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Review of fuel Consumption and Development of Fuel Consumption Equation for Traction using Dimensional Analysis

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Abstract

Fuel consumption in agricultural machinery is the main factor in selecting machinery. During tractor operation, there are different factors that affect fuel consumption in tillage equipment operation. These include the level of power used, working speed, cutting width, soil strength, moisture content, working depth, rolling resistance, and dynamic load on the wheel. This paper reviews the application of dimensional analysis in traction studies and applies dimensional analysis to develop a general equation for fuel consumption for traction. A fuel consumption equation using dimensional analysis with the Buckingham pi theorem developed for traction by considering tyre diameter, tyre width, cone index, wheel dynamic load, rolling resistance, slip, bulk density, and forward speed of the tractor. The developed fuel consumption model equation ($FC = d^2V * \left(\frac{w}{d}, \frac{W}{d^2V^2\rho}, \frac{\text{CI}}{V^2\rho}, \frac{R}{d^2V^2\rho}\right) * S$. where, FC is fuel conception ,d is the tire diameter, w, is the tire width, CI is the cone index, W is the wheel dynamic load, R is the rolling resistance, slip, ρ is the soil bulk density, S is the slope and v is the forward speed of the tractor. The developed fuel consumption model considers as the basic fuel consumption affecting parameters and it needs further study and experiments to validate the model.

Keywords: Fuel consumption, dimensional analysis, Buckingham Pi theorem

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1. INTRODUCTION

Farm machinery information helps the designers, researchers and users to get necessary data to design and select machinery in simple ways. The farm mechanization related soil character to test the performance of off-road vehicles in the field different parameter selectees. The factor of off road parameter may have direct or indirect factor. That relation should clarify for researchers, designers and farm machinery users. This need extends all the way from engineers designing off-road vehicles to the ultimate users (Pandey, 2004). The function of any off-road vehicle is to provide mobility to itself and power to an implement. One very important factor that limits vehicle mobility is the drive tyre to surface interaction (Carman, 2004).The gross traction developed by the drive tire under self-propelled conditions at zero pull with the applied torque simply overcoming the motion resistance of the wheel. This characterizes the system input requirements for a powered wheel operating on a level surface without developing additional pull (Pa. The analysis, the performance of the tire-soil system considered by the pull exerted by the powered wheel at a pre-selected slip in the pull-slip relation, the sink age of the powered wheel into the soil at the selected slip.

The torque required to turn the powered wheel at the selected slip, the force required to the loaded and free-rolling wheel on the soil (Freitag, 1966). Know days, there is an attempt to better utilize the energy consumption in agricultural production, particularly for high energy requiring operations. Fuel consumption is the primary diagnostic parameter in identifying the condition of the vehicle (Michalski, 2014).moreover, because of the continuous rise in fuel prices; energy consumption has become one of the most important factors in agricultural economy (Nkakini, 2019).Fuel is the source of energy for the tractor providing for the performance of work and propelling the tractor to overcome implement draught. There are many parameters in tillage operation that affect the fuel consumption of a tractor, such as soil texture, climate, relative humidity, tractor type (two or four-wheel drive), tractor size and tractor implement relationship (Karparvarfard, 2015).

The factors that fundamentally affect fuel consumption in tillage equipment use is the increase in power consumption by increasing the working speed, actual width of cut, soil strength, moisture content and the working depth (Leghari, 2016). Therefore, tractor fuel consumption is not constant and varies from one to another situation Fountas et al., 2015).

The basis of dimensional analysis is to condense the number of separate variables involved in a particular type of physical system into a smaller number of non-dimensional groups of the variables. The arrangement of the variables in the groups generally chosen so that each group has a physical significance where dimensional analysis was used to judiciously simplify the parameters controlling motion filtering by dimensional analysis (Garcia et al., 2019).

The dimensionless response comes given as a function of dimensionless groups of parameters. Using the dimensional analysis different equation developed based on the Buckingham's theorem (Garcia et al., 2019). Studying such things is the basic under understanding factor of parameter off road selection and design machinery. The objective of this review is:

- To review the application of dimensional analysis in traction studies
- To apply dimensional analysis to develop general equation for fuel consumption for traction

2. LITERATURE REVIEW

There are different works related with the performance of tractors and traction studies, which includes tractor tractive performance, fuel consumption, torque and power requirement and draft or pull force on different soil conditions like clay soil, sandy soil and loamy soil.

