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Navigating the Liminal Space: Strategies to Support Learners in Calculus at Chishinga Secondary School in Kawambwa District

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Abstract
This study investigates the challenges and pedagogical strategies associated with calculus learning at Chishinga Secondary School in Kawambwa District, Zambia, focusing on the concept of "liminal space", a transitional phase where learners grapple with difficult, transformative concepts that define their mastery of the subject. The research addresses three key questions: the factors influencing the emergence of liminal space in calculus, the specific barriers students encounter within this space, and the instructional strategies that can support students in overcoming these challenges. A mixed-methods approach was used, incorporating surveys, pre and posttests for 100 learners and, interviews with three (3) mathematics teachers to gain a nuanced understanding of the calculus learning experience. The findings reveal that both internal factors, such as prior algebra and trigonometry knowledge (foundational knowledge) and motivation, and external factors, like teaching methods and curriculum structure, contribute significantly to students experiences in the liminal space. Students frequently reported cognitive overload, misconceptions, and emotional challenges, including anxiety and frustration, particularly with threshold concepts such as limits, derivatives, and integrals. Teachers, too, observed that students often misunderstand these foundational ideas, which hinders their progress and affects their overall attitude toward calculus.The study identifies a range of effective instructional strategies for helping students navigate this challenging phase. Active learning, inquiry-based methods, and peer-assisted activities were found to enhance engagement and deepen understanding. Formative assessments and especially constructive feedback emerged as critical tools in identifying misconceptions and providing real-time guidance, while differentiated instruction and scaffolding were noted as essential in helping learners' bridge foundational gaps. Additionally, the integration of technology and visual aids proved valuable in helping students visualize abstract calculus concepts, though equitable access remains a concern. This research underscores the importance of a student-centered, multifaceted approach that includes active engagement, regular assessment, individualized support, and collaborative learning environments. The findings provide actionable insights for educators, policymakers, and curriculum developers to enhance calculus instruction and support students in navigating the complexities of mathematical learning.
Keywords: Liminal space, calculus education, threshold concepts, formative assessment, Chishinga Secondary School.
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1. Introduction

Chishinga Secondary School in Kawambwa district in Zambia faces persistent underperformance in mathematics, particularly in calculus, a critical area of the curriculum. For instance, the Examination Council of Zambia (2022) reported that mathematics recorded the lowest mean score of 27.51% in 2022, with a failure rate of 42.08% at school certificate level. This disturbing trend is attributed to learners' inability to master fundamental calculus concepts such as differentiation and integration. Specifically, learners struggle with applying rules for integrating terms of integer powers, interpreting definite integrals, and utilizing correct mathematical notation (ECZ, 2022). Additionally, a number of learners fail to comprehend and interpret terms like "evaluate" and concepts such as finding the normal to a curve, demonstrating a lack of foundational skills necessary for conceptual understanding. The challenges learners face highlight the liminal space between understanding and confusion, where they experience oscillation between old and new knowledge, often leading to frustration, anxiety, and disengagement (Land, Rattray, & Viavian, 2014; Dweck, 2006). These emotional and cognitive barriers significantly impede learners' progression and overall academic achievement (Boaler, 2008). Despite efforts such as curriculum revisions and pedagogical improvements, the issue persists, underscoring the urgent need for innovative instructional strategies to guide learners through the liminal space and facilitate their mastery of calculus concepts (Keane, Flynn, & Kealy, 2023). Without addressing these gaps, learners risk continued academic underachievement and limited professional opportunities in STEM-related fields (Changwe & Mwanza, 2019). This research explored factors influencing the emergence of the liminal space, the barriers learners face within it, and the strategies that can support them.
The study addresses three key questions:

1. What factors influence the emergence of learners' liminal space within the learning and teaching of calculus at Chishinga Secondary School?
2. What are the key challenges or barriers that learners encounter in the liminal space as they are learning calculus?
3. What strategies can be employed to teach learners who are transitioning in the liminal space?

2. Methodology

2.1 Research design

The choice of the design depended on the nature of the investigation. For this study, a mixed method approach was used to provide understanding of the strategies to support learners in learning calculus. This allowed for both qualitative exploration of learner experiences and quantitative analysis of strategies that can be employed in helping learners navigate the liminal space.

The qualitative component employed an interview with the teachers of these learners, who have experienced or are currently experiencing the liminal space of threshold concepts in calculus at Chishinga secondary school. The questions from the interviews allowed for an in-depth exploration of individual experiences, challenges, and coping strategies that teachers see their learners go through.

The quantitative component involved the distribution of a structured questionnaire to a larger sample of learners who were subjected to a pre and post-test. The questionnaire was designed to gather quantitative data on challenges, strategies, and the perceived strategies that can be employed. A 5 level likert scale questions was utilized to allow participants to express their opinions on a scale.

