that there is no difference between the post test scores in the two groups arising from the two interventions. There are indeed marked differences with experimental Group-(CRA) results higher than those of control group -(TA) in the delayed post test scores.

Table 12: Delayed Post test scores Test Statistics

	Post test scores
Mann-Whitney	221.000
Wilcoxon W	465.000
Z	-1.722
Asymp.Sig. (2-tailed)	.001.
Exact Sig. [2*(1-tailed Sig.)]	.001[a]

a. Not corrected for ties
b. Grouping Variable: Group - Experimental Group-(CRA)Instructional Approach and Control Group-(Traditional)Approach

Summary of Mean improvement by groups

As indicated above a repeated measures ANOVA summarised below was performed to answer the question "*What difference is there in the performance in circle theorems of Grade 11 pupils taught using Concrete-Representational- Abstract (CRA) Instructional Approach compared to its counterpart method of Traditional approach?*" With this question, one- way ANOVA was performed (see (see Tables 13a and 13b) to determine if there were no significant differences between CRA treatment and control groups.

Table 13a ANOVA (Descriptive)

		Mean	Std deviation	Std Error	95% Confidence interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Post- test (%)	Experimental (CRA)	83.87	12.429	1.853	80.13	87.60	53	100
	Control (Traditional)	52.13	17.307	2.580	46.93	57.33	28	96
	Total	68.00	21.887	2.307	63.42	72.58	28	100
Delayed Post- test (%)	Experimental (CRA)	85.58	8.882	1.324	82.91	88.25	58	100
	Control (Traditional)	52.47	18.300	2.728	46.97	57.96	20	88
	Total	69.02	21.948	2.314	64.43	73.62	20	100

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Table 13b: Results of One- Way ANOVA

	Sum of Squares	df	Mean Squares	F	sig
Between Groups	22657.600	1	22657.600	99.811	
Post- test (%)	19976.400	88	227.005		.000
Within Groups	42634.000	89			
Total					
Between Groups	24667.778	1	24667.778	119.232	
Delayed Post- test (%Within	18206.178	88	206.888		.000
Groups **Total**	42873.956	89			

To validate the results, the researcher performed a one- way Analysis of Variance (ANOVA) using the SPSS version 23.0. One- way ANOVA test was applied to observe whether there was a significant difference in performance between the two groups (a group taught using Concrete – Representational – Abstract (CRA) Instructional Approach and the other one taught using traditional approach). Results presented in tables (13a and 13b) showed a significant main effect for instruction based on simple comparisons from the post- test and the delayed post- test assessment. According to the results of the analysis, there was a significant difference in scores between the pupils who were taught circle geometry using Concrete – Representational – Abstract (CRA) Instructional Approach and the pupils who were taught the same topic using the traditional approach for both post- test and delayed post- test (for post- test: F= 99.811; p= .000 < 0.05, for delayed post- test: F= 119.232, p= .000<0.05) according to means.

In trying to investigate further on the effect of Concrete- Representational- Abstract Instructional Approach on Grade 11 pupils' performance in circle theorems and the difference in performance of pupils taught using Concrete – Representational – Abstract (CRA) Instructional against the counterpart method of Traditional approach, the researcher also performed the Chi Square analysis on the following hypotheses:

H_0=There is no significant mean difference in the performance in post-test and delayed post-test between the experimental group taught with Concrete – Representational – Abstract (CRA) Instructional Approach and the control group taught with traditional method.

A performance of the two groups (treatment and control) was done using the Chi- Square analysis for post- test and delayed post- test. According to the results of the analysis, there was a significant difference in scores between the pupils who were taught Circle Geometry using Concrete – Representational – Abstract (CRA) Instructional Approach and the pupils who were taught the same topic using the traditional approach for both post- test and delayed post- test (for post- test: Value= 65.667; sig= .004 < 0.05, for delayed post- test: Value = 78.333, sig= .000<0.05) according to means. Therefore, we fail to reject and conclude that there is significant difference in performance between the group which was taught circle geometry using CRA instruction approach and the group which was taught using traditional approach (see tables 14a and 14b).

