Journal of Education and Practice
ISSN 2222-1735 (Paper) ISSN 2222-288X (Online)
Vol.4, No.17, 2013 www.iiste.org

Modelling the Relationship between Mathematical Reasoning Ability and Mathematics Attainment

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Abstract
In this article, the author seeks to explain the indicators of mathematical reasoning ability and examines the relationship, using structural regression modeling technique, between mathematical reasoning ability and students' attainment in mathematics.
The sample consisted of 240 Senior Secondary School One Students (Age 14-16 years) who were randomly selected from four senior secondary schools in Isokan and Irewole Local Government Areas of Osun State, Nigeria. A 24-item Mathematics Reasoning Ability Test (MRAT) and a 36-item Attainment in Mathematics Test (AMT) were constructed to explain the indicators of mathematical reasoning ability and to assess how far these indicators enter into success in school mathematics work. Students' scores in MRAT and AMT were analysed by using maximum likelihood estimates of LISREL version 8.88. Results from the structural regression model showed that four fundamental notions (viz: class, variable, order and classification) were measures of mathematical reasoning ability and two, success in mathematics reasoning ability reliably predicted success in mathematics attainment. These findings suggest the need for mathematics teachers to mount intervention programmes that will help students develop and improve their mathematical reasoning ability and ultimately improve their attainment in mathematics.
Keywords: Mathematical reasoning ability; Attainment in Mathematics; Mathematics learning in Nigeria; Structural regression modeling

1. Introduction

Mathematics is an excellent vehicle for the development and improvement of a person's intellectual competence in logical reasoning, spatial visualization, analysis and abstract thought. Students develop numeracy, reasoning, thinking skills, and problem solving skills through the learning and application of mathematics. These are valued not only in science and technology, but also in everyday living and in the workplace. The development of a highly skilled scientifically- and technologically `based manpower requires a strong grounding in mathematics.

The Federal Government of Nigeria realizes that she needs citizens who can demonstrate adequate knowledge of Mathematics in order for the country to cope with increasing complex nature of economic activities in the world of today. This perhaps suggests why mathematics is made a compulsory subject for all students at the secondary school level in Nigeria. This is reflected in the Nigerian National Policy on Education (Federal Government of Nigeria, [FGN], 2004). No doubt, an emphasis on functional mathematics education will ensure that Nigeria has an increasingly competitive workforce to meet the challenges of the 21st century.

However, despite the importance of mathematics, secondary school students' level of achievement, continued to be low (Uwudiae, 2012). On the average, between 2009 and 2012, less than 50% of the students who sat for Mathematics in the Senior Secondary School Certificate Examination (SSSCE) conducted by West African Examination Council (WAEC) and National Examination Council (NECO) obtained a minimum of credit pass in Mathematics. The poor trend in achievement in mathematics among secondary school students continues to attract attention from the major stakeholders (parents, researchers, teachers, and examining bodies) in education in Nigeria (Uwadiae, 2012).

In their effort to improve students' level of achievement in mathematics, researchers (Adegoke, 2011; Awofala, Awoyemi, Fatade, & Nneji, 2012) have made several suggestions. Among such suggestions include the adoption of integrative teaching method, counseling strategies and use of indigenous language in the teaching of mathematics (Adegoke, 2011). Despite all these suggestions, little or no improvement has been observed. In the on-going search for ways of improving success in mathematics, a look at the influence of mathematical reasoning ability on students' achievement success in mathematics may suggest intervention programmes that may be embarked upon by mathematics teachers. In fact, some studies such as Choudhury and Das, 2012; Heng – Yuku and Sullivan (2000) and Nunes, Byrant, Barnes, and Sylva (2012) have suggested that there is a link between mathematical reasoning ability and attainment in mathematics. Specifically Nunes, Byrant, Barnes, and Sylva (2012) in their study found out that mathematical reasoning ability reliably predicted students' achievement in mathematics. Similarly, Choudhury and Das, 2012 in their study in Malaysia found out that geometrical ability (ability to reason with spatial figures) was a good predictor of students' achievement in mathematics. Results of these studies have suggested the need for changes in the curriculum of mathematics.

