

Unit Instructional Design on Graphs and Properties of Exponential Functions Based on Big Ideas

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Abstract

In the era of knowledge explosion, students urgently need deep understanding and transferability skills, and the integration of Big Ideas with unit teaching has become a breakthrough for educational reform. As a core component of mathematics, the teaching of functions directly impacts students' core competencies, and the graph and properties of exponential functions serve as a key medium for developing mathematical thinking. Despite a rich research foundation, the application of Big Ideas in function teaching still requires breakthroughs. This paper takes the graph and properties of exponential functions as an example, and through theoretical analysis and practical design, constructs a unit teaching path guided by Big ideas. The aim is to provide a feasible paradigm for frontline teaching, enhancing the quality of mathematics instruction and students' core competencies.

Keywords: Big Ideas, Unit Teaching Design , Exponential Functions, Core Competencies

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1. Introduction

The "Big Idea" refers to the core concepts or principles within a discipline, characterized by centrality, transferability, and connectivity. It is generally understood as a deep, overarching idea. Unit teaching design is a systematic arrangement of instruction aimed at helping students comprehensively understand and master knowledge by integrating teaching content, methods, and assessment.

Research on unit-based instructional design has been conducted relatively early and extensively. In the early stages, Ovide Decroly (1992) proposed the holistic principle in education, emphasizing that teaching activities should revolve around knowledge units. John Dewey (1910) developed the "unit teaching method" in practice, while William Kilpatrick (1918) specified concrete steps for implementing unit teaching. Henry Morrison (1956) introduced the "Five-Step Unit Teaching Method," and Benjamin Bloom established operational norms for unit instruction. Research on big ideas also emerged relatively early. Jerome Bruner (1960) argued that students should be guided to promote skill transfer through the fundamental structure of disciplines. Scholars like Grant Wiggins (2005) proposed that big ideas can organically connect fragmented knowledge points. Notably, big ideas not only possess deep-level and transferable characteristics but can also be applied flexibly across various contexts, providing new perspectives for curriculum design.

Scholars believe that Big Ideas can effectively guide the structure and design of unit teaching. Although research on unit teaching and Big Ideas has made certain progress, there are still shortcomings in both theoretical depth and practical breadth, especially regarding how to effectively combine Big Ideas with unit teaching and explore more operational teaching models. This study, based on Big Ideas in high school mathematics unit teaching design, aims to enhance the overall and systematic nature of mathematics instruction through the application of Big Ideas. The research delves into the teaching design of the functions unit, helping teachers overcome fragmented knowledge points and construct a systematic mathematical thinking framework, thereby supporting the improvement of classroom quality and the implementation of core mathematical literacy.

2. Analysis of the Lesson on "The Graph and Properties of Exponential Functions."

Under the guidance of Big Ideas teaching theory, this lesson design aims to help students form a systematic understanding of the learning content. The teaching design aligns with students' cognitive development, advocates for independent thinking, self-directed learning, cooperative communication, and other diverse learning methods, all of which promote the development of students' innovative thinking. Multiple teaching methods are employed simultaneously in this design, leveraging information technology to improve teaching efficiency and enhance students' ability to solve real-world mathematical problems. At the same time, the lesson design includes layered learning content, allowing students at different levels to develop mathematically in varied ways.

2.1 Analysis of Lesson Content

Drawing an analogy with the process of studying the properties of power functions, this lesson focuses on studying the graph of exponential functions from both the "number" and "shape" perspectives. Moving from the specific to the general, GeoGebra is used to graph several exponential functions with different bases, allowing students to observe the position, common points, and trends of the graphs. By identifying the commonalities, students can summarize the properties of exponential functions. The next step is to apply these properties. The content of this lesson involves observing the graph, generalizing the properties, and then using these properties to further understand the graph. In other words, it emphasizes the idea of "using shape to assist number" and "using number to assist shape," highlighting the concept of integrating numerical and graphical representations. Through the connections between analytical expressions, graphs, and properties, students gain a deeper understanding of the nature of functions and the characteristics of function models.

2.2 Analysis of Learner's Situation for the Lesson

The target students for this lesson are first-year high school students who have already studied quadratic functions, second-degree functions, and inverse proportional functions. Through the first three chapters of high school mathematics, students have gained a preliminary understanding of the concept and properties of functions and are able to apply functions to solve real-world problems. The concept of exponentiation is not unfamiliar to the students, so the learning of the concept of exponential functions follows naturally. Guided by the Big Ideas approach, it is therefore appropriate for us to study the graph and properties of exponential functions next.

