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Analyzing of Middle School Students' Perceptions of Concepts and Mathematical Language Skills Related to Dimension

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Abstract

This study aims to investigate secondary school students' perceptions of dimension-related concepts and mathematical language skills. The sample of the study consists of students attending seventh and eighth grade of a middle school in terms of socio-economic and socio-cultural conditions in Kocasinan district of Kayseri. In the study, which utilizes a mixed-method approach, data were collected using tests developed by the researcher with expert opinion support to measure students' mathematical language skills in the dimension domain and to determine their perceptions of related concepts. The data were collected and statistically analyzed for evaluation. According to the findings, there is a moderate, positive, and significant relationship between middle school students' perceptions of concepts related to dimension and their total scores on the mathematical language skills tests. Mathematical language skill scores were found to be close to the mean, and there was no statistically significant difference in mathematical language skill scores based on gender and grade level. Students had difficulty in making sense of the dimension and defining with words geometric concepts related to the subject, and generally resorted to explanation with figures. However, it was observed that their knowledge level was not sufficient to express the concepts.

Keywords: concept, dimension, mathematical language skills, secondary school student.

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1. Introduction

Mathematics is a universal language that improves people's ability to think logically, systematically, analytically, and critically. Mathematics has unique expressions, symbols and terms. Mathematical language is used to communicate and understand the rules and scientific ideas, including mathematical concepts, operations and symbols (Çalıkoğlu Bali, 2003). For effective communication and productivity in mathematics education, teachers and students should be able to understand each other's language and use it correctly. It is necessary to know the symbols and what they represent, to choose the appropriate symbols for the given situations, and to use them in a way that is consistent for everyone to understand mathematics (Boulet, 2007; cited in Yalvaç, 2019). Even a concept conveyed using the correct language does not create the same schema in the minds of teachers and students; thus, concepts conveyed incorrectly can lead to misinterpretations and cause misconceptions.

While learning mathematical language, students acquire language skills and mathematical concepts simultaneously (Ministry of National Education [MoNE], 2009). Mathematical definitions are one of the main factors that constitute the structure of mathematics. Definitions are essential in the characteristic axiomatic structure of mathematics (Edwards & Ward, 2008). Çakıroğlu (2013) stated that when making a mathematical definition, it is not enough to list the properties of the concept; selecting the necessary and sufficient properties that can specify the concept by making logical inferences among them is essential. Regarding mathematical language, definitions are important in expressing concepts' meanings and providing content integrity. A concept is a form or structure of knowledge that finds meaning in the human mind and reflects the common properties of different objects and phenomena (Ülgen, 2004). Knowledge of the concepts of a subject and the relationships between these concepts form the basis of the subjects to be learned or taught. For this reason, a problem in understanding basic concepts can affect other related subjects and cause problems. Therefore, it is necessary to emphasize concept teaching and use appropriate expressions and precise mathematical language to define, explain, and enable students to understand many abstract concepts related to the domain. The correct and active use of mathematical language is believed to be effective in the conceptual understanding of mathematics and in developing students' thinking skills (Toptaş, 2015).

Geometry is one of the fields in which concept teaching and mathematical language play an active role.

Geometry includes definite and indefinite concepts, axioms, and theorems (Doğan, 2015; Küçük & Demir, 2009). One of the approaches used in geometry education is to introduce concepts by moving from parts to wholes, that is, from points to objects (Kılıç et al., 2015). This approach creates defined geometry concepts using undefined basic concepts such as 'point', 'line', 'plane', and 'space' (Doğan, 2015). One topic that forms the basis of these defined and undefined concepts is dimension. Dimension is a fundamental concept that forms the idea of length, area and volume in the geometric sense (Doğan, 2015). In the mathematical context, dimension is defined in the TDK dictionary as "each of the three directions of length, width and depth that can be measured in lines, surfaces or objects." It is a concept we frequently encounter daily and use in different settings (TDK, 2024). Studies on the concept of dimension begin with Euclid's (300 BC) definition of boundary notation (Manin, 2006). In Euclidean geometry, an object is defined regarding its length, width, or height; it is expressed as one-dimensional if it has only length; two-dimensional if it has length and height; three-dimensional if it has length, height and depth (Ural, 2011). Similarly, in Plato's Republic - Book 7: "After the plane surfaces ... second dimension, the third dimension comes next ... the cube and everything has a depth" (Mandelbrot, 1983; as cited in Peker & Karakuş, 2015, p.186). Neither Euclid nor Plato gave a direct definition for the dimensions of geometric objects, adopting a more intuitive understanding (Peker & Karakuş, 2015). This approach defines the dimension as length, height, and width (Skordoulis et al., 2009).

Alexandroff (1932; 1961) and Pears (1975) state that the in-depth study of the concept of dimension by mathematicians began in 1905 when Poincare introduced topology. Until then, an object's dimension was considered an empirical way of extending it in different directions (Skordoulis et al., 2009). With the concept of topology, different perspectives and definitions of the concept of dimension have emerged. A systematic approach to the concept of dimension was realized by Freudenthal (2002). Freudenthal suggested that the concept of dimension can be considered from three perspectives: Euclidean geometry, analytic geometry, and topology. In addition to Freudenthal's dimension classification, Fractal dimensions can also be considered under Topological dimensions.

In the study, the focus was on the Euclidean dimension, which is examined in the most basic sense at the secondary school level, taking into account age and cognitive skills.Parallel to the dimension definitions in Euclidean geometry, Freudenthal (2002) defines line, surface, and space in the Cartesian system as below;

- A line is a moving point.

- A surface is a moving line.

- Space consists of a moving surface.

Here, Freudenthal indicates the direction with the concept of movement. Thus, line, surface, and space can be defined by adding the movement parameter (direction) to a new point.

Considering the dimensional definitions of Euclid, Plato, and Freudenthal, the following conclusions can be drawn.

- There is the idea that only lines are one dimensional. However, a line is also a curve, and curves are one dimensional. In other words, when we say one dimensional, we should not think of lines alone.
- Again, these definitions lead to the idea that planes and surfaces of solids are two dimensional. Although the terms face and surface are often used interchangeably in solids, it should be noted that this is not always correct. A face is the surface of a solid that is a plane. Although a face is also called a surface, the term face cannot always be used interchangeably with surface. For example, the base of a cone is a face, but its side surface cannot be called a face. A face is formed by the movement of a line, but a surface can be formed by the movement of a curve.
- From the definitions of dimensions by Euclid, Plato, and Freudenthal, the idea is that a space composed of faces and surfaces is three dimensional (Peker & Karakuş, 2015).

In summary, the movement of a point creates a line or curve (one dimensional), the movement of a line creates a surface or plane (two dimensional), and the movement of a face or plane creates an object (three dimensional).

Although dimension is an essential topic in mathematics education, it is not included in textbooks much (Skordoulis et al., 2009). During the period when the 2009 mathematics curriculum was taught in Turkey, this concept which we first encounter at school in the 5th- grade, is defined as the dimension of objects that can be measured in any direction (such as length, width, height, depth) in the 5th-grade mathematics textbook. It is also mentioned that the dimension concept is informally emphasized to develop the sense of space and that shapes and objects are classified according to their dimensions (MoNE, 2009). This concept, which was included in the mathematics curriculum in the same way in 2004 and 2009, was not mentioned later. Only in the 2018 secondary school mathematics curriculum, the concept of dimension was mentioned in the 7th grade gain of ‘Draws two-dimensional views of three-dimensional objects from different directions’ (MoNE, 2018). Similarly, in the newly published 2024 middle school mathematics curriculum, under the theme of seventh grade geometric quantities, the explanation of the learning outcome of being able to analyze the relationship between structures formed with identical cubes and their appearances includes the phrase, ‘Students are made to feel the two dimensional appearances of three dimensional objects’ (MoNE, 2024). Based on the latest curriculum, this subject mentioned in the seventh grade can actually be used in the pre-school period as a level for the development of intelligence, attention and talent and in measurement activity books to develop the visual spatial intelligence of students. For this reason, the fact that such a fundamental concept used in geometry and measurement learning is not included sufficiently in primary, secondary, and even high school mathematics curricula may lead to difficulties understanding the subject. Therefore, it is thought that this study will support the understanding of other geometry subjects. In addition, it is among the aims of this research to bring the concept of dimension, which is the basis of many geometric knowledge, to the agenda and to ensure that this concept is given the necessary importance in mathematics teaching programs.

Dimension-related studies that have been done in this area are given below.

Skordoulis et al. (2009) investigated whether pre-service teachers' use of the coordinate plane when determining the dimensions of a shape leads to misconceptions. As a result of their study, they found that the coordinate system affects understanding the dimensional concepts both epistemologically and didactically.

In the study conducted by Vitsas and Koleza (2000) to examine the determination of the dimensions of geometric shapes by students studying mathematics and the criteria they used in determination, it was observed that one of the preferred criteria was the coordinate system and that the students determined the dimension more accurately in the Euclidean plane compared to the Cartesian plane.

In the study by Ural (2011) measuring pre-service mathematics teachers' and master's degree students dimension determination skills, participants were asked how many dimensions the named geometric shapes or objects had and the criteria they used to determine their dimensions. According to study results, some participants made mistakes in determining the dimensions for various reasons, such as misinterpreting the coordinate system, having wrong ideas about the concepts of length and height, and using incorrect terminology in teaching geometry. The participants criteria for determining dimensions were area-volume, number of axes, width-length-height, and plane-space position.

Tat (2021) conducted a study to examine the dimension determination processes of 12th-grade students regarding their success, methods used, and errors made and to determine the role of the dimension concept in measuring length, area, and volume. Students were found to be most successful in determining the dimensions of three-dimensional objects. Although the methods used to determine dimensions varied, they generally focused on length, area, volume, emptiness or filling, and the number of axes used in the coordinate system. When these studies are examined, it is seen that the coordinate system used in determining the dimension affects the understanding of the concept of dimension. Therefore, it can be interpreted that the findings obtained from the studies support each other. In addition, it can be stated that the criteria used in determining the dimension in the studies of Ural (2011) and Tat (2021) are similar.

In the Duatepe Paksu et al. (2012) study determining conceptual images of dimension concept, pre-service elementary teachers' were given various geometric shapes. First, they were asked to choose a geometric shape with different dimensions. Secondly, they were asked to determine the dimensions of the given geometric shape by justifying them. The findings showed that pre-service teachers' knowledge of the concept of dimension was insufficient. When determining the number of dimensions, they focused on several criteria, such as the number of sides, vertices, diagonals, and visible faces.