In order to place this study in a very clear perspective, it is important to distinguish between ability (potential success) and attainment (actual success) in the study of any subject. According to Lee (1967) ability to succeed in a subject was defined as the power of an individual to grasp and manipulate the fundamental notions required for its study while attainment in the study of a subject, on the other hand, was taken as the acquired degree of mastery over the complexities of its conventional structure, starting from the simplest beginnings and constantly proceeding to more difficult work. Under this definition, ability is related to school work and is certainly necessary for achievement in it. This is so, because, some grasp of fundamental notions (though not explicitly demanded) is implicitly contained in the study of any subject. Lee further emphasized that attainment in a school subject entails a sequence of work, which rests on a prior acceptance of certain basic elements, concepts, and thought. At the school level, it therefore, seemed probable that an assessment of the student's success in grasping these basic notions might prove a useful pointer to his or her likely success in the study of the subject and hence of an estimate of his or her ability in the subject itself.

One of the earliest works on mathematical reasoning ability was carried out by Brown in 1916 (Adegoke, 2003). However, Brown's (1961) work had, in reality, judged mathematical attainment and not mathematical reasoning ability. Most of the items contained in Brown's work reflected normal class work on mathematics. The later works of Hamley in 1934 (Adegoke, 2003) provided a basis for mathematical tests which were unrelated to school work. A succession of small pieces of work (e.g. Blackwell, 1940, Bennett, 1948; Jenkins, 1939) and which had followed Hamley's work suggested that such tests could function as mathematics reasoning ability tests.

Hamley (Adegoke, 2003) hypothesized that three stages could be distinguished in the process of mathematical reasoning. These are;

a) Dividing the given material into classes, each determined by some unifying characteristics.
b) Detecting a prevailing order within each class.
c) Seeking out correspondence of relationship between the members of two or more classes.

Hamley felt that the ability to succeed in the study of mathematics could correspond to the ability to carry out these three processes of classification, order, and recognition of correspondences in dealing with the simple materials which is basic to mathematics, namely, arithmetical numbers, algebraic symbols and spatial figures. Jenkins (1939) introduced an additional stage into this scheme of mathematical reasoning, when he suggested that the ability to recognise some constant variable within a group preceded the identification of that group as a class. From the analysis of the batteries of tests constructed and administered to groups of children aged 13 to 15 years in London, Lee (1967) showed that there was sufficient evidence to suggest that the tests actually measured mathematical ability rather than attainment. Choudhury and Das (2012), Berrett and Williams (1997) Fischbein and Nachieli (1998) and Adegoke (2003) used the ideas contained in Hamley's and Jenkin's works to develop their mathematical reasoning ability tests.

The works of Hamley and Jenkins (Adegoke, 2003) rest on the assumptions that the fundamental notions of mathematics are of two kinds:

a) The notions of *arithmetical numbers*, *algebraic symbols* and *spatial figures*. The study of mathematics cannot proceed at all without these, for they provide the material with which mathematics work. They also designate the subject field called mathematics.
b) The notions of *variable*, *class*, *order*, and *correspondence*. These notions play a large part in mathematical work, since most problems are solved by recognizing classes amongst given data, creating orders within these classes and picking out correspondences to lead to unique conclusions.

A *variable* is a quantity which can take different values although its basic structure remains the same throughout; an even number is a variable, since there are many such numbers all having the property of exact divisibility by two. A *class* is a group of quantities having a common characteristics, a group of even numbers is thus a class, the common characteristics being the even-ness of the numbers or the property that each is exactly divisible by two. An *order* is obtained when quatities are arranged in sequence according to a fixed rule or law; thus even number can be arranged in ascending or descending order. A *correspondence* is obtained when two ordered classes of quantities are placed side by side in such a way that each pair is bound by a law which can be enunciated, thus, the sequence of odd and even numbers show a correspondence in which each member of one class differs from the corresponding member of the other class.

The notions of *variable*, *class*, *order* and *correspondence* can be seen in every phase of mathematical work, and hence can be regarded as the fundamentals required for the study of mathematics. Together with the notions of arithmetical numbers, algebraic symbols and spatial figures, they provide the basis for tests of mathematical ability (Lee, 1967). From the psychological angle, this is equivalent to the hypothesis that four stages can be distinguished in the dynamic process of mathematical reasoning, namely those of recognizing *variable*, *classifying*, *ordering* and *recognising correspondences* when solving school mathematical problems. The first objective of this study was, therefore, to determine the extent to which these four dynamic processes are indicators and sufficient measures of mathematical reasoning ability.