2.2 Participants

The selection of participants was crucial to ensure the study captures a diverse range of perspectives and experiences. Learners were selected using the stratified random sampling, I had put the learners in their respective grades and use a random number to select participants proportionally from each strata. The school had a population of 350 learners in higher grades, grade 10 to grade 12. A larger sample, comprised of at least 100 learners, was selected for the pre-test, post-test and questionnaire phase (Cresswell, 2013). Participants were drawn from both grade 11 and grade 12 to ensure representation and diversity in the responses.

As for teachers the interviews was conducted to all of them as there were only 3 mathematics teachers at Chishinga Secondary School. This was in line with (Cresswell, 2013), who pointed out that, if we are limited to a small sample size, qualitative research methods such case studies, intervews and focus group discussions might be appropriate. Why is it so, because it allows for an in-depth exploration of participants experiences and perspectives.

2.3 Data collection and treatment

To gather relevant data, a combination of pre and post-test, surveys and interviews was used. The qualitative component employed interviews, which were recorded using an audio recording application and analyzed using thematic analysis to identify recurring patterns, themes, and insights. The structured questionnaire and pre test and post-tests were distributed physically to the participants. The Likert scale responses were collected and subjected to statistical analysis, including descriptive statistics and inferential tests, to identify trends and correlations.

2.4 Instrumentation.

The selection of appropriate instruments for data collection and analysis was essential to ensure the validity and reliability of findings. To check the validity of the information triangulation was applied, which is referred to as the use of multiple methods or data sources in qualitative research to develop a comprehensive understanding of phenomena (Denzin, 1978). This helps in validating the documents that has to be reviewed. A pilot study was conducted to ensure the validity and reliability of the instruments. The pilot study was conducted at Kanengo secondary school in Kawambwa district which is the nearest school to the researcher, and it was conducted to a class of grade 12 pupils which brought about the survey questionnaire and pre and posttest to be in refined form.

3. Results of the study

To comprehensively address the research questions, the findings are presented thematically, drawing on both quantitative data from the survey, pretest and posttest and qualitative insights from interviews. This thematic approach allowed for triangulation of data, enabling a richer understanding of the phenomena under study. Each research question was addressed through corresponding themes that emerged from both data sources.

3.1 Research question one: What factors influence the emergence of liminal space in Calculus learning and teaching?

To answer this question the pretest and posttest were administered to the 100 learners and the following were the results.

Table 1. Descriptive statistics for pre and posttests

Descriptive Statistics					
	N	Minimum	Maximum	Mean	Std. Deviation
pretest marks	100	.00	84.00	41.6000	18.99069
Posttest mark	100	.00	91.00	51.5100	18.92062

The pretest mean score of 41.60% suggested limited prior understanding of calculus. The high standard deviation (SD = 18.99) indicated substantial variability. The posttest mean of 51.51% reflected notable improvement in understanding after instruction, with a slightly lower SD (18.92), indicating more consistent performance across learners.

Table 2. Paired samples t test

Paired Samples Test										
		Paired Differences					t	df	Significance	
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				One-Sided p	Two-Sided p
					Lower	Upper				
Pair 1	pretest marks - posttest mark	-9.91000	17.20588	1.72059	-13.32402	-6.49598	-5.760	99	<.001	<.001

To determine whether the improvement in scores from the pre-test to the post-test was statistically significant, a paired t-test was conducted and the following were the results. The statistical hypothesis considering the Null: There is no significant difference between the pre and posttests, while the Alternative: There is a significant difference between the pre and posttests.

Table 2 shows that the calculated p-value of t-test(-5.760) is 0.001 which is less than 5% level of significance so we reject the null hypothesis and which means the results indicate a statistically significant improvement in scores from the pre-test to the post-test. This suggests that the teaching intervention had a positive effect on learners' calculus understanding.

The analysis of pre- and post-test data reveals a marked improvement in calculus understanding among learners following the teaching intervention. The post-test scores display not only a higher mean but also a reduced variability, indicating a more consistent level of improvement across the sample.

Theme 1: Foundational knowledge in mathematics

Table 3. Mean and Standard deviation for foundational knowledge in mathematics

Descriptive Statistics			
	N	Mean	Std. Deviation
Foundational knowledge in mathematics	100	3.4800	0.70164

Learners reported a moderate level of foundational mathematical knowledge (M = 3.48, SD = 0.70). However, variability in responses suggested differing levels of preparedness among learners.

Table 4. Independent t-test for foundational knowledge in mathematics

Results showed a statistically significant gender difference (t(80.77) = 2.799, p = .006). This suggests foundational knowledge levels may differ between male and female students, potentially influencing their navigation through liminal space in calculus

Theme 2. Abstract nature of Calculus

The descriptive analysis revealed a mean score of 2.90 (SD = 1.17) for the abstract nature of calculus concepts based on responses from a sample of 100 participants. This suggests that participants generally perceive calculus as moderately abstract. These results align with concerns expressed during interviews, where participants highlighted the challenges posed by the abstractness of calculus. Many noted that this abstraction often alienates students, making it difficult for them to connect mathematical theories to real-world applications. This perceived disconnect not only diminishes the relevance of calculus in students' daily lives but also negatively impacts their engagement and overall understanding of the subject.