In a study conducted by Tuluk (2014) to learn the ideas and methods used by pre-service elementary teachers in expressing their knowledge of point, line, surface, and space concepts and to interpret these representations in terms of content knowledge and content teaching knowledge, Tuluk (2014) found that pre-service teachers only showed one and two dimensional shapes on the plane while expressing the concepts, that verbal explanation of dimension was insufficient to convey the meaning of the concept, and that comprehensibility could be increased by using algebra, drawings, and dynamic geometry software. In parallel with the suggestions in this study, in the study conducted by Altıkardeş and Yiğit Koyunkaya (2020) to examine the effects of various mathematics-based technology applications (such as virtual manipulatives, Geogebra and Cabri 3D dynamic mathematics-geometry software) on high school students' perceptions of solids and dimensions, it was determined that there was a positive change in the perceptions of all students regarding the concepts of solids and dimensions. While it was seen that the students' knowledge about solids and dimensions was insufficient before the research, it was seen that all the students used mathematical expressions correctly about solids and dimensions at the end of the research. However, it can be concluded from these studies that prospective teachers and high school students' knowledge about dimensions is not at a sufficient level.

In the study conducted by Karaca (2023), it was aimed to examine the level of comprehension of two and three dimensional geometric shapes and objects of fourth-grade students within the framework of primary school geometry achievements in the context of six different dimensions designed as defining and naming, describing, comparing, classifying, visualizing and building simple structures, It was observed that students' geometric reasoning regarding two and three dimensional shapes and objects varied according to the dimensions of the study. It has been determined that students cannot demonstrate the desired performance in geometric reasoning in comprehending two and three-dimensional shapes and objects from the primary school level geometry objectives included in the curriculum. The themes that emerged regarding students' geometric reasoning in the study were generally based on reasoning by counting, identifying prototypes, labeling relevant features, and comparing known shapes. It is thought that students' incorrect geometric reasoning about two and three dimensional shapes and objects is generally due to the use of incorrect terminology, misunderstandings about the differences in the dimensions of shapes, failure to show the desired performance in combining and separating skills, use of incorrect analogies, and inability to make formal reasoning. The findings also revealed that students faced similar difficulties across the six dimensions on which the study was assessed. In a study conducted by Piaget and Inhalder (1956) to examine children's knowledge about three dimensional objects, Children between the ages of four and thirteen were given the shapes of a cube, pyramid, cylinder and cone, and then were asked to first draw whatever they saw as three dimensional objects and then to draw the shape by imagining it. Similar results were obtained from these studies, with the finding that the younger age group (primary and secondary school students) cannot comprehend three dimensional objects.

In a study conducted by Ebersbach (2009) to investigate the properties used to define three dimensions by estimating the volumes of rectangular prisms with the help of unit cubes, conducted by kindergarten students, primary school students and adults, it was observed that there was no significant difference in the degree of use of the attributes defining the three dimensions in volume estimation within the sample. On the other hand, it was determined that the participants took into consideration and used the concepts of length, width and height at equal levels.

When the literature is examined, there are many separate studies on mathematical concepts (Ata, 2013; Aydın Karaca, 2014; Başışık, 2010; Birgin & Özkan, 2012; Bozkurt & Koç, 2012; Cilavdaroğlu, 2012; Çetin, 2009; Erbay, 2016; Ergün, 2010; Georgius, 2008; Gökçe Özdemir, 2014; Gümüş, 2019; Kaplan & Hızarcı, 2005; Karahan, 2021; Kaşıkçı, 2022; Larson, 2007; Man, 2019; Nalbant, 2015; Orhan, 2022; Önder, 2019; Pickreign, 2007; Sayma, 2021; Soğancı, 2006; Süzer, 2011; Tan Şişman & Aksu, 2009; Temel & Eroğlu, 2014; Yaman et al., 2003; Yenilmez & Demirhan, 2013; Yılmaz, 2015; Zazkis & Leikin, 2008,…) and mathematical language skills (Adams et al., 2005; Akarsu, 2013; Akgün, 2009; Albayrak, 2023; Aydın & Yeşilyurt, 2007; Aydoğan Belen, 2018; Capraro & Joffrion, 2006; Çakmak, 2013; Çakmak et al., 2014; Çalıkoğlu Bali, 2002; Çalıkoğlu Bali, 2003; Doğan & Güner, 2012; Dur, 2010; Gezgin, 2023; Gray, 2004; Greenes et al., 2004; Güldal, 2022; Kula Yeşil, 2015; Mercer & Sams, 2006; Özdüvenci Yavuz, 2022; Pazarbaşı, 2015; Spiridonova & Savvinova, 2015; Ünal, 2013; Woods, 2009; Yalvaç, 2019; Yeşildere, 2007; Yıldız, 2016; Yılmaz & Güzel, 2020; Yüzerler, 2013,…), but it is seen that the studies on dimension are limited to those given above and these studies are mostly conducted with teacher candidates(Duatepe Paksu et al., 2012; Skordoulis et al., 2009; Tuluk, 2014; Ural, 2011; Vitsas & Koleza, 2000). Thereupon secondary school students were preferred as the study group. Additionally, no research has been found in which dimension, concept and mathematical language skills are studied together. Therefore, it is thought that the study will contribute to the teaching of geometry learning field by filling the gap in the literature.

1.1 Purpose of the Study

This study aims to examine secondary school students' conceptions of dimension and mathematical language skills. The problem of this study was formulated as:

"What are secondary school students' perceptions of dimensional concepts, and what is the level of their mathematical language skills?"

For this purpose, the following sub-problems were addressed:

- How do secondary school students understand mathematical terms about dimension at a conceptual level?
- What is the level of secondary school students' mathematical language usage skills about dimensions?
- Do students' mathematical language usage skills about dimension differ significantly by grade or gender?
- Is there a significant relationship between secondary school students' conceptual understanding of dimensions and their mathematical language usage skills about dimensions?

2. Methodology

2. 1. Study Model

A mixed method model that combines qualitative and quantitative methods was used in this study. The mixed method is defined as research in which data are collected and analyzed using different approaches, findings are integrated, and future predictions are made (Tashakkori & Creswell, as cited in Yıldırım & Şimşek, 2016). According to Yıldırım and Şimşek (2016), the essential feature of the mixed-methods model is that the data collected using different approaches are used to verify each other, which increases the validity and reliability of the research findings and makes them stronger. Creswell's (2003) simultaneous transformational design of the mixed method was preferred because qualitative and quantitative data were collected at the same time and analyzed separately, the priority of data types was not important, the combination was mostly made at the data interpretation stage or during data analysis if the data was transformed, and it provided a better understanding of the phenomenon under study.

2. 2. Universe-Sample

While the universe of the study consisted of seventh and eighth grade students of a secondary school in Kocasinan district of Kayseri province, the sample of the study was determined as the seventh and eighth grade students of a public school in Kocasinan district. A total of 150 students, 78 girls and 72 boys, from the 7th and 8th grades, who were selected in accordance with the purpose of the research and taking into account age and cognitive skills, were studied in the process of collecting both quantitative and qualitative data. In the study, convenience sampling method, which is one of the non-random sampling methods, was used. Christensen et al. (2015) stated that the convenience sampling method is sampling from individuals who are easy to reach, available, and willing to participate in the research. In educational research, it is generally very difficult, and sometimes impossible, to determine a random or systematic sample. In such cases, convenience sampling method may be preferred.

2. 3. Data Collection Tools

The data collection tools used in this study consist of a test for concepts and a test for mathematical language skills, prepared by the researcher after receiving expert opinion and taking into account the curriculum of the 7th and 8th grades. The test for concepts consists of three open-ended questions, and the first question asks for the definition of the concepts of dimension, one dimensional, two dimensional, three dimensional, length, width, depth, solid object, prism, line, line segment, surface, edge, lateral edge , polygon, polygonal region, circle, circle region, circumference, area and volume. In the second question of the test, the given six shapes are asked to be divided into three groups and to state how they are classified, and in the third question, the calculation of perimeter, area and volume is expected to be made by writing the dimensions of the given shapes. The first sub-problem of the research was answered by qualitatively analyzing the test regarding concepts, the answers given in the same test were scored, the qualitative test was converted to quantitative and analyzed, and the fourth sub-problem was answered by examining its relationship with mathematical language skills. The test for mathematical language skills consists of nine open-ended questions with sub-items related to measuring length, volume, posing problems, and drawing the appearances of three-dimensional objects. This test was analyzed quantitatively and the second, third and fourth sub-problems of the research were answered.

2. 4. Data Collection

In the study, firstly, open-ended tests developed by the researcher after receiving the opinions of experts consisting of a faculty member and two mathematics teachers were applied to 35 seventh and eighth grade students as a pilot study in order to measure the students' mathematical language skills on the subject of dimension and to determine their perceptions of the concepts related to the subject. The reliability of the concept test was found to be 0.820, and the reliability of the mathematical language skills test was found to be 0.683. For both tests, item discrimination and difficulty values were calculated and no changes were made to the concept test, but in the mathematical language skills test, six items were removed because they were deemed necessary and reduced reliability. In the last case, the reliability of the mathematical language skills test was calculated as 0.702. Since these values are greater than 0.70, it can be concluded that the reliability coefficient is at a sufficient level, in short, the scores obtained from these items are reliable (Akbulut, 2010). Following the pilot study, the tests were prepared for a total of 150 students, 75 of whom were in the 7th grade and 75 of whom were in the 8th grade, and were administered collectively in the classes after a certain period of time had elapsed after the subject was covered. Two class hours were given for the concept test and one class hour was given for the mathematical language skills test. Reliability coefficients of the data obtained from the applied tests are given below.

Table 1. Reliability coefficients of concept test and mathematical language skills test

Test	Cronbach's Alpha	Number of items	N
Concept	0.828	50	150
Mathematical Language Skills	0.801	20	150

As seen in Table 1, the reliability coefficients of the data obtained from the concept and mathematical language skills tests applied to 150 students were found to be 0.828 and 0.801, respectively, so it can be said that the scores obtained from the tests are reliable.

2. 5. Data Analysis

The findings obtained from the tests on concepts and mathematical language skills were analyzed and evaluated qualitatively and quantitatively.

The answers given in the qualitative part of the research were categorized and the students' perceptions of the concepts were analyzed using frequency and percentage tables and content analysis. During content analysis, the data were coded independently by two separate coders, and then the percentage of agreement between the coders was calculated using the Miles and Huberman (1994) reliability formula (Reliability = Consensus / (Consensus + Disagreement)). Accordingly, the agreement percentage between the coders was found to be 90%. This value shows that the results obtained are reliable.

In the quantitative part of the study, questions answered correctly for the test of mathematical language skills and concepts were evaluated as one point, and questions answered incorrectly or left blank were evaluated as zero points. The highest score that students can get for the mathematical language skills test is 20, and it can be interpreted that students who get a high score from this test have a high level of mathematical language skills, and the highest score that students can get for the concept test is 50, and it can be interpreted that students who get a high score from this test have a high level of perception of the concepts.