In Nigeria, the curriculum of secondary school subjects is normally prepared by the Nigerian Educational Research and Development Council (NERDC). The mathematics curriculum as prepared by NERDC in 2005 consists of four major sections. These are Number and Numerations, Algebra, Geometry, and Statistics. Scrutiny of the curriculum shows that within these four sections, a total of about fifty distinct topics are taught including percentages in arithmetic, quadratic equations in algebra, circles theorems in geometry and measures of central tendency, dispersion, and probability theories in statistics. These are the fundamental concepts of mathematics which Nigerian students must learn during their secondary school education. Students' level of attainment in mathematics is the extent to which they are able to master these fundamental concepts of mathematics. In this study emphasis was, however, on three sections, namely, Number and Numerations, Algebra, and Geometry. This is because, this study centers on senior secondary school one students (Ages 14 - 16). In Nigeria, usually, in-depth study of statistics starts with students in senior secondary two (Ages 16 -18).

In this study, therefore, the second objective was to present structural model of the linkages existing between the fundamental notions of mathematical reasoning ability and attainment in mathematics. In order to study the relationships between the fundamental notions of mathematical reasoning ability and attainment in mathematics the author developed a Structural Regression model (SR) (see Figure 1). SR is a variant of structural equation models. The other examples are Path Analysis (PA) and Confirmatory Factor Analysis (CFA) (See Adegoke, 2012; Kline, 2005 for more detailed information on Structural Equation Modelling). An SR model results from the synthesis of path model (PA) and CFA measurement models.

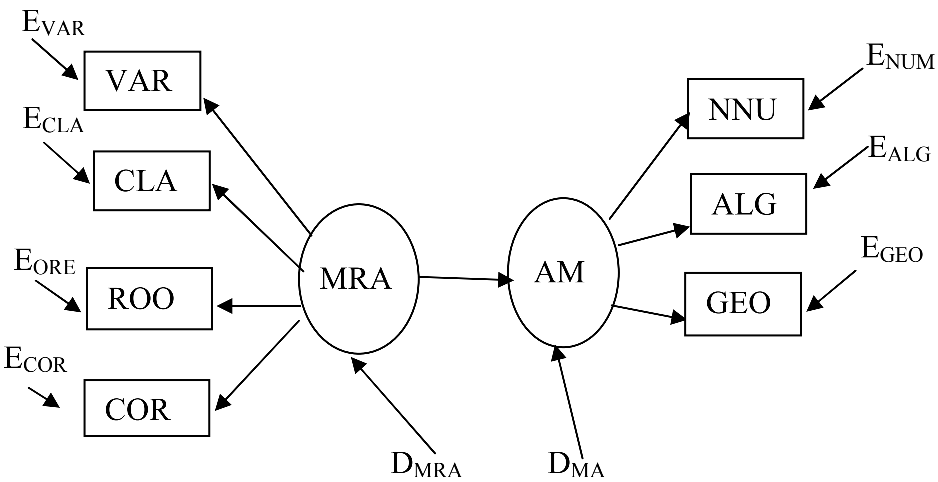

Figure 1: Structural regression model of Mathematics reasoning ability and Mathematics Achievement.

As in PA, SR allows test of hypotheses about direct and indirect causal effects. However, unlike PA, these effects incorporate a measurement component that represents observed variables as indicators of underlying factors as in CFA. In SR models, the researcher can test hypotheses about structural and measurement relations within a single model (Adegoke, 2012; Kline, 2005).

In figure 1, MRA and AM are factors or latent traits and they are not directly measured but rather assessed indirectly using scores from the indicators. For example, MRA is assessed through students' ability to recognize the four processes of recognition of variable (VAR) classification (CLA), order (ROO), and recognition of correspondences (COR) in dealing with the simple materials which is basic to mathematics, namely, arithmetic number, algebraic symbols, and spatial figures. AM is in turn predicted by MRA, a second factor, which is assessed through scores in number and numeration (NNU), algebra (ALG), and geometry (GEO). In the figure Es represent errors in the measurement of observed variables and Ds represent Disturbance (variance) in the latent traits (or factors). Because the causes of exogenous variables are not represented in the models, they are free to vary and covary (Kline, 2005). However, note that the Ds associated with latent traits in the figure reflect omitted causes of latent variable rather than observed variables.