Theme 3. Emotional factors and self-efficacy

Descriptive statistics revealed a mean score of 2.84 (SD = 0.67) for emotional factors and self-efficacy, based on a sample of 100 participants.

Qualitative data from interviews further illuminated these findings. It was observed that students often experience frustration, particularly when they repeatedly make mistakes, leading to a sense of defeat and diminished self-efficacy and that anxiety is another critical factor affecting self-efficacy. Students often feel overwhelmed during assessments, facing pressure to quickly grasp complex concepts.

Theme 4. Teaching and classroom management

Descriptive statistics indicated a mean score of 3.11 (SD = 0.99) for teaching and classroom management strategies, based on responses from 100 participants. This suggested that participants generally perceive these strategies as moderately effective. An independent samples t-test revealed no statistically significant differences between groups, $t(98)=-0.87, p=.389$, with a mean difference of -0.17 (95% CI [-0.56, 0.22]). This indicates uniformity in perceptions of teaching and classroom management across different classroom environments.

Qualitative data provided deeper insights into the strategies employed by teachers to support student learning. One teacher emphasized the use of step-by-step examples to break down complex calculus problems, helping students build confidence incrementally. Additionally, formative assessment techniques, such as quick quizzes and exit tickets, were used to gauge understanding and tailor subsequent instruction. He also highlighted the importance of visual learning tools, including graphs and calculators, to help students connect abstract calculus concepts to real-world applications. A supportive environment was fostered by providing immediate, specific feedback, which focused on highlighting correct responses before addressing errors to maintain student motivation.

Another teacher complemented these strategies with the use of analogies to simplify abstract concepts, such as describing limits as "approaching a goal," which made the material more relatable. Regular formative assessments, including mini-projects, allowed him to track student understanding throughout each unit and provide timely interventions. Visual learning tools, were employed to help students visualize functions and their changes, enhancing their comprehension of calculus concepts. Differentiated practice catered to diverse learning needs, offering varying levels of difficulty to accommodate both struggling and advanced students. Constructive feedback was a continuous process, with detailed comments encouraging improvement and fostering a growth mindset. Additionally, he focused on creating a positive classroom culture where students felt comfortable sharing challenges, thus promoting an open and collaborative learning environment.

3.2 Research question two: What key challenges or barriers do learners encounter in the liminal space?

Theme 5. Cognitive challenges

Descriptive statistics revealed a mean score of 2.88 (SD = 0.92) for cognitive challenges, based on responses from a sample of 100 participants. This suggests that students experience moderate levels of cognitive difficulties in mastering calculus concepts.

Insights from interviews highlighted the specific nature of these cognitive challenges. A recurring issue identified was students' weak algebra skills, which critically impede their ability to manipulate equations, a fundamental requirement for solving calculus problems. This deficiency not only makes it harder for students to grasp calculus concepts but also creates a cycle of frustration, where repeated struggles lead to disengagement from the subject. The abstract nature of calculus further exacerbates these challenges, as students often struggle to connect theoretical mathematical principles with practical applications. Without a solid foundation in prerequisite skills like algebra and logical reasoning, students find it difficult to interpret and solve complex calculus problems, which hinders their overall academic progress.

Theme 6. Emotional and psychological barriers

Descriptive statistics revealed a mean score of 3.21 (SD = 0.93), indicating that participants perceive these barriers as moderately impactful. An independent samples t-test found no statistically significant differences between groups, $t(98)=0.88, p=.383$t(98) = 0.88, p = .383, with a mean difference of 0.16 (95% CI [-0.21, 0.54]). This suggests that emotional and psychological barriers are experienced consistently across different student groups.

Qualitative data provided a deeper understanding of these challenges. The interviewee described adopting a proactive approach to identifying emotional barriers by actively observing signs of confusion or frustration during classroom discussions and individual tasks. To mitigate these barriers, peer support systems, such as tutoring and small group discussions, were implemented. These strategies not only reinforce conceptual understanding but also provide emotional support, particularly for students transitioning from algebra to calculus.

Another interviewee noted that students often struggle to connect calculus concepts with their prior knowledge, especially with derivatives and integrals. This disconnect can lead to feelings of anxiety and overwhelm, further impeding their learning process. A specific misconception identified was the belief that derivatives are only applicable to linear relationships, limiting students' understanding of their broader applications and exacerbating psychological barriers.

Theme 7. Difficulty grasping abstract concepts

Descriptive statistics revealed a mean score of 3.15 (SD = 0.89), suggesting that participants perceive this challenge as moderately significant. An independent samples t-test showed no statistically significant differences between groups, $t(98)=0.18, p=.858$, with a mean difference of 0.03 (95% CI [-0.32, 0.39]). These results suggest that difficulties in understanding abstract concepts are consistently experienced across participant groups.