SPSS 25.0 was used for statistical operations in the quantitative analysis of the data. The mathematical language skill levels of secondary school students on the subject of dimension are shown descriptively in the table. In order to investigate whether there is a significant difference in secondary school students' mathematical language skill scores regarding dimension according to gender and grade level, firstly the skewness coefficient and Kolmogorov-Smirnov test results, which provide information about whether the data showed a normal distribution, were examined. Then, in this direction, whether the mathematical language skills of secondary school students created a significant difference according to gender and class level was evaluated with the Independent group t-test. The relationship between students' mathematical language skills and their perceptions of the concepts was examined using Pearson Correlation.

3. Findings

This section presents statistical analyses of the data obtained from the tests for dimensional concepts and mathematical language skills.

3. 1. Findings of the First Sub-Problem

In order to find an answer to the first sub-problem of the research, "How do middle school students understand mathematical terms about dimension at a conceptual level?", the students were first asked the question "Define the concepts given below" in the three-question test on concepts. The results of this question are shown below.

Table 2. Findings related to the definitions of the dimension concept

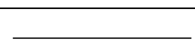
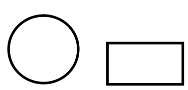
Term	Codes	f	%
Dimension	Length of the object in any direction	12	8.00
	Height of the object	4	2.66
	Appearance of the object	24	16.00
	Size of the object	7	4.66
	Length of the object	13	8.66
	Surface, part of an object	8	5.33
	Display unit	4	2.66
	Realism	1	0.66
	Perspective	3	2.00
	Blank	74	49.33

As shown in Table 2, half of the students had difficulty defining the dimension and left it blank. 8.66% of the students defined dimension as the object's length and 8.00% defined it as the object's length in any direction. Accordingly, most students failed to define dimension, an abstract concept. They perceived it as size and magnitude as used in everyday life. If we accept "the length of an object in any direction", "the length of an object", and "the height of an object" as correct for secondary school level, only 19.32% of them achieved to explain the concept of dimension. Below are examples of correct and incorrect answers belonging to students.

"The length of a shape in any direction (width, length and height)."

"Whether an object is small or large is called a dimension."

Table 3. Findings related to the definition of one dimensional

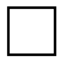
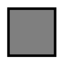
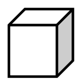
Term	Codes	f	%
One Dimensional		13	8.66
		48	32.00
	View from one side	17	11.33
	An object with only length, no thickness, no height (area & volume cannot be calculated)	7	4.66
	Blank	65	43.33

As seen in Table 3, students also had difficulty defining one dimensional. 43.33% of the students left it blank, and 40.66% failed to express one dimensional verbally and used shapes to explain it. In general, those who used a shape or expressed it as an object with only length, without thickness and height (area and volume cannot be calculated) achieved to define one dimensional correctly (45.32%). Below are examples of correct and incorrect answers belonging to students.

"We can think of shapes that consist only of length, without area or volume, as one-dimensional."

"A single view of an object means it is one-dimensional."

Table 4. Findings related to the definition of two dimensional

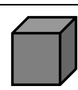
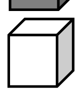
Term	Codes	f	%
Two Dimensional		15	10.00
		13	8.66
		26	17.33
	View of two sides	10	6.66
	Objects with only width and length	7	4.66
	Depth	2	1.33
	Blank	77	51.33

As seen in Table 4, students also had difficulty defining two dimensional. Half of the students left it blank, and one-third failed to express it verbally as in 1-dimensionality and used shapes to explain it. Those who drew a rectangular shape or defined it as objects with width and length expressed two dimensional correctly (13.32%). Below are examples of correct and incorrect answers belonging to students.

"If we add width to the length (height) in one dimension, we will have passed to two dimensional."

"If we can see two sides of an object when we look at it, we can say that this object is two-dimensional."

Table 5. Findings related to the definition of three dimensional

Term	Codes	f	%
Three Dimensional		3	2.00
		46	30.66
	Volume can be calculated; filled; has width and height	7	4.66
	The view of 3 sides	11	7.33
	Size of the object	4	2.66
	The most realistic version of the object	12	8.00
	Prism, box, cube	3	2.00
	Blank	64	42.66

As seen in Table 5, students also had difficulty defining three dimensional.42.66% of the students left it blank, and 32.66% failed to express it verbally and used shapes to explain it. Those who drew a filled cube or defined it as a cube whose volume can be calculated or a filled object with width, length, and height, expressed three dimensional correctly (6.66%). Below are examples of correct and incorrect answers belonging to students.

"Geometric objects whose volume we can find with width, length and height are three dimensional."

"In the three dimensional form of the shape looks more vivid and realistic."

Table 6. Findings related to the definition of length

Term	Codes	f	%
Length	Height	45	30.00
	Can be measured with a ruler	25	16.66
	Height above the ground (vertical length)	20	13.33
	Size	2	1.33
	Expressed by drawing a shape	21	14.00
	Blank	37	24.66

Table 6 shows that students who had difficulty defining dimension, an abstract concept, did not experience much difficulty explaining the concept of length, which they encounter and use more frequently in everyday life. The number of students who left it blank dropped to 24.66%. 30% of the students defined length as height, 16.66% defined it as something that can be measured with a ruler, and 13.33% defined it as height above the ground. In general, those who defined length as height, which can be measured with a ruler, and those who responded by drawing a shape defined length correctly (60.66%). Below are examples of correct and incorrect answers belonging to students.

"For example, we can calculate the length of something using a ruler."

"Like the size, I couldn't write it exactly."

Table 7. Findings related to the definition of width

Term	Codes	f	%
Width	Length, thickness	41	27.33
	Covered area, size of the object	22	14.66
	Vertical length	6	4.00
	Expressed by drawing a shape	25	16.66
	Blank	56	37.33

As shown in Table 7, 27.33% of the students defined width as thickness, 16.66% defined it by drawing a shape, and 14.66% defined it as the object's size. In general, those who defined width as thickness and those who responded by drawing a shape expressed width correctly (44%). Below are examples of correct and incorrect answers belonging to students.

"The width of an object."

"The area covered by any object in the environment in which it is located."

Table 8. Findings related to the definition of depth

Term	Codes	f	%
Depth	Height: the distance of an object from the surface; the length measured from the ground up.	61	40.66
	The hole inside the object	4	2.66
	Length of the inside of the object	8	5.33
	Expressed by drawing a shape	46	30.66
	Blank	31	20.66

As shown in Table 8, 40.66% of the students defined depth as height, while 30.66% explained it by drawing a shape. Those who defined it as height, the distance of the object from the surface, the length measured from the ground and those who responded by drawing a shape expressed depth correctly in the language of this age group (71.32%). Below are examples of correct and incorrect answers belonging to students.

"The measured distance of the object from the horizontal plane we are located downwards."

"Voids in matter."

Table 9. Findings related to the definition of solid object

Term	Codes	f	%
Solid Object	The state in which objects particles are closest to each other.	4	2.66
	With volume and mass	11	7.33
	We can feel with one of our five senses (touch, etc.).	21	14.00
	Expressed with shapes and examples	18	12.00
	Rigid object	32	21.33
	Non-fluid object	32	21.33
	Having a particular volume and having all the components of width, length and depth (3D)	2	1.33
	Blank	30	20.00

Table 9 shows that 42.66% of the students defined a solid object as a rigid, non-fluid object, while 20% failed to define it and left the question blank. Those who defined it as having volume and mass, having a particular volume, or having all the components of width, length, and depth, and those who responded by drawing a shape or giving an example expressed solid objects correctly (20.66%). Below are examples of correct and incorrect answers belonging to students.

"A three dimensional geometric object with a width, length, height, whose volume we can find."

"Objects that we can hold with our hands and see with our eyes are called solid objects. That's what we saw in science class."

Table 10. Findings related to the definition of prism

Term	Codes	f	%
Prism	Geometric shape (triangle, square, circle)	13	8.66
	Objects with a polygonal bases and rectangular lateral faces	4	2.66
	Expressed with shapes and examples	56	37.33
	3-dimensional object	32	21.33
	With types such as triangular prism, square prism	9	6.00
	Blank	36	24.00

As shown in Table 10, 37.33% of the students explained the prism with shapes and examples, 21.33% defined it as a 3-dimensional object, and 24% failed to define it and left it blank. Those who defined it as an object with a polygonal bases and rectangular lateral faces and those who responded with drawings and examples explained prism correctly (40%). Below are examples of correct and incorrect answers belonging to students.

"We call a prism an object whose base is a polygon and its sides are rectangles."

"Three-dimensional objects are prism."

Table 11. Findings related to the definition of line

Term	Codes	f	%
Line	————	22	14.66
	⟷	47	31.33
	A set of points extending infinitely at both ends	42	28
	Points that start at a point and go on infinitely (——→)	19	12.66
	Blank	20	13.33

Table 11 shows that 45.99% of the students explained the line with shapes, 28% defined it verbally, and 12.66% explained it both verbally and by drawing a shape. 59.33% of the respondents who defined the line as a set of points extending infinitely at both ends or responded with a shape (⟷) expressed the line correctly (59.33%). Below are examples of correct and incorrect answers belonging to students.

"A straight line that extends indefinitely at both ends."

"A line with one side fixed and the other side extending to infinity."

Table 12. Findings related to the definition of line segment

Term	Codes	f	%
Line Segment	————	44	29.33
	⟷	17	11.33
	A set of points bounded at both ends	43	28.66
	Points that start at a point and go on infinitely (——→)	22	14.66
	A straight line whose length can be measured	2	1.33
	Blank	22	14.66

As shown in Table 12, 40.66% of the students showed the line segment with a shape, 28.66% defined it verbally, and 14.66% explained it both verbally and by drawing a shape. Students who defined the line segment as a set of points bounded at both ends, or a straight line whose length can be measured and those who responded with a drawing (——) expressed the line segment correctly (59.33%). Below are examples of correct and incorrect answers belonging to students.

"A straight line whose length we can calculate (sets of point)."
"One of the two ends is a piece of the line that goes to infinity."

Table 13. Findings related to the definition of surface

Term	Codes	f	%
Surface	Any part of an object	5	3.33
	Every point of an object in contact with the air	6	4.00
	Top of an object	22	14.66
	Expressed with shapes	57	38.00
	The shape formed by joining the edges	2	1.33
	1-dimensional shape	2	1.33
	Shape whose area can be calculated	3	2.00
	Polygons forming the prism	6	4.00
	The flat and outer side	14	9.33
	Blank	33	22.00

Table 13 shows that 38% of the students showed the surface with a shape, 14.66% defined it as the top of an object, and 9.33% explained it as the flat and outer side. Those who defined the surface as the polygons forming the prism, whose area can be calculated, or responded with a shape expressed the surface correctly (44%). Below are examples of correct and incorrect answers belonging to students.