The third objective of this study was to determine the fitness of the two-factor model of Mathematical reasoning ability and attainment in Mathematics and determine the extent to which the indicators of mathematical reasoning ability predict attainment in mathematics. These objectives were realized through the assessment of the maximum likelihood parameters of the hypothesized model, and the relationships between the indicators and their corresponding latent traits and the relationship between the latent traits.

Specifically, five research questions were answered. These were:

1) What is the nature of the relationship among the indicators of each of the latent variables?

2) Does this model fit the observed data?
3) If the model fits the observed data, what are the models fit statistics?
4) What is the nature of the factor loadings of each of the latent traits?
5) To what extent does mathematical reasoning ability predict the attainment in Mathematics?

2. **Method**

2.1 Participants

The participants were 240 Senior Secondary One (Age 14 – 16 years) students randomly drawn from 10 Senior Secondary Schools in Isokan, and Irewole Local Government Areas, Osun State, Nigeria. In this study, only the students in science classes took part. From each school, intact science classes were used. Among the 240 students sampled, 127 (52.9%) were boys, while 113 (47.1%) were girls. Their ages ranged between 14 and 16 years (Mean age = 15.43; SD = 1.79).

2.2 Materials

For this survey, two instruments were used. These are:
a) Attainment in Mathematics Test (AMT)
b) Mathematical Reasoning Ability Test (MRAT)

AMT: This consists of 36 multiple choice items with four options (ABCD). The author of this article developed these items from three major sections of mathematics (Number and numeration, Algebra, Geometry, and Geometry) as prescribed by the Nigerian Educational Research and Development Council, (NERDC, 2005). Initially there were 60 items. These items were subjected to pilot testing among 30 SSII students in Ayedade local Government Area, Osun State, Nigeria. Twenty-four items whose difficulty indices were either below 0.30 or above 0.71 were deleted. For the remaining 36 items, each of the four major sections consists of nine items. Test blue print placed under Knowledge, Understanding and Thinking were used to establish the content validity of the final 36 items. The difficulty indices of each item ranged between 0.38 and 0.75, while the discriminating indices ranged between 0.43 and 0.51. The reliability indices of each section: Number and numeration, Algebra, and Geometry were 0.71, 0.76, and 0.69 respectively. This was established by using Kuder Richardson 20 formular. The maximum obtainable score in each section was 12, that is, each item attracted a score of 1 for right response and 0 for wrong response.

MRAT: The author of this article constructed this instrument. It was constructed by combining the fundamental processes of recognition of variable, class, order and correspondence with the three basic types of materials employed in the study of Mathematics, these being arithmetical numbers, algebraic symbols, and spatial figures. As suggested by Lee (1967), separate tests were designed to estimate capacity for handling each of the four processes in terms of each of the three types of materials making twelve sub tests in all. Initially under each sub test, there were four items, that is, the draft copy consisted of 48 items. These items were subjected to pilot testing among 35 students in Ayedaade Local Government Area, Osun State, Nigeria. In each of the sub tests, items with low difficulty levels ($p < .30$) were eliminated. In the final draft, there were two 24 items. The difficulty and discriminating indices of each item ranged between 0.37 and 0.70; and 0.30 and 0.57 respectively. The reliability indices of each section ranged between 0.69 and 0.81. The maximum obtainable score in MRAT was 24, that is, each item correctly answered attracted a score of 1 while wrong responses attracted 0.

Examples of item used in the MRAT include:

1. Classification-Arithmetical Number (CLA-AN): In each of the rows of numbers, there are five numbers on the left. Four have something in common and one is different. Underline the one which is different. When you have done this, place a circle on one of the numbers on the right which could best take the place of the one you have underlined.