Qualitative findings provided nuanced insights into these challenges. The interviewed teacher highlighted specific struggles with understanding limits and derivatives, noting that students often find it difficult to conceptualize limits as approaching a value rather than reaching it. Additionally, a lack of strong algebra skills was identified as a significant barrier, making calculus problems seem daunting for students who struggle with foundational mathematical operations. Common misconceptions, such as viewing derivatives as simple differences or misunderstanding limits as finite values, were also frequently observed.

Another teacher noted similar challenges, particularly in relation to derivatives. Students often confuse them with operations from earlier mathematics courses, which stems from insufficient algebra foundations. Furthermore, the abstract terms and symbols used in calculus were described as intimidating, leading to hesitation and disengagement.

Theme 8. Overreliance on memorization

The study explored the impact of overreliance on memorization in learning calculus. The descriptive statistics showed a mean score of 3.59 (SD = 0.87), indicating that participants perceived this as a moderately significant issue. An independent samples t-test found no significant differences across groups, $t(98)=0.73, p=.469$, with a mean difference of 0.13 (95% CI [-0.22, 0.48]). These results suggest that the tendency to rely on memorization is consistent among students.

Qualitative data shed light on the implications of this behavior. Many students mistakenly equate success in calculus with the ability to memorize formulas, neglecting the understanding of underlying mathematical principles. The teacher noted that this overreliance on rote memorization often leads to frequent errors when students are required to apply formulas in unfamiliar contexts. For example, students may correctly recall a formula but fail to understand how or why it is used, which limits their ability to solve complex problems. Similarly another teacher observed similar patterns, emphasizing that students who focus solely on memorization tend to struggle with higher-order thinking tasks that require the integration of concepts. This approach not only hinders their ability to apply calculus in practical scenarios but also prevents them from appreciating the subject's broader significance and utility.

3.3 Research question three: What strategies can be employed to help learners who are transitioning?

Theme 9. Utilizing Conceptual and visual approaches

The study investigated the role of conceptual and visual approaches in learning calculus. Descriptive statistics revealed a mean score of 3.23 (SD = 0.89), suggesting moderate importance placed on these methods by participants. An independent samples t-test showed no significant differences across groups, $t(98)=-0.57, p=.570$, with a mean difference of -0.10 (95% CI [-0.46, 0.25]). These results indicate a consistent perception of the value of conceptual and visual approaches among participants.

Qualitative insights further highlighted the importance of these strategies. As one teacher emphasized the effectiveness of dynamic graphing tools, such as graphing calculators, in helping students visualize functions and their transformations. These tools bridge the gap between abstract concepts and tangible representations, enabling students to better comprehend complex ideas like derivatives, integrals, and limits. For example, visualizing the slope of a tangent line in real time helps students grasp the concept of derivatives beyond mere symbolic representations. While another teacher similarly noted that conceptual approaches, such as using real-world analogies and interactive simulations, foster deeper understanding. For instance, describing limits as "approaching a goal" or using animations to illustrate area under a curve provides students with relatable and intuitive ways to engage with calculus concepts.

Theme 10. Building on foundational knowledge

The role of foundational knowledge in calculus learning was examined, with descriptive statistics showing a mean score of M=3.23 (SD = 0.89). This indicates a moderately high perception of the importance of foundational knowledge among participants. Independent samples t-test results revealed no significant differences between groups, $t(98)=-0.57, p=.570$ with a mean difference of -0.10 (95% CI [-0.46, 0.25]). These findings suggest that participants across different groups share a uniform view on the importance of building foundational knowledge.

The interviewee emphasized the necessity of reinforcing key algebraic and trigonometric concepts before delving into calculus topics. Students with a solid grasp of algebraic manipulation and function properties were found to transition more smoothly into understanding derivatives and integrals. For example, revisiting the concept of slope in algebra and linking it to the derivative as a rate of change helped demystify the abstract nature of calculus while, another interviewee highlighted the importance of scaffolding, where lessons are structured to gradually build complexity. By introducing fundamental principle-such as limits-as intuitive concepts before progressing to formal definitions and applications, students were better able to grasp the material. Additionally, practice sessions focusing on prerequisite skills, such as solving equations or understanding geometric interpretations, were found to significantly boost student confidence.

Theme 11. Incorporating Formative assessment

The importance of formative assessment in teaching practices was analyzed, with participants reporting a mean score of M=3.73 and (SD = 0.99), indicating a high perception of its value in educational settings.

Independent samples t-test results showed no significant differences in perceptions across groups, $t(98)=-0.31, p=.757$, with a mean difference of -0.06 (95% CI [-0.46, 0.33]). This suggests consistent recognition of the importance of formative assessments among participants.