2.3 Key Points and Difficult Points of the Lesson

Key focus: Guide students to understand and master the concept of exponential functions, be able to plot the graph of an exponential function, and understand the properties of exponential functions through its graph.

Difficulty: Using the method of combining number and shape, move from the specific to the general, and generalize the properties of exponential functions.

3. The Teaching Design of the Graph and Properties of Exponential Functions.

3.1 Review previous knowledge and introduce the topic

Question 1: By analogy with power functions, what is the general path of studying functions that we have learned so far?

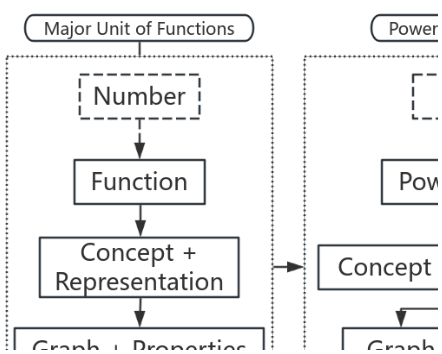

Figure 1. Mind map

Question 2: In this chapter, we first learn about exponents, then move on to exponential functions. What is the concept of an exponential function?

Tip: Generally, a function in the form of is called an exponential function, where the exponent is the independent variable and is the dependent variable.

Question 3: After learning the concept, what should we study next according to our experience?

Tip: The graph and properties of exponential functions.

Design Intention: Through a series of question strings, students can form a broad concept of function learning. By making an analogy with power functions, they can clarify the general approach to exploring the graphs and properties of exponential functions. This can enhance students' logical thinking ability, permeate the idea of the combination of numbers and shapes, and make the learning of this class come naturally.

3.2 Scenario-Driven Image Exploration

Question 4: By analogy with the process of studying power functions, draw the graphs of $y = 2^x$ and $y = 1/2^x$ in the same rectangular coordinate system, and try to summarize the properties of the function graphs.

x	…	−3	−2	−1	0	1	2	3	…
$y=2^x$	…	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	…
$y=\left(\frac{1}{2}\right)^x$	…	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	…

Question 5: Observe the graphs of and. What's the relationship between them? Can we draw the graph of using the graph of the exponential function?

Question 6: On the basis of the known graph of $y = 3^x$, can you draw the graph of $y = (1/3)^x$ without using the point-plotting method? Besides the graph, can you use the analytical formula to explain the above- mentioned symmetry relationship from an algebraic perspective?

Design intention: Draw the graph by the point-plotting method to cultivate the core literacy of intuitive imagination. Let students summarize on their own through the method of combining numbers and shapes, and permeate the ideas of classification discussion and inductive reasoning. Perceive the symmetry of the graphs of exponential functions whose bases are reciprocal to each other.

3.3 Compare, analyze and summarize the properties.

Question 7: Put the graphs of and $y = 2^x, y = (1/2)^x, y = 3^x, y = (1/3)^x$ in one graph. Conduct a preliminary exploration of the properties of specific functions, as well as the influence of the magnitude of the base on the position of the graph.

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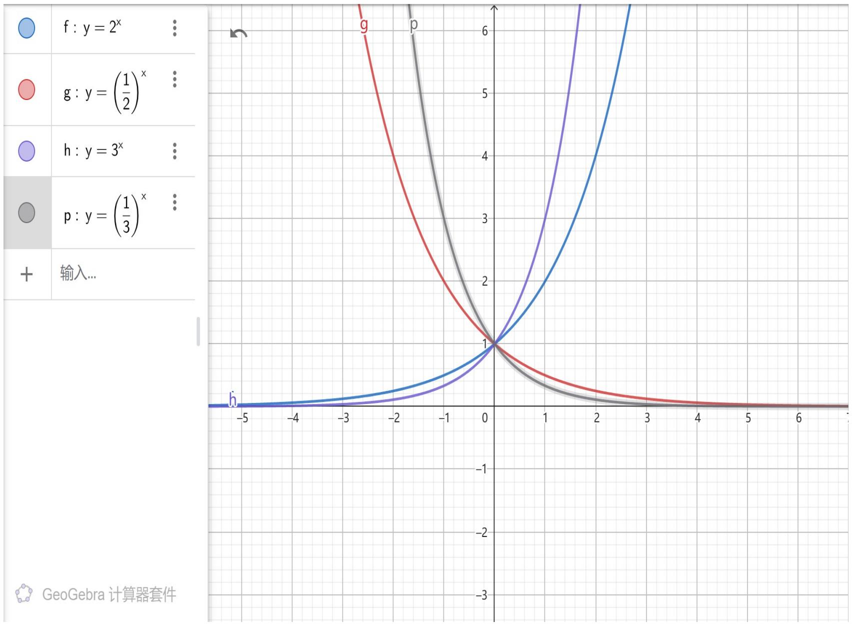