Table 14a: Post- test Analysis Using the Chi Square

	Value	df	Asymptotic significance (2- sided)
Pearson chi- Square	65.667	38	.004
Likelihood Ratio	89.541	38	.000
Linear- by- Linear Association	47.299	1	.000
No. Valid Cases	90		

Table 14b: delayed Post- test Analysis Using the Chi Square

	Value	df	Asymptotic significance (2- sided)
Pearson chi- Square	78.333	39	.000
Likelihood Ratio	106.682	39	.000
Linear- by- Linear Association	51.207	1	.000
No. Valid Cases	90		

Discussion

The study aimed at determining the causal difference in performance in circle theorems when concrete-representational- abstract sequence of instruction as a conventional pedagogical method of instruction among Grade 11 pupils and compared it to the traditional method of instruction. Findings revealed post-test changes in performance with a better performance of learners who were subjected to concrete-representational- abstract sequence of instruction when compared to the traditional approach. As shown by Mann-Whitney U test, the medians of the post test score were independent – and statistically significantly higher with values being 83.8 for the experimental group when concrete-representational-abstract instructional approach roach was employed compared to the value 52.13 for the control group and U was 110 and p was .024 when the conventional approach was employed. In addition, Mann-Whitney U test for the delayed post test score were independent – and were statistically significantly higher with values being 85.5 for the experimental group compared to 52.47 and U was 221 and p was 0.001.

The variation in performance between the two groups can only be attributed to concrete-representational-abstract sequence of instruction as a conventional pedagogical method used in the intervention process in this study. The approaches focus on building conceptual understanding and then procedural knowledge. We can attest as previous research has done that conceptual and procedural knowledge is useful in developing understanding to solve geometric problems and that conceptual knowledge is important and strengthens procedural knowledge (Rittle - Johnson et al., 2001). Both are necessary components for mathematical competence as shown in previous quasi experimental designs (Milton et al., 2019[20]; Rittle - Johnson et al., 2001[21]). We would vouch for teachers to use much more often concrete-representational- abstract sequence of instruction as a conventional pedagogical method. This call is buttressed by previous research (Hinton & Flores, 2019; Flores et al., 2020[22]; Morano et al., 2020[23]; Zhang et al., 2021[24]). A closer examination of both the post-test and delayed post test scores of the traditional method suggests that learners had less ability to employ circle theorems when constructing geometric shapes. Some of the possible reasons that might have accounted for the low scores in the control group could be that learners were not exposed to the concept of discussion, group work, problem- solving hands-on-activities and guided discovery. The limited conceptual instruction provided very little opportunity for learners in the control group to understand mathematical concepts and techniques involving meaningful definitions as well as helping them know the reasons behind executing every step of the procedure.

Study Limitations

We take cognisance of the limitations of this design that the sample was not representative of schools in Katete and generalisation is not possible. In case our readers have concerns that randomisation was not done, there are replete studies that have employed quasi experimental designs to answer causal comparative questions, though not necessarily as an "experiment" with an intervention. Obviously, it was not feasible to randomize learners and teachers. We attempted to control biases by ensuring that the two groups comparable on important sociodemographic and learning environment and we did not find during the design stage the two groups differing on, for example, age, availability of learner support like libraries and family income.

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Contributions to Research Practice, Research Implications

This is a novel study in this part of the world. As such, the study adds new knowledge to the existing pool of knowledge in pedagogy. In terms of teaching geometry, this study stresses the importance of employing.

Conclusion

The aim of the current study was to determine the causal difference in performance in circle theorems when concrete-representational- abstract sequence of instruction as a conventional pedagogical method of instruction among Grade 11 pupils and compared it to the traditional method of instruction. The intervention was successful as a way to demonstrate possible improvement in performance in geometry especially circle theorem. This success was achieved by the teacher who employed concrete-representational-abstract instructional approach as a method of instruction through a short intervention period that included accessible materials and required no professional development. The findings of the current study are also important in that they demonstrate the efficacy of combining the three stages of concrete-representational-abstract instructional approach. Therefore, this could be a promising intervention for teaching geometry and future research is needed to apply this approach in other areas within Mathematics.

Recommendations

In reference to the foregoing conclusion, we render the following recommendations:

1) This study has provided an instruction model for the application of concrete-representational- abstract sequence of instruction that can prove successful. It is crucial that Heads of Departments develop creative scheduling that allows teachers to provide additional instruction in Mathematics to struggling students. In this study, intervention students continued to attend to their regular instruction in Mathematics and received their tiered instruction.

2) School managers should foster the use of instructional strategies and techniques where mathematics teachers continuously propose instructional strategies and techniques that would be effective in helping more learners learn and develop their skills and increase achievement on Mathematical analysis.

3) The researcher proposes the need for writing modules in Mathematics in schools. Heads of Departments should superintend the writing of modules that cover areas in Mathematics that would require the application of concrete-representational- abstract sequence of instruction.

4) The findings may be used to modify instruction of particular topics in Mathematics using the characteristics of concrete-representational- abstract sequence of instruction approach. For future researchers, the same strategy may be tested involving an item analysis in the pre and post-test results to design more valid measures of determining the usefulness of concrete-representational- abstract sequence of instruction and eventually devise approaches that would further broaden the mastered skills with the approach.

5) In order to avoid bias, future researchers may involve extending the study to more schools.

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