The first teacher reported using formative assessment tools such as exit tickets, short quizzes, and group discussions to evaluate students' understanding of calculus concepts. Immediate feedback on these assessments provided opportunities to clarify misconceptions, ensuring that students had a solid grasp of foundational concepts before progressing while, the second highlighted the use of project-based formative assessments, where students collaboratively worked on calculus problems to demonstrate their understanding. This approach not only facilitated peer learning but also allowed the teacher to identify and address common areas of difficulty. Incorporating formative assessment practices into classroom teaching offers several benefits: Real-Time Feedback, customized Instruction and increased Engagement.

Theme 12. Collaborative and peer learning

The results of the analysis on collaborative and peer learning revealed a mean score of 3.76 (SD = 0.80) across the 100 participants. This score indicates a moderate to high level of engagement with collaborative learning practices among the respondents.

To assess whether there were any significant differences in collaborative and peer learning between different groups, an independent samples t-test was conducted. The results showed no significant differences between the groups, $t(98) = 1.01$, $p = .315$. This suggests that the perceived benefits of collaborative and peer learning did not significantly vary between the groups being compared. The mean difference between the two groups was 0.16, with a 95% confidence interval ranging from -0.16 to 0.48. Given that the p-value is greater than the conventional significance level of 0.05, it can be concluded that there were no significant variations in the responses related to collaborative and peer learning.

While the statistical analysis did not reveal significant differences, the literature highlights the importance of fostering collaborative and peer learning environments. Encouraging peer interactions in educational settings allows students to support one another in understanding difficult concepts, promoting a sense of community and shared responsibility for learning. As suggested by previous studies, pairing students for collaborative tasks can enhance learning outcomes by creating a more interactive and supportive classroom dynamic.

Theme 13. Providing constructive feedback

Table 5. Mean and SD for providing constructive feedback

Descriptive Statistics			
	N	Mean	Std. Deviation
Providing constructive feedback	100	3.4550	1.44965
Valid N (listwise)	100		

The analysis of participants' responses regarding the provision of constructive feedback revealed a mean score of 3.46 (SD = 1.45). This suggests that participants generally view constructive feedback practices moderately favorably, though there is notable variability in their responses, as indicated by the standard deviation

4. Discussion

4.1 Discussion of Research Question 1: What Factors Influence the Emergence of Liminal Space in Calculus Learning?

The increase in learners' mean scores post-intervention suggests that a clear and focused teaching approach can positively influence learners as they navigate theliminal space" a transitional learning phase where learners move from limited to proficient understanding. This result aligns with threshold concept theory, which highlights the importance of clear conceptual teaching to help learners overcome initial confusion in new topics. The discussion of research question one will be summarized in one theme: **Navigating the multifaceted pathways to Calculus mastery.** This theme will encapsulate the interconnected factors influencing learners' journey through calculus, emphasizing the interplay of foundational knowledge, the abstract nature of calculus, emotional resilience and self-efficacy and instructional strategies. As such we discuss each of the components of this theme.

4.1.1 Foundational Knowledge in Mathematics

Foundational knowledge in mathematics is critical for learners to successfully transition through liminal spaces when engaging with advanced topics like calculus. A lack of foundational understanding prevents learners from developing relational understanding, leading them to rely on procedural methods without conceptual clarity. This aligns with the findings of (Tall, 2013) who emphasized that difficulties in calculus often arise from an insufficient grasp of basic mathematical principles, such as algebra, functions, and arithmetic. Without a strong foundation, learners are unable to build the cognitive connections required to navigate the abstract nature of calculus.

The statistically significant difference between the groups suggests that certain external factors, such as **prior mathematical exposure and teaching quality**, play a role in learners' foundational knowledge. For instance, (Kilpatrick, Swafford, & Findell, 2001) argue that disparities in foundational mathematical skills often stem from differences in instructional practices, resource availability, and learners' prior experiences with mathematics. Similarly, (Anthony & Walshaw, 2009) highlight that gaps in foundational understanding are further exacerbated when learners are introduced to abstract concepts like limits, derivatives, and integrals, which require a deeper conceptual and procedural comprehension.

The moderate performance observed in this study underscores the need for interventions that focus on strengthening foundational knowledge as a prerequisite for calculus learning. (Feldman & Newcomb, 2017) advocate for instructional strategies that emphasize conceptual understanding over rote memorization, as this approach better prepares learners for the cognitive challenges of higher-level mathematics. Targeted support, such as formative assessments and tailored feedback, could help identify and address individual gaps in foundational knowledge, ultimately improving learners' ability to navigate the liminal space in calculus.

In conclusion, the findings highlight the importance of addressing foundational mathematical skills as a core component of calculus instruction. Consistent with the literature, the results demonstrate that learners' ability to engage meaningfully with calculus is directly influenced by their foundational knowledge. Addressing these gaps through effective teaching strategies and supportive learning environments can enhance learners' conceptual understanding and facilitate smoother transitions through liminal spaces in mathematics learning (Tall, 2013).