Left	Right
45, 54, 63, 27, **30**	33, 36, 66, 93,

In this problem, on the left hand side, 30 have been underlined, because it is not divisible by 9. On the right hand side, 36 have been circled because it is the only one out of the four numbers on the right which is divisible by 9

2. Order-Arithmetic Number (ROO-AN). The following rows show the beginnings of series of numbers, and some numbers are missing. Fill in the blank spaces with the numbers you think should be there.

1, 3, 5, 7, 9, ---, ---

2.3 Procedure

Three research assistants were recruited for this study. They were all graduate students in the Institute of Education, University of Ibadan, Nigeria. They have received formal training in test administration. However, the author of this article explained the purpose of the study to them. The author also went round to monitor how the tests were administered. The data collection lasted four weeks. The administration of each of the tests took place during the normal time scheduled for mathematics on the official school time table. This was to avoid disruption to the school programmes. For MRAT, the students completed it on the average, in 50 minutes, though the time allowed was one hour. The data collected were analysed using maximum likelihood estimates of LISREL Version 8.80 (Linear Structural Relations; Jöreskog & Sörbom, 2003).

3. Results

The descriptive statistics (Pearson Correlation coefficient, mean, and standard deviation) were calculated using LISREL Version 8.80. Also variance-covariance statistics were obtained. Table 1 presents the Pearson product-moment correlation coefficients between the variables. Mean and standard deviations of each of the six variables are as presented on the table.

Table 1: Pearson correlation coefficient, mean, and standard deviation

	VAR	CLA	ROO	COR	NNU	ALG	GEO
VAR	1.000						
CLA	.630*	1.000					
ROO	.537*	.535*	1.000				
COR	.833*	.623*	.516*	1.000			
NNU	.860*	.633*	.540*	.705*	1.000		
ALG	.589*	.821*	.736*	.586*	.592*	1.000	
GEO	.636*	.751*	.794*	.582*	.640*	.589	1.000
Mean	6.49	5.92	5.13	7.08	6.87	5.21	6.41
SD	2.85	2.57	2.35	2.92	2.83	2.40	2.50

Note *p < .05 *Key: VAR = recognition of variables, CLA = classification of given materials, ROO = recognition of order, COR = recognition of correspondences, NNU= Number and Numeration, Algebra = Algebra, GEO = Geometry.*

Research Question One: What is the nature of the relationship among the indicators of the latent variables?

In this study, as specified in the preceding sections, the hypothesized indicators of mathematical reasoning ability were the fundamental processes of recognition of variable, class, order and correspondence and for attainment in mathematics they were number and numeration, algebra and geometry. It is noted from Table 1 that, one, all the variables are moderately and significantly correlated with one another. Two, the relationships between measures of mathematical ability and measures of attainment in mathematics are moderate and positive. In fact, a perusal of the relationships between each of the measures of mathematical reasoning ability and each of the measures of mathematics attainment shows that mathematical reasoning ability is a good predictor of attainment in mathematics.

The highest correlation (r = .860) is between recognition of variables in a given material, and number and numeration. The observed high and positive relationship between recognition of variables and number and numeration suggests that a student with high ability of recognizing some constant variable within a group of numbers is likely to do well in problems dealing with number and numeration.

The lowest correlation (r = .516) is between recognition of correspondences in a given material and spatial figures. Though in this study, the correlation coefficient may appear low, it is moderate and more importantly, statistically significant. The obtained variance-covariance statistics lend credence to the fact that there existed positive and high relationships among the indicators of numerical and attainment in mathematics.

Research Question Two: Does the hypothesized model fit the observed data?

From the maximum likelihood estimates, the independence model Chi square analysis shows that the variables in this study are correlated χ^2 (13, N = 240) = 352.52, $p < 0.05$. The value of the minimum fit function Chi Square is χ^2 (21, N = 240) = 635.55, $p < 0.05$. Although in a very good model fit situation, the hypothesized model fit function Chi Square statistics should not be significant, other fit indices when examined, suggested reasonable overall model fit.

Research Question Three: If the model fit the data, what are the model fit statistics?

The model fit statistics examined include: Root-Mean-Square-Error of Approximation (RMSEA) = 0.33 with the 90% Confidence interval (1.18 – 1.69); Comparative Fit Index (CFI) = 0.71; Normed Fit Index (NFI) = 0.71; and Parsimony Goodness of Fit Index (PGFI) = 0.44. These values indicate overall satisfactory model fit (See Hu and Bentler, 1999; Kline, 2005).