Figure 2. Comparative Analysis of Four Exponential Function Graphs

Question 8: We've already drawn the graphs of four functions. Now, using information technology, select different base values and plot the corresponding exponential function graphs in the same coordinate system. Observe these graphs. What common features do they have? Can you summarize the properties of exponential functions?

Design intention: The teacher uses the Geogebra software to dynamically display the changing graphs of exponential functions, creating a natural transition from the specific to the general. This enables students to intuitively and independently summarize the general properties of exponential functions.

3.4 Analyze the example problems to deepen the application.

Question 9: Compare the magnitudes of the following two values.

$1.7^{2.5}$ ____ 1.7^{3} , $1.7^{0.3}$ ____ $0.9^{3.1}$, $0.8^{-\sqrt{2}}$ ____ $0.8^{-\sqrt{3}}$, $0.8^{\frac{1}{2}}$ ____ $0.7^{\frac{1}{2}}$

Design intention: Through sample problems, consolidate students' understanding of the knowledge, guide them to reflect on the model assumptions, and cultivate a scientific attitude.

3.5 Summarize the class to sublimate the ideas.

Question 10: What are the basic approaches, research contents, and research methods for studying a function that we have learned in this class?

Tip:

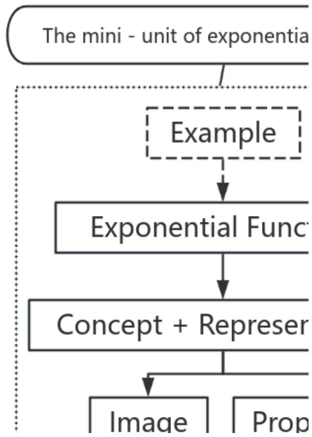

Figure 3. Exponential function learning framework

Design intention: Through mind maps, it helps students form broad concepts, targets the core competencies, and lays a foundation for unit learning.

3.6 The Design of Blackboard Writing for a Class Period

<table>
<tr><td colspan="3">(1) The graphs of the exponential functions $y = a^x$ and $y=\left(\frac{1}{a}\right)^x$ are symmetrical about the axis
(2) The graphs and properties of exponential functions.</td></tr>
<tr><td></td><td>$a>1$</td><td>$0<a<1$</td></tr>
<tr><td>Graph</td><td></td><td></td></tr>
<tr><td rowspan="6">Characteristic</td><td>(1) Domain:</td><td></td></tr>
<tr><td colspan="2">(2) Range:</td></tr>
<tr><td colspan="2">(3) Fixed point:</td></tr>
<tr><td>(4) Monotonicity:</td><td>(5) Monotonicity:</td></tr>
<tr><td>(6) Parity:</td><td>(7) Parity:</td></tr>
<tr><td>(8) $X>0$, y
(9) $X<0$, y</td><td>(10) $X>0$, y
(11) $X<0$, y</td></tr>
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4. Conclusion

In summary, this paper explores the application of Big Ideas in high school mathematics instruction, using "The Graph and Properties of Exponential Functions" as a specific case. The study emphasizes how Big Ideas can guide the design and structure of unit teaching, promoting a more comprehensive understanding of mathematical concepts. This paper utilizes the framework of Big Ideas for unit introduction and, at the same time, employs dynamic tools like GeoGebra to analyze and visualize the graphs of exponential functions. Students are able to deduce general properties from both algebraic and geometric perspectives, thereby deepening their understanding of mathematical functions. This approach not only supports the development of core competencies but also encourages independent thinking, problem-solving, and the application of mathematical concepts to real-world situations. Ultimately, the goal of this paper is to enhance the quality of mathematics instruction, cultivate a systematic approach to learning, and provide teaching materials for educators.

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