4.1.2 Abstract Nature of Calculus Concepts

The findings of this study underscore the significant role the abstract nature of calculus plays in shaping learners' perceptions and experiences. The study indicated a moderate abstraction, as it aligns with the literature emphasizing the cognitive demands imposed by calculus' threshold concepts such as limits, derivatives, and integrals. (Meyer & Land, 2003) describe threshold concepts as transformative yet troublesome, often creating cognitive dissonance that complicates learners' ability to reconcile new ideas with prior knowledge. This abstraction, as identified by (Tall, 2013), is exacerbated by the need to navigate between conceptual, symbolic, and formal-axiomatic representations of calculus.

The statistically significant differences in group perceptions suggest that external factors, such as teaching methods and curriculum design, could influence how abstraction is experienced. Interviews with participants further highlighted that abstraction often alienates students, making calculus seem disconnected from real-world applications. This detachment aligns with findings by (Land, Rattray, & Viavian, 2014), who argue that abstraction can amplify learners' frustrations, especially when concepts are presented in isolation without linking them to relatable contexts.

4.1.3 Emotional Factors and Self-Efficacy

The emotional landscape of calculus learning reveals moderate challenges, these findings resonate with the literature, particularly (Cousin, 2006) and (Savin-Baden, 2008), who emphasize the emotional dimensions of liminal spaces. Negative emotions such as frustration and anxiety, exacerbated by repeated failures, contribute to a diminished sense of self-efficacy. For instance, the teacher's observations on students frustrations when making repeated mistakes align with (Vygotsky, 1978) assertion that learners require consistent scaffolding to bridge the Zone of Proximal Development (ZPD). Without this support, the liminal space becomes a site of oscillation between old and new understandings, as described by (O'mahony, Buchanan, O'Rourke, & Higgs, 2011).

The study's qualitative insights into anxiety particularly its link to assessments and symbolic representations underscore the significance of emotional barriers in mathematics education. Learners' struggles with abstract terms and their inherent intimidation reflect (Perkins, 2006) characterization of troublesome knowledge. To mitigate these emotional barriers, educators must foster supportive environments that prioritize incremental successes, as highlighted by (Dweck, 2006) growth mindset framework. Providing constructive feedback focused on effort and progress rather than outcomes can bolster resilience and self-efficacy.

4.1.4 Teaching Strategies and Classroom Management

The role of teaching strategies in mitigating the abstract and emotional challenges of calculus was evident, with participants rating their effectiveness at a moderate level, however, the lack of significant group differences suggested a uniform perception of teaching practices across different contexts. This aligns with literature highlighting the critical need for tailored interventions, as ineffective pedagogical approaches exacerbate learners' struggles in navigating threshold concepts (Sierpinska, 1994).

The qualitative data reveal how scaffolding, visual aids, and formative assessments enhance comprehension. Teachers' use of step-by-step examples and visual tools corresponds with Sweller's (1988) Cognitive Load Theory, which emphasizes reducing extraneous cognitive load to optimize germane processing. Additionally, the use of analogies and differentiated practice aligns with Vygotsky (1978) ZPD framework, ensuring that support is appropriately calibrated to individual learner needs.

Moreover, the integration of real-world applications and collaborative environments is crucial for bridging the abstraction-relevance gap identified in the findings. Active learning strategies, such as group problem-solving and technology-enhanced instruction, resonate with Sfard (2008) commognitive framework, which advocates for discourse-rich classrooms to facilitate transitions through liminal spaces. These strategies not only improve conceptual understanding but also address the emotional dimensions of learning by fostering a supportive and engaging classroom culture.

The findings highlight the intertwined nature of cognitive, emotional, and pedagogical factors in shaping learners' calculus experiences. Addressing the abstract nature of calculus requires integrating visual aids, analogies, and real-world applications to make concepts more tangible and relatable. As Meyer & Land (2003) argue, scaffolding transitions through threshold concepts demands instructional designs that prioritize coherence and relevance.

Emotional factors, particularly frustration and anxiety, necessitate targeted interventions to build resilience and self-efficacy. Growth mindset strategies, such as emphasizing effort and incremental progress, can help learners overcome perceived barriers. The inclusion of collaborative activities and peer support structures further reduces feelings of isolation and enhances motivation, as supported by Vygotsky's social constructivist theory.

4.2 Discussion of Research Question 2: What Challenges Do Learners Encounter in the Liminal Space of Calculus?

The pre-test findings, with a relatively low mean score and a high standard deviation reflect the initial challenges learners faced with calculus concepts, likely due to inadequate foundational knowledge or difficulties with complex ideas like limits, derivatives, and integrals. This phase of struggle is characteristic of the liminal space in threshold learning, where the learners learning process is marked by confusion, frustration, and uncertainty.

The improvement in post-test results indicates that the intervention may have addressed some of these barriers by using structured instruction, clarifying foundational knowledge, and offering additional practice in problem solving. However, the reduced but still notable standard deviation suggests that some learners continued to experience challenges, possibly due to different levels of mathematical background or varying individual cognitive strategies.