Research Question Four: What is the nature of the factor loadings of each of the latent traits?

Reported in Table 2 are the unstandardised and standardized maximum likelihood estimates for all parameters of the two-factor (Mathematical Reasoning Ability and Attainment in Mathematics) SR model.

Table 2: Maximum Likelihood Parameter Estimates of the Structural Regression Model

Parameter	Standardardised Estimate	Unstandardised	Std. Error	Z
		Factor Loadings		
MRA→VAR	0.55	1.56	0.15	10.39
MRA→CLA	0.84	2.17	0.13	16.43
MRA→ROO	0.83	1.96	0.12	16.09
MRA→COR	0.30	0.88	0.14	06.42
AM→NNU	0.53	1.50	-	-
AM→ALG	0.83	2.00	0.18	10.93
AM→GEO	0.83	2.07	0.19	10.91
		Measurement error variance		
E_{VAR}	0.70	5.70	0.46	12.45
E_{CLA}	0.29	1.90	0.15	12.35
E_{ROO}	0.31	1.72	0.14	12.42
E_{COR}	0.91	7.76	0.68	11.49
E_{NNU}	0.72	5.79	0.47	12.33
E_{ALG}	0.31	1.77	0.15	11.92
E_{GEO}	0.32	1.99	0.17	11.96

Also presented in Figure 2 are the standardized solutions of the hypothesized regression model.

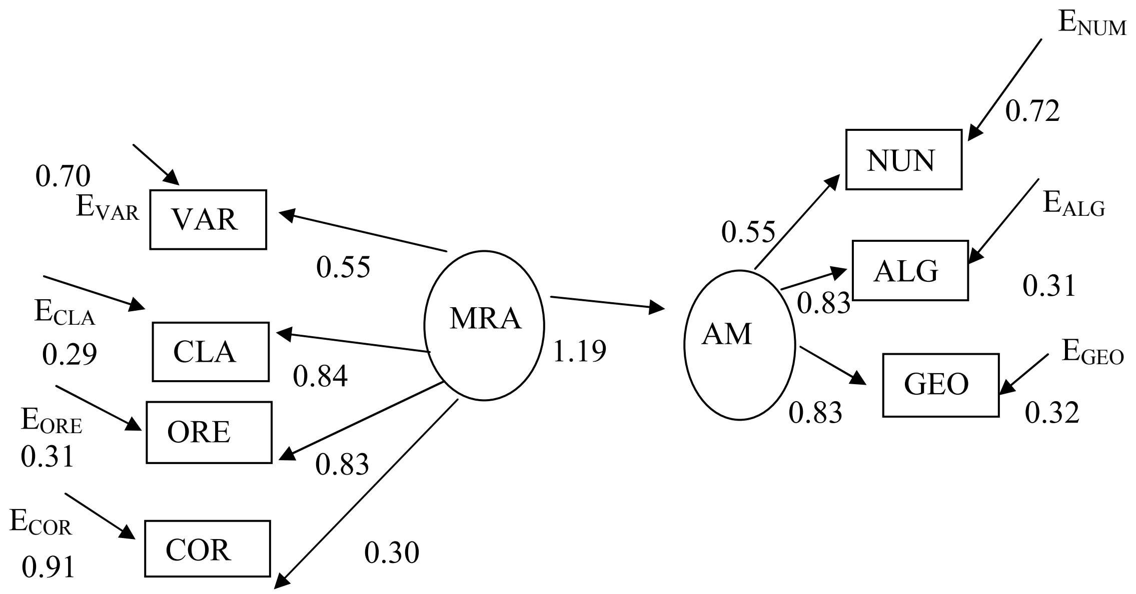

Figure 2: Standardized solutions of the hypothesized structural regression model.

From Table 2 and Figure 2, it can be observed that almost all the factor loadings (standardized coefficient) range from being moderate (0.30, mathematical reasoning ability to recognition of correspondence among given materials) to being high (0.84, mathematical reasoning ability to classification) and statistically significant, $p < .05$. The z-statistic in each case is greater than 1.96. This shows that the four indicators viz: recognition of variable, class, order and correspondence can validly measure mathematical reasoning ability.