"The shapes that form the base and side faces, where we can find the area of geometric objects such as prisms, pyramids and cylinders, are called surfaces."

"The top of something."

Table 14. Findings related to the definition of edge

Term	Codes	f	%
Edge	The line segment connecting two corners	21	14.00
	Corner	13	8.66
	Where two surfaces meet	14	9.33
	Expressed with shapes	75	50.00
	Blank	27	18.00

As shown in Table 14, half of the students expressed the edge with a shape, 14% defined it as a line segment connecting two corners, and 9.33% explained it as where two surfaces meet. Those who defined it as a line segment connecting two corners and those who responded with a drawing expressed the edge correctly (64%). Below are examples of correct and incorrect answers belonging to students.

"A straight line segment connecting two corners of a geometric shape, such as a triangle, quadrilateral, or pentagon."

"Corners outside an object."

Table 15. Findings related to the definition of lateral edge

Term	Codes	f	%
Lateral Edge	Edge	32	21.33
	Line segment separating two surfaces	13	8.66
	Separate things	7	4.66
	Expressed with shapes	23	15.33
	The edges of prisms	17	11.33
	Blank	58	38.66

Table 15 shows that 38.66% (1/3) of the students had difficulty defining the lateral edge and left it blank. 21.33% defined the lateral edge as an edge, while 15.33% expressed it by drawing a shape. Those who defined it as a line segment separating two surfaces from each other or the edges of prisms and those who responded by drawing a shape expressed it correctly (35.33%). Below are examples of correct and incorrect answers belonging to students.

"Line connecting the corners of three dimensional objects."

"They are things that are separate from each other."

Table 16. Findings related to the definition of polygon

Term	Codes	f	%
Polygon	Shapes with many sides	33	22.00
	Shapes with at least 3 sides and 3 corners	25	16.66
	Closed shapes with at least 3 sides	11	7.33
	Expressed with shapes	50	33.33
	Lines with edges and corners	4	2.66
	Blank	27	18.00

As shown in Table 16, 1/3 of the students expressed polygons by drawing a shape, while 22% defined polygons as shapes with many sides. Only 7.3% mentioned the property of having at least 3 sides and being closed. Those who defined polygons as closed shapes with at least three sides and those who responded by drawing shapes expressed polygons correctly (40.66%). Below are examples of correct and incorrect answers belonging to students.

"They are geometric shapes with at least three sides. The shape must be closed."

"Shapes with many sides are polygons."

Table 17. Findings related to the definition of polygonal area

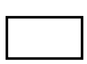
Term	Codes	f	%
Polygonal Area	Filled polygon	19	12.66
	The surface formed by multiple polygons	4	2.66
		11	7.33
		6	4
	Blank	110	73.33

Table 17 shows that 3/4 of the students failed to define the polygonal region. 12.66% expressed it as a filled polygon, while 11.33% used a shape to explain it. Those who defined polygonal area as a filled polygon and those who drew a filled polygon expressed the polygonal region correctly (16,66%). Below are examples of correct and incorrect answers belonging to students.

“Filling of a one-dimensional polygon.”

“It's probably something like a surface formed by combining polygons.”

Table 18. Findings related to the definition of circle

Term	Codes	f	%
Circle	A shape formed by joining the points equidistant from a point	9	6
	Closed round with interior filled	2	1.33
	Empty circle	60	40
	Expressed with shapes	58	38.66
	Circular	8	5.33
	Blank	13	8.66

As shown in Table 18, 40% of the students defined the circle as a hollow circle, while 38.66% expressed it by drawing a shape. Those who defined the circle as a hollow circle or as a shape formed by joining the points equidistant from a point and those who responded by drawing a shape expressed the circle correctly (84.66%). Below are examples of correct and incorrect answers belonging to students.

“The empty interior of the circle.”

“Filled round shapes.”

Table 19. Findings related to the definition of circular region

Term	Codes	f	%
Circular Region	Empty circle	23	15.33
	The shape formed by the circle and the inner region of the circle	6	4
	Round	5	3.33
	Filled circle	67	44.66
	Expressed with shapes	40	26.66
	Circular region	1	0.66
	Blank	8	5.33

Table 19 shows that 44.66% of the students defined the circle as a filled circle, while 26.66% expressed it by drawing a shape. Those who defined the circle as a shape formed by the union of the circle and its inner region or the circular region or a filled circle, and those who responded by drawing a shape expressed the circle correctly (76%). Below are examples of correct and incorrect answers belonging to students.

“Considering the circle and the inner region of the circle together.”
“Round things.”

Table 20. Findings related to the definition of circumference

Term	Codes	f	%
Circumference	The surroundings, the outer region of an object	23	15.33
	The sum of the side lengths of a geometric shape	31	20.66
	Explained through example	32	21.33
	Expressed with shapes	20	13.33
	Blank	44	29.33

As shown in Table 20, 29.33% of the students had difficulty defining the circumference and left it blank. 21.33% explained it through example, while 20.66% expressed it as the sum of the side lengths of a geometric shape. Those who defined the circumference as the sum of the side lengths of a geometric shape or explained it by drawing a shape or through an example expressed the circumference correctly (55.32%). Below are examples of correct and incorrect answers belonging to students.

"For example, when finding the perimeter of a square with side length a br , we add the lengths of the sides that forms the square and find 4a br.In other words, we call all the edges that forms the shape as the perimeter of the shape."

"The outer region of an object."

Table 21. Findings related to the definition of area

Term	Codes	f	%
Area	The space occupied by an object	18	12
	Multiplication of sides	34	22.66
	Explained through example	32	21.33
	Inner region	25	16.66
	Number of unit squares	1	0.66
	Blank	40	26.66

Table 21 shows that ¼ of students had difficulty defining the area and left it blank. 22.66% explained it as the product of the sides, while 21.33% expressed it by giving an example. Those who defined the area as the space occupied by the object or the interior region and those who explained it through example expressed the area correctly (50%). Below are examples of correct and incorrect answers belonging to students.

"The space occupied by an object in 2 dimensions, region."

"If we multiply the sides, we find the area."

Table 22. Findings related to the definition of volume

Term	Codes	f	%
Volume	Mass, weight	17	11.33
	The space occupied by an object in space	42	28
	Explained through example	17	11.33
	m^3	4	2.66
	Blank	70	46.66

As shown in Table 22, almost half of the students had difficulty defining volume and left it blank. 11.33% thought it was the same as mass and weight, while another 11.33% explained it through an example. Those who defined it as the space occupied by an object in space or explained through example expressed the volume correctly (39.33%). Below are examples of correct and incorrect answers belonging to students.

"The place an object occupies in space in 3 dimensions."

"The weight of an object is equal to its volume."

The second question of the test is "Divide the shapes below into three groups. It is expressed as "specify your classification criteria. The findings regarding this question are given below.

Table 23. Categories formed according to classifications

Categories	f	%
Those who form only 1 group correctly	88	58.66
Those who form all groups correctly	34	22.66
Those who fail to form groups	16	10.66
Blank	12	8

As shown in Table 23, 58.66% of the students formed only one group correctly, and 22.66% formed all groups correctly. 65 students who formed only one group put triangle and rectangle made of cardboard together and did not form any other group. As classification criteria, 21 students stated they were made of the same material. The remaining 44 students categorized rope as a group, triangle and rectangle as another group, and eraser, sugar cube, and triangular prism as another, saying that they were categorized according to the number of dimensions. Of the 88 students who formed only one group correctly, 22 put sugar cubes and erasers in a group, stating they were three dimensional and did not form any other group. The remaining one student formed a group by putting the triangular prism and the rope and gave no reason.

Of the 34 students who formed all groups correctly, only five mentioned dimensions as a reason, 10 stated that they were similar in shape and structure, and 19 gave no reason.

The third question of the test is expressed ,"By writing the number of dimensions of the following shapes, determine which one or which of them we can calculate the circumference, area and volume." The findings regarding this question are given below.

Table 24. Findings on the number of dimensions

Geometric Shape	Number of Dimensions	f	%
Circle	1	105	70
	2	5	3.33
	3	0	0
	Blank	40	26.66
Rectangular region	1	80	53.33
	2	23	15.33
	3	0	0
	Blank	47	31.33
Pentagonal region	1	81	54
	2	24	16
	3	0	0
	Blank	45	30
Triangle	1	99	66
	2	8	5.33
	3	10	6.66
	Blank	33	22
Square Prism (Wire)	1	4	2.66
	2	24	16
	3	57	38
	Blank	65	43.33
Rectangular Prism (Cardboard)	1	0	0
	2	42	28
	3	63	42
	Blank	45	30
Cube (Wood)	1	0	0
	2	23	15.33
	3	79	52.66
	Blank	48	32

As shown in Table 24, 70% of the students correctly stated the number of dimensions of the circle, 66% of the triangle, and 52.66% of the wooden cube. However, the percentage of correct answers for the number of dimensions of other geometric shapes is relatively low. Rectangular region was answered correctly by 15.33%, pentagonal region by 16%, rectangular prism (cardboard) by 28% and square prism (wire) by 2.66%. Most students thought the rectangular and pentagonal regions were one dimensional, and the rectangular prism (Cardboard) was three dimensional. The square prism (wire) was left blank because they could not determine its dimensions.

Table 25. Findings on the circumference of the shape

Geometric Shape	Circumference	f	%
Circle	Calculable	101	67.33
	Incalculable	23	15.33
	Blank	26	17.33
Rectangular region	Calculable	116	77.33
	Incalculable	8	5.33
	Blank	26	17.33
Pentagonal region	Calculable	115	76.66
	Incalculable	10	6.66
	Blank	25	16.66
Triangle	Calculable	118	78.66
	Incalculable	6	4.00
	Blank	26	17.33
Square Prism (Wire)	Calculable	101	67.33
	Incalculable	18	12.00
	Blank	31	20.66
Rectangular Prism (Cardboard)	Calculable	102	68.00
	Incalculable	21	14.00
	Blank	27	18.00
Cube (Wood)	Calculable	100	66.66
	Incalculable	16	10.66
	Blank	34	22.66

Table 25 shows that most students stated that circumference is calculable for all one, two and three dimensional geometric shapes whose length can be measured, which can be interpreted as students know about measuring length.