The discussion of research question one will be summarized in one theme: **Overcoming barriers to deep mathematical understanding.** This theme unifies the various challenges learners face in developing a robust understanding of mathematics, particularly calculus. It address the cognitive demands of mastering complex topics, the emotional and psychological barriers that hinder learning, the struggle with abstract conceptualization and the reliance on rote memorization at the expense of conceptual depth. We now discuss this research question in line with the components of the theme.

4.2.1 Cognitive Challenges

The findings reveal that cognitive challenges are a significant factor in students' struggles with calculus, with a mean score suggesting moderate difficulties. The statistically significant difference between groups underscores the pervasive impact of cognitive barriers on learning outcomes. These findings align with (Tall, 2013) assertion that weak foundational skills, particularly in algebra, exacerbate difficulties in calculus by hindering students' ability to manipulate equations and understand abstract concepts. This deficiency creates a cycle of frustration, leading to disengagement and reinforcing negative attitudes toward mathematics.

Qualitative insights provided further depth, identifying specific areas where students struggle. The Teacher's observations of students' weak algebra skills resonate with Perkins (2006) findings that fragmented prior knowledge hampers progression through the liminal space of calculus. Similarly, a note on students struggles to connect theoretical principles with practical applications highlights the need for contextualized learning. Addressing these challenges demands targeted support in foundational skills, step-by-step problem-solving guidance, and the integration of real-world examples to foster understanding and engagement.

4.2.2 Emotional and Psychological Barriers

Emotional and psychological barriers emerged as moderately impactful. While no significant differences between groups were observed, qualitative data emphasized the consistency of these barriers across contexts. The Teacher's proactive identification of frustration and confusion aligns with Cousin (2006) characterization of emotional challenges in liminal spaces. Strategies such as peer support systems and small group discussions were effective in addressing these barriers, reinforcing both conceptual understanding and emotional resilience and insights into students misconceptions about derivatives and their limited application of calculus concepts further underscore the role of psychological factors. Anxiety stemming from these misconceptions can lead to disengagement and a sense of inadequacy. This finding echoes (Savin-Baden (2008) work on the emotional turmoil of transitioning through threshold concepts. By connecting calculus to prior knowledge and clarifying misconceptions, educators can alleviate these barriers, fostering a more supportive learning environment that enhances confidence and participation.

4.2.3 Difficulty Grasping Abstract Concepts

The study highlights the challenge of abstract concepts in calculus. The lack of significant group differences suggests a uniform struggle among participants. Teachers' observations of students' difficulties with limits and derivatives resonate with Meyer and Land's (2003) characterization of threshold concepts as transformative yet troublesome. Common misconceptions, such as interpreting limits as finite values or derivatives as simple differences, highlight the need for pedagogical interventions, and an emphasis on students' intimidation by abstract terms and symbols further aligns with Sweller's (1988) Cognitive Load Theory, which identifies the burden of extraneous cognitive demands. Traditional teaching methods that prioritize procedural knowledge over conceptual clarity exacerbate these challenges, leaving students ill-equipped to apply calculus concepts in varied contexts. Strategies such as using visual aids, scaffolding complex ideas, and linking theory to practical applications are critical for fostering deeper understanding and engagement.

4.2.4 Overreliance on Memorization over conceptual understanding

The study identifies overreliance on memorization as a pervasive issue, qualitative data revealed the detrimental impact of this approach. It was noted that frequent errors stemming from students' superficial understanding of formulas, which aligns with Sfard's (2008) emphasis on the importance of conceptual discourse in mathematics. Similarly, Teacher B's observations of students' struggles with higher-order tasks highlight the limitations of rote learning.

Traditional instructional methods that emphasize memorization over critical thinking fail to equip students with the skills needed for problem-solving and application. This finding corroborates Sierpinska (1994) critique of procedural teaching. Addressing this issue requires a shift toward inquiry-based learning and conceptual teaching strategies, such as emphasizing the reasoning behind formulas and integrating real-world applications. These approaches not only reduce reliance on memorization but also enhance students' ability to engage with calculus meaningfully and confidently.

4.3 Discussion of Research Question 3: What Strategies Can Be Employed to Support Learners in the Liminal Space?

The discussion of research question one will be summarized in one theme: **Facilitating deep learning through collaborative and conceptual teaching strategies.** This theme integrates key approaches that support meaningful mathematical understanding. It emphasizes the importance of conceptual and visual methods to aid comprehension, the need to build on foundational knowledge, the use of formative assessment to guide instruction, fostering collaborative and peer learning for shared insights, and delivering constructive feedback to reinforce progress and address gaps. We now discuss this research questions with regard to the components of the theme.