Research Question Five: To what extent does mathematics reasoning ability predict attainment in Mathematics?

From Figure 2, the obtained coefficient of 1.19 ($z = 11.25$) shows that reasoning ability predicted attainment in mathematics quite well. The obtained coefficient of 1.19 implies that for every one full unit increase in reasoning ability, attainment in mathematics increases by 1.19 units.

4. Discussion

The major goals of this study were to explain the four basic indicators of mathematics reasoning ability test and to develop a model that could help explain the relationship between mathematical reasoning ability and attainment in secondary school mathematics. Factor loadings of the structural regression analysis showed that the four fundamental notions of recognition of variable, classification, recognition of order, and recognition of correspondence, among materials given in mathematical problems, are measures of mathematical reasoning ability. These findings were consistent with the works of earlier philosophers such as Lee (1967) and Hamley (1934).

Empirical evidence from literature (e.g. Adegoke, 2003; Berret & Williams, 1997 Fischbein & Nachieli, 1998; Heng-Yuku & Sllivan, 2000) suggests that ability to succeed in the study of mathematics could correspond to the ability to carry out the four processes of recognition of variable, classification, ordering, and recognition of correspondence. This is because these notions play a large part in Mathematical work, since most problems are solved by recognizing classes amongst given data, creating orders with these classes, and picking out correspondence to lead to a unique conclusion. The ability to carry out these processes is needed in dealing with the simple mathematics which is basic to mathematics, namely arithmetical numbers, algebraic symbols, and spatial figures.

For mathematics achievement, number and numeration, algebra, and geometry, as the factor loadings of structural regression model showed are valid indicators of attainment in mathematics. The earlier works of philosophers such as Hamley (1934) and Jenkins (1939) had shown that arithmetical numbers, algebraic symbols, and spatial figures are the fundamental notions of mathematics. In fact, the study of mathematics cannot proceed at all without these, for they provide the material with which mathematicians work. They also designate the subject field called mathematics. In all Anglo-phone countries (Ghana, Sierra Leone, Gambia, Nigeria and Senegal) in West Africa, curriculum of secondary school mathematics includes topics in algebra, geometry and statistics. More over items for the public examinations are usually drawn from these topics (see Mathematics Syllabus of West African Examination Council, 2011).

Examination of the raw test scores showed that no student obtained consistently low marks for the tests of mathematical ability while obtaining consistently high marks for the corresponding tests of mathematical attainment. On the other hand, quite a number of the students scored well above average on the ability tests and below average on the attainment tests. Analysis of data showed that the two batteries of ability and attainment tests in Mathematics share a common ground in that success in the former seems to predict success in the latter, in the sense in which ability and attainment in the study of a subject were defined in the earlier sections of this article. This finding corroborates the empirical evidence from literature (e.g. Choudhury and Das, 2012; Heng-Yuku & Sullivan, 2000; Nunes, Byrant, Barros, & Sylva, 2012). For example, in their study, Nunes, Byrant, Barros, and Sylva (2012) found that mathematical reasoning ability did made independent contributions to the prediction of mathematical achievement. Similarly, Choudhury and Das (2012) in their study found that geometrical ability contributed significantly to students' achievement in mathematics in Malaysia. These findings show that ability to succeed in the study of mathematics could correspond to the ability to carry out the four processes of recognition of variable, classification, ordering, and recognition of correspondence. This is because these notions play a large part in Mathematical work, since most problems are solved by recognizing classes amongst given data, creating orders with these classes, and picking out correspondence to lead to a unique conclusion. The ability to carry out these processes is needed in dealing with the simple mathematics which is basic to mathematics, namely arithmetical numbers, algebraic symbols, and spatial figures.

5. Implications of Findings and Recommendations

The findings of this study showed that students' level of mathematics reasoning ability plays a major role in their attainment in mathematics. Therefore it is important that teachers take note of this, and more importantly mount intervention programmes that can help students develop their reasoning ability. This can be achieved by giving them cognitive tasks that are not necessary curriculum based. Examples of such tasks include finding missing numbers in an array of numbers as stated in one of the problems used in this study (See the two examples given under AMT)

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