Table 26. Findings on the area of the shape

Geometric Shape	Area	f	%
Circle	Calculable	74	49.33
	Incalculable	40	26.66
	Blank	36	24.00
Rectangular region	Calculable	124	82.66
	Incalculable	3	2.00
	Blank	23	15.33
Pentagonal region	Calculable	112	74.66
	Incalculable	9	6.00
	Blank	29	19.33
Triangle	Calculable	92	61.33
	Incalculable	28	18.66
	Blank	30	20.00
Square Prism (Wire)	Calculable	91	60.66
	Incalculable	24	16.00
	Blank	35	23.33
Rectangular Prism (Cardboard)	Calculable	103	68.66
	Incalculable	21	14.00
	Blank	26	17.33
Cube (Wood)	Calculable	105	70.00
	Incalculable	19	12.66
	Blank	26	17.33

Table 26 shows that, regarding the two and three dimensional geometric objects whose area can be calculated, students marked the following shapes with the following percentages: 82.66% for the rectangular region, 74.66% for the pentagonal region, 68.66% for the rectangular prism (cardboard), and 70% for the wooden cube. In addition, they marked calculable for the following one dimensional shapes: 49.33% for circle, 61.33% for triangle and 60.66% for square prism (wire).

Table 27. Findings on the volume of the shape

Geometric Shape	Volume	f	%
Circle	Calculable	27	18.00
	Incalculable	78	52.00
	Blank	45	30.00
Rectangular region	Calculable	38	25.33
	Incalculable	72	48.00
	Blank	40	26.66
Pentagonal region	Calculable	40	26.66
	Incalculable	69	46.00
	Blank	41	27.33
Triangle	Calculable	24	16.00
	Incalculable	79	52.66
	Blank	47	31.33
Square Prism (Wire)	Calculable	97	64.66
	Incalculable	20	13.33
	Blank	33	22.00
Rectangular Prism (Cardboard)	Calculable	108	72.00
	Incalculable	8	5.33
	Blank	34	22.66
Cube (Wood)	Calculable	111	74.00
	Incalculable	10	6.66
	Blank	29	19.33

As shown in Table 27, 74% of the students answered that the volume of the wooden cube, one of the three dimensional geometric shapes, is calculable. 64.66% of the students gave the same answer for the one dimensional wire prism and 72% for the two dimensional rectangular prism (cardboard).

3.2. Findings of the Second Sub-Problem

In order to answer the second sub-problem of the study, "What is the level of secondary school students' mathematical language usage skills about dimensions?", the descriptive values related to the scores obtained from the mathematical language skills test were analyzed. The arithmetic mean of 7^{th}- and 8^{th}-grade students' skills in using mathematical language about dimension ($\bar{X}$ = 10.31, SD = 4.045) was approximately equal to the mean score obtainable from the scale ($\bar{X}_{scale}$= 10). Accordingly, it can be said that the mathematical language skills of 7^{th}- and 8^{th}-grade secondary school students on the dimension are close to the average.

3.3. Findings of the Third Sub-Problem

The data obtained from the Mathematical Language Skills Test were statistically analyzed to answer the third sub-problem of the study, "Do 7^{th}- and 8^{th}-grade secondary school students' mathematical language usage skills about dimension differ significantly by grade or gender?". First, the Kolmogorov-Smirnov test was used to determine whether the data were normally distributed. The p-value was below 0.05 (p=0.000), showing that the data were not normally distributed. Normality was also checked by using the skewness coefficient. According to Huck (2012), Hair et al. (2013), kurtosis and skewness values between -1 and +1 show normal distribution; this range is between -1.5 and +1.5 for Tabachnick and Fidel (2013). The Table below shows descriptive values of 7^{th}- and 8^{th}-graders' mathematical language skill scores on dimension.

Table 28. Descriptive values of mathematical language skill scores

Statistics	Mathematical Language Skill Test
Mean	10.31
Standard error of the mean	0.330
Median	11.00
Mod	11.00
Standard deviation	4.045
Variance	16.362
Skewness coefficient	-0.109
Standard error of skewness	0.198
Kurtosis coefficient	-0.564
Standard error of kurtosis	0.394
Range	18.00
Minimum score	2.00
Maximum score	20.00
Overall	1546

The skewness of the Mathematical Language Skills test was -0.109 in Table 28; therefore, it was assumed that the data were normally distributed.

Kolmogorov-Smirnov test was applied for each grade level to determine whether 7^{th}- and 8^{th}-grade students' mathematical language skills about dimension differed according to their grade level. The p-value of the 7^{th} graders was below 0.05 (p=0.000), and the p-value of the 8^{th} graders was above 0.05 (p=0.200), indicating that the data were not normally distributed. Normality was also checked using the skewness coefficient. The descriptive values of the mathematical language skills scores by grade are shown in Table 29.

Table 29. Descriptive values of mathematical language skill scores by grade

Grade	Statistics	Mathematical Language Skill Test
7^{th} Grade	Mean	10.19
	Median	11.00
	Standard deviation	3.597
	Variance	12.938
	Skewness coefficient	-0.274
	Standard error of skewness	0.277
	Kurtosis coefficient	-0.005
	Standard error of kurtosis	0.548
	Range	18.00
	Minimum score	2.00
	Maximum score	20.00
8^{th} Grade	Mean	10.43
	Median	11.00
	Standard deviation	4.470
	Variance	19.978
	Skewness coefficient	-0.051

Standard error of skewness	0.277
Kurtosis coefficient	-0.940
Standard error of kurtosis	0.548
Range	17.00
Minimum score	2.00
Maximum score	19.00

Table 29 shows that the skewness coefficient of the Mathematical Language Skills Test is -0.274 for 7th grade and -0.051 for 8th grade, indicating normal distribution. Thus, the Independent Samples t-test, one of the parametric tests, was used. The significance of the Independent Sample t-test was above 0.05 (p= 0.718), so there was no significant difference between students' mathematical language skills according to grade level [t(150)= -0.362; p=0.718]. On the other hand, statistical significance is affected by sample size (Fan, 2001); a more accurate interpretation of the results can be made by calculating the effect size. Cohen's d formula (Cohen, 1988) is widely used to calculate effect size. The d value was found to be 0.0595 using this formula. Since this value is between -0.20 and 0.20, it is considered low.

Kolmogorov-Smirnov test was applied for gender to determine whether 7th- and 8th-grade students' mathematical language skills about dimension differed according to gender. The p-values were below 0.05 (p=0.004 for girls and p=0.026 for boys), indicating that the data were not normally distributed. Normality was also checked using the skewness coefficient. The descriptive values of the mathematical language skills scores by gender are shown in Table 30.

Table 30. Descriptive values of mathematical language skill scores by gender

Gender	Statistics	Mathematical Language Skill Test
Girls	Mean	10.64
	Median	11.00
	Standard deviation	4.058
	Variance	16.467
	Skewness coefficient	-0.088
	Standard error of skewness	0.272
	Kurtosis coefficient	-0.308
	Standard error of kurtosis	0.538
	Range	18.00
	Minimum score	2.00
	Maximum score	20.00
Boys	Mean	9.94
	Median	10.00
	Standard deviation	4.028
	Variance	16.222
	Skewness coefficient	-0.142
	Standard error of skewness	0.283
	Kurtosis coefficient	-0.845
	Standard error of kurtosis	0.559
	Range	16.00
	Minimum score	2.00
	Maximum score	18.00

Table 30 shows that the skewness coefficient of the Mathematical Language Skills Test is --0.088 for girls and -0.142 for boys, indicating normal distribution. Thus, the Independent Samples t-test, one of the parametric tests, was used. The significance of the Independent Sample t-test was above 0.05 (p= 0.294), so there was no significant difference between students' mathematical language skills according to gender [t(150)= 1.054; p=0.294]. The Cohen's d value was found to be 0. 1731. Since this value is between -0.20 and 0.20, it is considered low.

3.4. Findings of the Fourth Sub-Problem

The data obtained from the Concept Test and Mathematical Language Skills Test were statistically analyzed to answer the fourth sub-problem of the study, "Is there a significant relationship between secondary school students' conceptual understanding of dimension and their mathematical language usage skills about dimension?". First, the Kolmogorov-Smirnov test was used to determine whether the data were normally distributed. The p-value was below 0.05 (p=0.013 for the Concept Test and p=0.000 for the Mathematical Language Skills Test), showing that the data were not normally distributed. As in the whole study, normality was also checked by using the skewness coefficient. The skewness coefficients of the Concepts and Mathematical Language Skills tests were -0.215 and -0.109, respectively. Since the assumption of normal distribution was accepted for both tests, Pearson's correlation was used to examine the relationship between the total scores obtained from the tests. The significance of Pearson's correlation coefficient (r=0.406; n=150; p=0.000) for the scores of the tests was $p<0.05$, so there was a significant relationship. The relationship between the scores is $0.30<r<0.70$, indicating a moderate strength (Büyüköztürk et al., 2010). No further calculation was done as the correlation coefficient expresses the effect size (Kotrlik & Williams, 2003).

4. Discussion and Conclusion

The results of the data analysis in this study, which examined secondary students' perceptions of dimensional concepts and mathematical language skills in many aspects, are discussed for each subproblem.

The first subproblem of the study examined secondary students' conceptual understanding of mathematical terms related to dimension. For this purpose, the students were asked to "define the terms given below". In defining dimensionality, only 19.32% of the students could explain the concept of dimensionality; 49.33% of the students had difficulty defining it and left it blank. In defining one dimensional, 43.33% of the students left the answer blank, and 40.66% could not express one dimensional verbally and resorted to explaining it with shapes. In total, 45.32% of the students could define one dimensional correctly. For two dimensional, 51.33% of the students left the answer blank, and 36% could not express it verbally. They used shapes to explain. In total, 13.32% of the students could express two dimensional correctly. Regarding the definition of three dimensional, 42.66% of the students left the answer blank, and 32.66% could not express it verbally and used shapes to explain it. In total, 6.66% were able to express three dimensional correctly. The fact that students tend to leave questions blank and the number of correct answers is low indicates that they do not have sufficient knowledge about the concept of dimension. It can be said that this result is similar to the findings of the studies conducted by Tuluk (2014), Duatepe Paksu et al. (2012), Altıkardeş and Yiğit Koyunkaya (2020).

Students who had difficulty defining dimension, an abstract concept, did not have much trouble explaining the concepts of length, width, and depth, which they encounter and use more frequently in everyday life. The number of students who left the definition of length blank dropped to 24.66%. 60.66% of the students were able to define length correctly. For the width definition, 37.33% of the students left it blank, and 44% expressed it correctly. When defining depth, 20.66% of the students left it blank, and 76.65% defined depth correctly.