4.3.1 Building on Foundational Knowledge

The role of foundational knowledge in calculus is highlighted by a mean score of 3.23 (SD = 0.89), reflecting a shared understanding among participants of its importance. Teachers noted that reinforcing algebraic and trigonometric skills facilitates smoother transitions to calculus topics, such as derivatives. Revisiting concepts like slope in algebra and connecting them to calculus principles addresses misconceptions and strengthens foundational understanding. On the other hand Teacher proposal on scaffolding approach, that is introducing limits as intuitive concepts before delving into formal definitions, illustrates how gradual complexity builds competence and confidence. Targeted reviews of prerequisite skills and diagnostic assessments are essential strategies for addressing gaps in foundational knowledge.

4.3.2 Incorporating Formative Assessment

The study highlights the critical role of formative assessment, with a mean score of 3.73 (SD = 0.99), indicating high recognition of its value. All teachers emphasized the importance of immediate feedback in clarifying misconceptions. The use of tools like exit tickets and quizzes aligns with Sweller's (1988) principles of managing cognitive load through targeted intervention. Project-based assessments promote peer collaboration and active engagement, allowing for a deeper understanding of calculus concepts. Incorporating formative assessments into teaching practices provides real-time feedback, supports customized instruction, and fosters continuous learning.

4.3.3 Collaborative and Peer Learning

Collaborative and peer learning received a mean score of 3.76 (SD = 0.80), highlighting its perceived effectiveness. Qualitative data affirm its benefits. Encouraging peer interactions fosters shared responsibility for learning and enhances understanding of complex concepts. Vygotsky's (1978) social constructivist theory supports the role of collaboration in cognitive development, emphasizing the importance of discourse and shared problem-solving.

4.3.4 Providing Constructive Feedback

Constructive feedback was viewed favorably, with a mean score of 3.46 (SD = 1.45), though variability in responses suggests differences in implementation. The marginally significant difference between groups indicates some variation in perceptions. Teacher A highlighted the role of immediate, specific feedback in fostering a growth mindset, aligning with Dweck's (2006) emphasis on resilience. The detailed feedback in collaborative settings encouraged students to address errors and refine their understanding. Constructive feedback helps guide learners through challenges, promoting continuous improvement and deeper engagement with calculus.

5. Recommendations

To address the challenges in calculus learning at Chishinga Secondary School, a set of actionable recommendations is proposed to enhance both teaching and learning outcomes. First, educators should prioritize strengthening students' foundational knowledge in algebra and trigonometry. Conducting diagnostic assessments at the start of the academic year can help identify gaps in prerequisite skills, enabling targeted interventions through remedial lessons and tailored practice exercises. Instruction should be scaffolded by gradually introducing calculus concepts, ensuring students build on a solid base of prior knowledge. Second, fostering emotional resilience among learners is essential. Teachers should create supportive classroom environments where students feel comfortable expressing their difficulties, reducing anxiety and frustration. Strategies such as peer support systems, group discussions, and collaborative learning activities can promote a sense of community, while regular constructive feedback focused on progress and areas for improvement can build confidence and encourage a growth mindset.

Third, instructional methods must incorporate conceptual and visual approaches to make abstract ideas more tangible. Utilizing dynamic graphing tools like GeoGebra and integrating real-world applications into lessons can help students' bridge the gap between theory and practice. Visual aids, analogies, and interactive demonstrations can further clarify complex topics, enhancing comprehension. Fourth, embedding formative assessments into teaching practices is crucial for continuous monitoring of student progress. Tools such as exit tickets, quizzes, and project-based assessments enable early identification of misconceptions and allow for timely interventions. Immediate, specific feedback on these assessments ensures that students can address errors and refine their understanding. Finally, professional development programs should focus on equipping educators with effective strategies for teaching calculus. Workshops and training sessions on scaffolding techniques, dynamic visual tools, and emotional support strategies will empower teachers to create inclusive and supportive learning environments. Collaborative efforts among teachers to share best practices and resources can further strengthen instructional quality. By implementing these recommendations, Chishinga Secondary School can establish a robust framework that addresses both cognitive and emotional barriers, empowering students to navigate the liminal space of calculus with confidence and achieve academic success.

6. Conclusion

In conclusion, the emergence of learners' liminal space in calculus is shaped by a combination of foundational knowledge gaps, cognitive challenges, and emotional barriers (Tall, 2013). Weak algebraic and logical reasoning skills, coupled with the abstract nature of calculus concepts like limits and derivatives, often create cognitive overload and uncertainty. Emotional factors, including anxiety and frustration, further hinder progression, particularly when instructional approaches prioritize rote memorization over conceptual understanding. To address these challenges, educators must adopt targeted strategies that integrate conceptual and visual teaching methods, such as dynamic graphing tools and real-world applications, to enhance comprehension (Sfard,2008). Building on foundational knowledge through scaffolding and formative assessments ensures students are prepared for advanced topics, while constructive feedback and collaborative learning foster engagement, resilience, and a growth mindset (Dweck, 2006; Vygotsky, 1978). By creating supportive and dynamic learning environments, educators can effectively guide students through the liminal space, promoting deeper understanding and confidence in calculus

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