For solid objects, 20% of the students left it blank, and only 20.66% expressed solid objects correctly. For prism, 24% of the students left it blank, and 40% could describe it correctly.59.33% could correctly define line and line segment, 44% could correctly express area, and 64% could correctly express edge. It was observed that the percentage of correct answers increased for the concepts of line, line segment, and edge, which were frequently used in the lesson. This percentage decreased for the definition of side edge; 38.66% of the students left it blank, and only 35.33% correctly expressed side edge. Although 40.66% of the students explained polygons correctly, 73.33% did not define polygon areas, leaving them blank, and only 16.66% expressed them correctly. This low percentage shows that the students do not understand the concept. Again, 84.66% and 76% of the students could correctly express the concepts of circles and filled circles, which are frequently used in everyday life.

While 55.32% of the students could correctly express the perimeter and 50% the area, this percentage decreased to 39.33 in the definition of volume. The concept of polygon is generally explained as a "many-sided shape". In explaining the term in this way, the relationship between the number of edges and the word formed may be recognized and defined by the students. The fact that polygon is not clear enough to show the conceptual meaning of the word root may have caused students to make inadequate definitions. It can be said that in the studies conducted by Akuysal (2007), Başışık (2010), Ergün (2010) and Önder (2019), polygon was defined as a multi-sided shape, thus parallel results were obtained. The concept of circle and circle region is generally explained by emphasizing whether the inside is empty or filled and by drawing shapes. In the studies of Gülkılık (2008), Önder (2019), Yenilmez and Demirhan (2013), it was observed that the term round, which students mostly used for circle and circle region, was rarely used in the research. For this reason, it can be said that the findings obtained differ. However, it can also be interpreted that these studies are in parallel with the findings obtained from the research, since the number of definitions made using mathematical language for circle and circle region is low.

In the second question of the test, students were asked, "Classify the following shapes into 3 groups. State your classification criterion. 22.66% of the students classified all groups correctly. Of the 34 students who classified all groups correctly, only 5 gave dimension as a reason, 10 gave their similarity in shape and structure, and 19 gave no reason.

In the third question of the test, the students were asked, "Write down the number of dimensions of the following shapes and determine those whose circumference, area and volume can be calculated". 70% of the students answered correctly for the circle, 66% for the triangle, and 52.66% for the wooden cube. However, the percentage of correct answers for other geometric shapes was relatively low. The rectangular area was answered correctly by 15.33%, the pentagonal area by 16%, the rectangular prism (cardboard) by 28%, and the square prism (wire) by 2.66%. Most students thought the rectangular and pentagonal regions were one dimensional, while the rectangular prism (cardboard) was three dimensional. They could not determine how many dimensions the square prism (wire) had and left it blank. In parallel with the results of Ural (2011), Duatepe Paksu et al. (2012), and Tat (2021), students had problems determining the number of dimensions. In summary, the results of these studies conducted with pre-service teachers and high school students support the results of our study conducted with secondary school students.

In the circumference/area/volume calculation part, most students marked calculable for the circumference of all one, two and three dimensional geometric shapes whose lengths can be measured, which can be interpreted as students' knowledge of measuring length. Regarding the two and three dimensional geometric shapes, students marked calculable for the areas of the rectangular region (82.66 %), pentagonal region (74.66%), rectangular prism (cardboard) (68.66%) and the wooden cube (70%). However, students have the same opinion for one dimensional shapes. 49.33% of them answered that the area of a circle can be calculated, 61.33% answered the same for a triangle, and 60.66% answered the same for a rectangular wire prism. Contrary to length, it can be said that the students lack the knowledge to calculate the area. Among the three dimensional geometric shapes whose volume can be calculated, 74% of the students answered that the volume of the wooden cube can be calculated; on the other hand, 64.66% of the students answered the same for the one dimensional wire prism and 72% for the two dimensional rectangular prism (cardboard). From this point, it can be interpreted that the students consider all types of prisms three dimensional, whether made of wire or cardboard. Therefore, since they accepted the prisms as three dimensional, the computable volume view was dominant. In Tat's (2021) study, students identified the concept of length with one dimensional shapes, area with two dimensional shapes, and volume with three dimensional objects. The rate of correct calculation judgments was high for shapes whose size was correctly determined. At the same time, it decreased as the number of incorrect size judgments increased. This result is similar to the results of our study.

The ability to understand and express what students read, which is one of the areas where they were weak, was one of the areas where they had difficulty in this study. It has been observed that many students list features instead of definitions, resort to explanations with figures instead of verbal expressions, in short, have problems explaining concepts verbally, and their knowledge level in expressing concepts is not sufficient. Similar results were found in Taşpınar (2019), Ayaz (2017), Bayram and Duatepe Paksu (2018), Çakmak et al. (2014), Pickreign (2007), Rasslan and Tall (2002), Yenilmez and Demirhan (2013), Ergün (2010). Çetin (2009), Çıkla and Duatepe (2002), Gümüş (2019) and it was also found in the studies of Cilavdaroğlu (2012).

In the second sub-problem of the research, which examined the mathematical language skills of secondary school students on the subject of size, it was observed that the mathematical language skill scores of the 7th and 8th grade secondary school students on the subject of size had a value close to the arithmetic average. Ünal's

(2013) study, which examined the mathematical language skills of 7th graders in the field of geometry learning, supports this result obtained from the research. On the other hand, it can be said that students answered the questions in the mathematical language skills test more easily than in the test related to concepts. This may be caused by the fact that basic geometric concepts, which started to be taught to students in primary school in geometry teaching, and geometric shapes and objects are given less space at the conceptual level (Uygun & Akyürek Tay, 2023) but the frequent use of mathematical language even in interdisciplinary relationships (Uygun & Aşıkcan, 2023).

Concerning the third sub-problem of the study, which dealt with the differentiation of the mathematical language skills of 7th and 8th graders in terms of dimension by grade level and gender, no statistically significant difference was found. Cognitively, for the levels of 7th and 8th grade students are close to each other, it can be interpreted that similar results emerged from the tests as expected. In addition, it was observed that similar results were obtained with studies Akarsu (2013), Yılmaz and Güzel (2020), Pazarbaşı (2015), Ünal (2013) and Yüzerler's (2013), where mathematical language skills were examined according to the gender variable, but different results were obtained with studies Yüzerler (2013) and Yıldız (2016), depending on the gender variable.

According to the results obtained in the fourth subproblem of the study, in which secondary school students' mathematical language skills and their concept perceptions of dimension were examined, a moderate, positive, and significant relationship was found between these two variables. Since mathematical language is directly related to the concepts used, they are interrelated regarding the results obtained from the tests.

There is information about the fact that the time used in the generally applied and newly implemented teaching programs is short and geometry subjects are compressed to the end of the semester and not given the necessary importance, (change was made in the 2024 curriculum and geometry subjects were distributed throughout the semester.) It was observed that students did not have sufficient knowledge about the dimension due to the lack of emphasis on teaching at the conceptual level. Therefore, it is thought that awareness should be raised in teaching mathematical concepts. It is explained that researchers should do more research on this subject in order to emphasize the basic dimension of many subjects in teaching geometry and to present the importance of this theoretical mathematics teaching programs. In addition, more courses on concept teaching can be included in education faculties or the scope of courses can be expanded in this direction. Teachers' competencies on the subject can be improved through in-service training. Technological materials can be used that can support the teaching of concepts and facilitate students' understanding by concretizing these concepts.

References

Adams, T. L., Thangata, F., and King, C. (2005). "Weigh" to go! Exploring mathematical language. *Mathematics Teaching in the Middle School, 10*(9), 444-448. https://doi.org/10.5951/MTMS.10.9.0444

Akarsu, E. (2013). *7. sınıf öğrencilerinin cebir öğrenme alanında matematiksel dil kullanımlarının incelenmesi.* [Yayımlanmamış yüksek lisans tezi]. Dokuz Eylül Üniversitesi,

Akbulut, Y. (2010). *Sosyal bilimlerde SPSS uygulamaları*. İdeal Kültür Yayıncılık.

Akgün, L. (2007). *Değişken kavramına ilişkin yeterlilikler ve değişken kavramının öğretimi.* [Yayımlanmamış doktora tezi). Atatürk Üniversitesi.

Akuysal, N. (2007). *İlköğretim 7. sınıf öğrencilerinin 7. sınıf ünitelerindeki geometrik kavramlardaki yanılgıları.* [Yayımlanmamış yüksek lisans tezi]. Selçuk Üniversitesi.

Albayrak, S. (2023). *Ortaokul öğrencilerinin matematik dilini anlama ve kullanma becerilerinin incelenmesi*. [Yayımlanmamış yüksek lisans tezi]. Bayburt Üniversitesi.

Altıkardeş, E. ve Yiğit Koyunkaya, M. (2020). Matematik temelli çeşitli teknoloji uygulamalarının lise öğrencilerinin katı cisim ve boyut konularındaki algılarına etkisinin incelenmesi. *Ondokuz Mayıs Üniversitesi Eğitim Fakültesi Dergisi, 39*(1), 40-68. https://doi.org/10.7822/omuefd.586755

Alexanderoff, P. (1961). *Elementary concepts of topology. (Çev. Alan E. Farley)*. Dover Publications.

Aşıkcan, M. ve Uygun, N. (2023). A taxonomic analysis of the questions prepared by prospective primary teachers for primary school mathematics and turkish language courses. *International Journal of Education and Literacy Studies*, *11*(3), 286-293. https://doi.org/10.7575/aiac.ijels.v.11n.3p.286

Ata, A. (2013). *Öğretmen adaylarının olasılık konusuna ilişkin kavramsal ve işlemsel bilgi düzeylerinin incelenmesi.* [Yayımlanmamış yüksek lisans tezi]. Eskişehir Osmangazi Üniversitesi.Ayaz, Ü. B. (2017).

Ortaokul öğrencilerinin dörtgenlere ilişkin kavram imajları. [Yayımlanmamış yüksek lisans tezi]. Necmettin Erbakan Üniversitesi.

Aydın Karaca, Ö. (2014). *8. sınıf öğrencilerin uzunluk, alan ve hacim ölçme kavramlarını anlamaya ilişkin yeterliliklerinin incelenmesi.* [Yayımlanmamış yüksek lisans tezi].Karadeniz Teknik Üniversitesi.

Aydın, S. ve Yeşilyurt, M. (2007). Matematik öğretiminde kullanılan dile ilişkin öğrenci görüşleri. *Elektronik Sosyal Bilimler Dergisi, 6*(22), 90-100.

Aydoğan Belen, N. (2018). *İlkokul dördüncü sınıf öğrencilerinin kullandıkları matematiksel dilin incelenmesi.* [Yayımlanmamış yüksek lisans tezi]. Ordu Üniversitesi,

Başışık, H. (2010). *İlköğretim 5. sınıf öğrencilerinin çokgenler ve dörtgenler konularındaki kavram yanılgılarının belirlenmesi* .[Yayımlanmamış yüksek lisans tezi]. Adnan Menderes Üniversitesi.

Bayram, G. ve Duatepe Paksu, A.. (2018). Sekizinci sınıf öğrencilerinin paralelkenara ilişkin yaptıkları çizimler bağlamında kavram imajları ve bu dörtgen için yaptıkları tanımlar. In *the 27th. International Congress on Educational Sciences*.

Birgin, O. ve Özkan, K. (2012). Farklı öğretim kademesindeki öğrencilerin "yamuk" kavramı konusundaki algılarının karşılaştırılması. *First International Symposium of Teacher Training and Development,* Usak University.

Boulet, G. (2007). How does language impact the learning of mathematics? Let me count the ways. *Journal of Teaching and Learning, 5*(1), 1-12. https://doi.org/10.22329/jtl.v5i1.125

Bozkurt, A. ve Koç, Y. (2012). İlköğretim matematik öğretmenliği birinci sınıf öğrencilerinin prizma kavramına dair bilgilerinin incelenmesi. *Educational Sciences: Theory & Practice, 12*(4), 2941-2952.

Büyüköztürk, Ş., Akgün, Ö. E., Demirel, F., Karadeniz, Ş. ve Çakmak, E. K. (2010). *Bilimsel araştırma yöntemleri.* Pegem.

Capraro, M. M. and Joffrion, H. (2006). Algebraic equations: Can middle-school students meaningfully translate from words to mathematical symbols? *Reading Psychology, 27* (2), 147-164. https://doi.org/10.1080/02702710600642467

Christensen, L. B. Johnson, B. R. and Turner, L. A. (2015). *Araştırma yöntemleri desen ve analiz* (Çev. Ed. A. Aypay). Anı Yayıncılık.

Cilavdaroğlu, A. K. (2012). *İlköğretim matematik öğretmenliği birinci sınıf öğrencilerinin bazı iki boyutlu geometrik kavramların tanımları ve şekillerine dair bilgilerinin incelenmesi.* [Yayımlanmamış yüksek lisans tezi], Gaziantep Üniversitesi.

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd ed.). Erlbaum.

Creswell, J. W. (2003). *Research design: Qualitative, quantitative, and mixed methods approaches* (2nd ed.). Sage.

Çakıroğlu, E. (2013). Matematik kavramlarının tanımlanması. İ. Zembat, M.F. Özmantar, E. Bingölbali, H. Şandır ve A. Delice (Editörler), *Tanımları ve tarihsel gelişimleriyle matematiksel kavramlar*. Pegem Akademi Yayıncılık.

Çakmak, Z. (2013). *Sekizinci sınıf öğrencilerinin istatistik konusundaki matematiksel dil becerilerine ilişkin değişkenlerin yapısal eşitlik modeli ile incelenmesi.* [Yayımlanmamış yüksek lisans tezi]. Erzincan Üniversitesi.

Çakmak, Z., Baş, F., ve Bekdemir, M. (2014). İlköğretim matematik öğretmen adaylarının örüntüler konusundaki matematiksel dil becerileri. *Erzincan Üniversitesi Eğitim Fakültesi Dergisi, 16*(1), 204-223. https://doi.org/10.17556/jef.17653

Çalıkoğlu Bali, G. (2002). Matematik öğretiminde dil ölçeği. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi*, *23*(23), 57-61.

Çalıkoğlu Bali, G. (2003). Matematik öğretmen adaylarının matematik öğretiminde dile ilişkin görüşleri. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi, 25*, 19-25.

Çetin, İ. (2009). *7. ve 9. sınıf öğrencilerinin oran ve orantı konusundaki kavram yanılgıları*. [Yayımlanmamış yüksek lisans tezi]. Selçuk Üniversitesi.

Çıkla, O. A.,ve Duatepe, A. (2002). İlköğretim matematik öğretmen adaylarının orantısal akıl yürütme becerileri üzerine niteliksel bir çalışma. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi*, *23*, 32-40.

Doğan, M. (2015). Nokta, doğru, doğru parçası, ışın, düzlem ve uzay kavramları. İ. Ö. Zembat, M. F. Özmantar, E. Bingölbali, H. Şandır ve A. Delice (Editörler), *Tanımları ve tarihsel gelişmeleriyle matematiksel kavramlar.*Pegem Akademi Yayıncılık.

Doğan, M. ve Güner, P. (2012). İlköğretim matematik öğretmen adaylarının matematik öğretimini anlama ve kullanım anlamlarını incelemek. *X. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi,* Niğde Üniversitesi.

Duatepe Paksu, A., Musan, M., İymen, E., ve Pakmak, G. S. (2012). Sınıf öğretmeni adaylarının boyut konusundaki kavram görüntüleri. *Dokuz Eylül Üniversitesi Buca Eğitim Fakültesi Dergisi*, (34), 52-68.

Dur, Z. (2010). Öğrencilerin matematiksel dili hikaye yazma yoluyla iletişimde kullanabilme becerilerinin farklı değişkenlere göre incelenmesi. [Yayımlanmamış yüksek lisans tezi]. Hacettepe Üniversitesi.

Ebersbach, M. (2009). Achieeving a new dimension: children integrate three stimulus dimension in volume estimations. *Developmental Psychology, 45*(3), 877-883. https://doi.org/10.1037/a0014616

Edwards, B. and Ward, M. (2008). The role of mathematical definitions in mathematics and in undergraduate mathematics courses. In M. Carlson ve C. Rasmussen (Eds.), *Making the connection: Research and teaching in undergraduate mathematics.* (s. 221-230). Mathematical Association of America. https://doi.org/10.5948/UPO9780883859759.018

Erbay, H. N. (2016). 6. sınıf öğrencilerinin açılar konusundaki kavram bilgilerinin incelenmesi. *Akademik Sosyal Araştırmalar Dergisi, 4*(36), 704-718. https://doi.org/10.16992/ASOS.11815

Ergün, S. (2010). *İlköğretim 7. sınıf öğrencilerinin çokgenleri algılama, tanımlama ve sınıflama biçimleri.* [Yayımlanmamış yüksek lisans tezi]. Dokuz Eylül Üniversitesi.

Fan, X. (2001). Statistical significance and effect size in education research: Two sides of a coin. *Journal of Educational Research, 94*(5), 275-283. https://doi.org/10.1080/00220670109598763

Freudenthal, H. (2002). *Didactical phnomenology of mathematical structures.* Kluwer Academic Publichers.

Georgius, K. (2008). *Improving communication about mathematics through vocabulary and writing.* Department of Teaching, Learning and Teacher Education University of Nebraska-Lincoln.

Gezgin, İ. (2023). *İlköğretim matematik öğretmenliği lisans öğrencilerinin has olmayan integraller konusunda matematiksel dil kullanımlarının incelenmesi*. [Yayımlanmamış yüksek lisans tezi]. Dokuz Eylül Üniversitesi.

Gökçe Özdemir, Z. (2014). *Ortaokul matematik terimlerinin semantik açıdan incelenmesi.* [Yayınlanmamış yüksek lisans tezi]. Sakarya Üniversitesi.

Gray, V. D. (2004). *The language of mathematics: a functional definition and the development of an instrument to measure teacher perceived self-efficacy.* [Unpublished Doctoral Dissertation]. Oregon State University.

Greenes, C., Ginsburg, H. P., and Balfanz, R. (2004). Big math for little kids. *Early Childhood Research Quarterly,19,* 159–166. https://doi.org/10.1016/j.ecresq.2004.01.010

Güldal, Ö. (2022). *8. sınıf öğrencilerinin özdeşlikler konusunda matematiksel dili kullanma becerileri ile matematik başarıları arasındaki ilişkinin incelenmesi.* [Yayımlanmamış doktora tezi]. Marmara Üniversitesi.

Gülkılık, H. (2008). *Öğretmen adaylarının bazı geometrik kavramlarla ilgili sahip oldukları kavram imajlarının ve imaj gelişiminin incelenmesi üzerine fenomenografik bir çalışma*. [Yayımlanmamış yüksek lisans tezi]. Gazi Üniversitesi.

Gümüş, H. K. (2019). *İlköğretim matematik öğretmen adaylarının ortaokul matematik dersi öğretim programı cebir öğrenme alanındaki kavramlara yönelik tanımları.* [Yayımlanmamış yüksek lisans tezi]. Gazi Üniversitesi.

Hair, J. F., Black, W. C., Babin, B. J., Anderson, R. E. and Tatham, R. L. (2013). *Multivariate data analysis*. Pearson Education Limited.

Huck, S. W. (2012). *Reading statistics and research* (6th ed.). Pearson.

Kaplan, A. ve Hızarcı, S. (2005). Matematik öğretmen adaylarının üçgen kavramı ile ilgili bilgi düzeyleri. *Atatürk Üniversitesi Kazım Karabekir Eğitim Fakültesi Dergisi*, (11), 472-478.

Karaca, E. T. (2023). *Fourth grade students' geometrical reasoning related to 2d and 3d shapes* [Ph.D. - Doctoral Program]. Middle East Technical University.

Karahan, V. (2021). *İlkokul ve ortaokul matematik ders kitaplarındaki temel geometrik kavramların incelenmesi.* [Yayımlanmamış yüksek lisans tezi]. Recep Tayyip Erdoğan Üniversitesi.

Kaşıkçı, M. (2022). *Ortaokul matematik öğretim programındaki sayılar ve işlemler ile ilgili matematiksel kelimelerin semantik (anlambilimsel) açıdan incelenmesi.* [Yayımlanmamış doktora tezi]. Dokuz Eylül Üniversitesi.

Kılıç, A. S., Temel, H.,ve Şenol, A. (2015). Öğretmen adaylarının "nokta, doğru, düzlem ve açı" kavramları hakkında bilgi düzeyleri ve kavram yanılgılarının incelenmesi. *Dicle Üniversitesi Ziya Gökalp Eğitim Fakültesi Dergisi,* (26), 205-229.

Kotrlik, J. W. and Williams, H. A. (2003). The incorporation of effect size in information technology, learning, and performance research. *Information Technology, Learning, and Performance Journal, 21*(1), 1-7.

Kula Yeşil, D. (2015). *Sekizinci sınıf öğrencilerinin dörtgenler bağlamında matematik dili kullanımları: Sentaks ve semantik bileşenler*. [Yayımlanmamış yüksek lisans tezi]. Anadolu Üniversitesi.

Küçük, A. ve Demir, B. (2009). İlköğretim 6-8. sınıflarda matematik öğretiminde karşılaşılan bazı kavram yanılgıları üzerine bir çalışma. *Dicle Üniversitesi Ziya Gökalp Eğitim Fakültesi Dergisi, 13*(2009), 97-112.

Larson, C. (2007). The importance of vocabulary instruction in everyday mathematics. *Math in the Middle Institute Partnership Action Research Project Report (s.60). Erişim adresi: https://digitalcommons.unl.edu/mathmidactionresearch/60*

Man, S. (2019). *Ortaokul matematik öğretmenlerinin geometrik cisimlere ilişkin kavram tanımlarının incelenmesi.* [Yayımlanmamış yüksek lisans tezi]. Adıyaman Üniversitesi.

Mandelbrot, B. B. (1983). *The fractal geometry of nature*. W. H. Freeman. https://doi.org/10.1119/1.13295

Manin, Y. I. (2006). The notion of dimension in geometry and algebra. *Bulletin of the American Mathematical Society, 43*(2), 139-161.Mercer, N. ve Sams, C. (2006). Teaching children how to use language to solve maths problems. *Language and Education, 20*(6), 507-528. https://doi.org/10.2167/le678.0

Miles, M. B.,and Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook*. Sage.

Millî Eğitim Bakanlığı (2009). *İlköğretim matematik dersi öğretim programı (6-8. sınıflar).* Talim ve Terbiye Kurulu Başkanlığı.

Milli Eğitim Bakanlığı (2018). *Matematik dersi öğretim programı (İlkokul ve ortaokul 1, 2, 3, 4, 5, 6, 7 ve 8. sınıflar).* Talim ve Terbiye Kurulu Başkanlığı.

Milli Eğitim Bakanlığı (2024). *Ortaokul matematik dersi öğretim programı (5, 6, 7 ve 8. sınıflar).* Talim ve Terbiye Kurulu Başkanlığı.

Nalbant, S. (2015). *9. sınıf öğrencilerinin matematiksel kavram ve sembolleri anlamlandırma yeterlikleri ile matematik problemlerini çözme başarıları arasındaki ilişkinin incelenmesi.* [Yayımlanmamış yüksek lisans tezi]. Karadeniz Teknik Üniversitesi.

Orhan, M. S. (2022). *8. sınıf öğrencilerinin üslü ve kareköklü ifadeler alt öğrenme alanlarında matematiksel kavram ve sembolleri anlamlandırma yeterlilikleri ile matematiksel problemleri çözme becerileri arasındaki ilişkinin incelenmesi.* [Yayımlanmamış yüksek lisans tezi]. Tokat Gaziosmanpaşa Üniversitesi.

Önder, S. (2019). *Ortaokul öğrencilerinin sayılar, geometri ve cebir özelinde kullanılan sözcüklere yükledikleri mananın semantik açıdan incelenmesi.* [Yayımlanmamış yüksek lisans tezi]. Erciyes Üniversitesi.

Özdüvenci Yavuz, B. (2022). *Ortaokul öğrencilerinin alan ve sıvı ölçme konularında alan dili kullanımlarının incelenmesi.* [Yayımlanmamış yüksek lisans tezi]. Dokuz Eylül Üniversitesi.

Pazarbaşı, B. N. (2015). *İlköğretim matematik öğretmen adaylarının analitik geometri alan dilini kullanma becerileri ve tutumlarının incelenmesi*. [Yayımlanmamış yüksek lisans tezi]. Gazi Üniversitesi.

Pears, A. R. (1975). *Dimension theory of general spaces.* Cambridge University Press.

Peker, M. ve Karakuş, F. (2015). Matematiksel anlamda boyut kavramı. İ. Ö. Zembat, M. F. Özmantar, E.

Bingölbali, H. Şandır ve A. Delice (Editörler). *Tanımları ve tarihsel gelişmeleriyle matematiksel kavramlar.* Pegem Akademi Yayıncılık.

Piaget, J. and Inhelder, B. (1956). *The child's conception of space*. London Routledge ve Kegan Paul.

Pickreign, J. (2007). Rectangles and Rhombi: How Well Do Preservice Teachers Know Them? *Issues in the undergraduate mathematics preparation of school teachers*, *1*.

Rasslan, S. and Tall, D. (2002, July). Definitions and images for the definite integral concept. *In PME Conference* (Vol. 4, pp. 4-089).

Sayma, B. N. (2021). *Ortaokul 8. sınıf öğrencilerinin aritmetik ve geometri sembollerine yükledikleri anlamların incelenmesi.* [Yayımlanmamış yüksek lisans tezi]. Gaziantep Üniversitesi.

Skordoulis, C., Vitsas, T., Dafermos, V., and Koleza, E. (2009). The system of coordinates as an obstacle in understanding the concept of dimension. *International Journal of Science and Mathematics Education,* 7, 253-272. https://doi.org/10.1007/s10763-008-9130-2

Soğancı, Ö. (2006). *Öğreniminde ve öğretiminde öğretmen adaylarının matematiksel tanımlara yaklaşımları üzerine fenomenografik bir çalışma*. [Yayımlanmamış yüksek lisans tezi]. Gazi Üniversitesi.

Spiridonova, N. I., and Savvinova, A. D. (2015). Distinctive features of formation of mathematical language in the context of bilingual education. *Procedia-Social and Behavioral Sciences*, 714-724. https://doi.org/10.1016/j.sbspro.2015.11.693

Süzer, V. (2011). *Dokuzuncu sınıf öğrencilerinin fonksiyon kavramı ile kavram tanımı ve imajı üzerine bir durum çalışması.* [Yayımlanmamış yüksek lisans tezi]. Gazi Üniversitesi.

Tabachnick, B. G., Fidell, L. S., and Ullman, J. B. (2013). *Using multivariate statistics* (Vol. 6, pp. 497-516). Pearson.

Tan Şişman, G. ve Aksu, M. (2009). Yedinci sınıf öğrencilerinin alan ve çevre konularındaki başarıları. *İlköğretim Online, 8*(1), 243-253.

Taşpınar, S. İ. (2019). *Sınıf öğretmenlerinin geometrik cisimler hakkındaki konu alan bilgilerinin incelenmesi.* [Yayımlanmamış yüksek lisans tezi]. Tokat Gaziosmanpaşa Üniversitesi.

Tat, N. (2021). *12. sınıf öğrencileri için boyut belirleme süreci ve boyut kavramının uzunluk, alan ve hacim ölçmedeki rolü* [Yayımlanmamış yüksek lisans tezi] Erciyes Üniversitesi.

Temel, H., ve Eroğlu, A. O. (2014). İlköğretim 8. sınıf öğrencilerinin sayı kavramlarını anlamlandırmaları üzerine bir çalışma. *Kastamonu Üniversitesi Kastamonu Eğitim Dergisi, 22*(3), 1263-1278.

Toptaş, V. (2015). Matematiksel dile genel bir bakış. *International Journal of New Trends in Arts, Sports & Science Education (IJTASE), 4*(1), 18-22.

Tuluk, G. (2014). Sınıf öğretmeni adaylarının nokta, çizgi, yüzey ve uzay bilgileri ve çoklu temsilleri. *Kastamonu Eğitim Dergisi*, *22*(1), 361-384.

Türk Dil Kurumu. (2024). Güncel türkçe sözlük. http://www.tdk.org.tr/(erişim tarihi: 29.06.2024).

Ural, A. (2011). Matematik öğretmen adaylarının boyut ölçütleri. *Pamukkale Üniversitesi Eğitim Fakültesi Dergisi, 30*, 13-25.

Uygun, N. ve Akyürek Tay, B. (2023). An analysis of mathematical concepts in the 4th grade social studies textbook. *Türk Akademik Yayınlar Dergisi (TAY Journal), 7(Special Issue)*, 160-180. https://doi.org/10.29329/tayjournal.2023.609.07

Ülgen, G. (2004). *Kavram geliştirme kuramlar ve uygulamalar*. Nobel Yayın Dağıtım.

Ünal, Z. (2013). *7.sınıf öğrencilerinin geometri öğrenim alanında matematiksel dil kullanımlarının incelenmesi.* [Yayımlanmamış yüksek lisans tezi]. Dokuz Eylül Üniversitesi.

Vitsas, T. and Koleza, E. (2000). Student's misconceptions on the concept of dimension. *In Proceedings of the 2nd Mediterranean Conference on Mathematics Education* ,108-119.

Woods, G. (2009). An investigation into the relationship between the understanding and use of mathematical language and achievement in mathematics atthe foundation stage. *Procedia Social and Behavioral Sciences, 1*, 2191-2196. https://doi.org/10.1016/j.sbspro.2009.01.385

Yalvaç, B. (2019). *Sekizinci sınıf öğrencilerinin cebir öğrenme alanında matematiksel dili kullanma becerilerinin incelenmesi*. [Yayımlanmamış yüksek lisans tezi] Hacettepe Üniversitesi.

Yaman, H., Toluk, Z., ve Olkun, S. (2003). İlköğretim öğrencileri eşit işaretini nasıl algılamaktadırlar? *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi, 24*, 142-151.

Yenilmez, K. ve Demirhan, H. (2013).6. sınıf öğrencilerinin bazı temel matematik kavramlarını anlama düzeyleri. *Dicle Üniversitesi Ziya Gökalp Eğitim Fakültesi Dergisi. 20*, 275-292.

Yeşildere, S. (2007). İlköğretim matematik öğretmen adaylarının matematiksel alan dilini kullanma yeterlikleri. *Boğaziçi Üniversitesi Eğitim Dergisi, 24*(2), 61-70.

Yıldırım, A. ve Şimşek, H. (2016). *Sosyal bilimlerde nitel araştırma yöntemleri.* Seçkin Yayıncılık.

Yıldız, F. (2016). *6. ve 7. sınıf öğrencilerinin matematiksel sözel, sembolik ve görsel dili anlama ve kullanma becerilerinin incelenmesi.* [Yayımlanmamış yüksek lisans tezi]. Marmara Üniversitesi.

Yılmaz, E. (2015). *İlköğretim matematik öğretim adaylarının katı cisimler ile ilgili kavram tanımı ve kavram imajlarının fenomenografik yaklaşımla incelenmesi.* [Yayımlanmamış yüksek lisans tezi], Erciyes Üniversitesi.

Yılmaz, S. ve Güzel, S. (2020). Sekizinci sınıf öğrencilerinin kareköklü ifadeler konusundaki matematiksel dil kullanım düzeyleri ve dile ilişkin görüşleri. *e- Kafkas Eğitim Araştırmaları Dergisi, 7*, 282-302. https://doi.org/10.30900/kafkasegt.733078

Yüzerler, S. (2013). *6. ve 7. sınıf öğrencilerinin matematiksel dili kullanabilme becerileri.* [Yayımlanmamış yüksek lisans tezi]. Necmettin Erbakan Üniversitesi.

Zazkis, R., ve Leikin, R. (2008). Exemplifying definitions: A case of a square. *Educational Studies in Mathematics, 69*(2), 131–148. https://doi.org/10.1007/s10649-008-9131-7