Table 1: Application of dimensional analysis in traction studies

Authors (Reference)	Title	Description	Repeating variables used	Equation developed
(A.M. Moeenifar et al., 2013)	Application of dimensional analysis in determination of traction force acting on a narrow blade	Predict the draught force required for narrow blade tool considering one dependent variable force (F) and 10 independent variables including bulk density (γ), soil angle of internal friction (φ), adhesion (C), cohesion (C_a), angle of soil metal friction (δ), tool width (w), rake angle (α), tool depth(d), surcharge (q) and speed (v)	Soil bulk Density (γ), Tool width (w)and speed (v)	$\frac{F}{V^2\gamma w^2}=f(\frac{c}{V^2\gamma}, \varphi, \delta, \alpha, \frac{C_a}{V^2\gamma}, \frac{d}{W})$
(Freitag Et al., 2016)	Application of similitude to soil machine systems	Study the tire-soil interaction in clay and sand soil using four dependent variables pull (P), towed force (P_T), torque (Q), sink age (z) and 13 independent variables tire diameter (d), section height (h), section width (w), deflection (δ), soil friction angle (φ), cohesion (c), specific weight (γ), spissitude (β), system load (W), translational velocity (v), slip (S), tire-soil friction (μ), acceleration (g), average cone index (C) and cone index gradient (G)	Tire diameter (d), translational velocity (v) and system load (W)	$\frac{P}{W}, \frac{P_T}{W}, \frac{Q}{dW}, \frac{z}{d} = f(\mu, \varphi, S, \frac{b}{d}, \frac{h}{d}, \frac{\Delta}{d}, \frac{cd^2}{W}, \frac{\gamma d^3}{W}, \frac{Bvd}{w}, \frac{gd}{c^2}$ for sand $\frac{P}{W}, \frac{P_T}{W}, \frac{Q}{dW}, \frac{z}{d} = f(\frac{Gd^3}{W}, \frac{h}{d}, \frac{\Delta}{h})$ Clay: $\frac{P}{W}, \frac{P_T}{W}, \frac{Q}{dW}, \frac{z}{d} = f(\frac{Cd^2}{W}, \frac{h}{d}, \frac{\Delta}{h})$

(Moeenifar et al., 2013)	Determination on of traction force acting on a wide blade using dimensional analysis Method	Predict the draught force required for narrow blade tool considering one dependent variable force (F) and 10 independent variables including bulk density (γ), soil angle of internal friction (φ), adhesion (C), cohesion (Ca), angle of soil metal friction (δ), tool width (w), rake angle (α), tool depth(d), surcharge (q) and speed (v)	Soil bulk Density(γ), tool width (w) and speed (v)	$\frac{F}{V^2\gamma W^2} = f(\frac{C}{V^2\gamma}, \varphi, \delta, \alpha, \frac{C_a}{V^2\gamma}, \frac{d}{w})$
(Fakhraei & Karparvarfar d, 2008)	Development of a general equation for estimation of tractive efficiency by dimensional analysis	Predict the general equation of tractive efficiency by using one dependent variable tractive efficiency (T.E) and 10 independent variables load on wheel (W), slip (S), cone index (CI), wheel width (b), wheel diameter (d), pull (P), rolling radius (r) and tractive force (T.F)	Weight of drive wheel (W) and wheel diameter (d)	T.E = $F_1 (\frac{P}{W})$ $F_2 (\frac{CI.bd}{W})$ F_3 (S) F4 $(\frac{r}{d})$ F5 $(\frac{T.F}{W})$.
(Karparvarfar d & Koushkaki, 2015)	Development of a fuel consumption equation: Test cases for a tractor chisel ploughing in a clay loam soil.	Predict the fuel consumption equation using experimental fuel consumption (FC) as dependent variable and independent variables of hourly fuel consumption (Qi), actual forward speed (Va), gravity (g), blade width (W), working depth (D), cone index (CI), slip (S), dynamic wheel load (W_d), draught force (F_d), rolling resistance (F_r), unloaded tire diameter (T_d) and unloaded tire width (T_w)	Gravity (g), blade width (W),dynamic wheel load (W_d) and hourly fuel consumption (Qi)	$\frac{FC}{Qi} = f(\frac{V_a}{g^{0.5}\, W^{\,0.5}}, \frac{D}{W}, \frac{CI\, T_w\, T_d}{W_d}, \frac{F_d}{W_d}, S, \frac{F_r}{W_d})$
(Upadhyaya, 2009)	Dimensional analysis and Similitude Applied to	Reviewed many works related with application of dimensional analysis and similitude in traction. He determined the interaction of soil-narrow tillage tools using one	Operation width (w), specific weight (γ)	$\frac{D}{\gamma W^3} = f(\rho, \beta, \alpha, \frac{k}{\gamma w}, \frac{\eta S}{\gamma W^2}, \frac{A}{\gamma w}, \mu, \frac{s^2}{gw}, \frac{d}{w})$

	Soil Machine Systems	dependent variable draft force (D) and 14 independent variables including working width (w), rake angle (ρ), cutting angle (β), Poisson's ratio (ν), and the slope (α), intercept (k), viscosity of soil (η), specific weight of soil (γ), gravitational constant (g), adhesion (A), soil-metal friction (μ), operating depth (d), operating speed (S) and young's modulus (E)	and gravity (g)	
(Harrison, 2013)	Dimensional analysis for vibratory tillage tools	Predict single general equation for vibratory tillage tools draft requirement using draft (d) dependent variable and independent variables of rake angle (α), share dimension (L), plane of oscillation (θ), amplitude of oscillation (A), travel rate (v), soil density (γ), cone index (Ci) and radius of oscillating crank or eccentricity (r)	Soil density (γ), share dimension (L)	$\frac{D}{\gamma L^3} = f(\alpha, \theta, \lambda, \frac{Ci}{\gamma L}, \frac{A}{L}$ Where, $\lambda = \frac{1}{2r}$
(Freitag, 1966)	A dimensional analysis of the performance of pneumatic tires on clay	Study the tire-soil interaction in clay soil using four dependent variables pull (P), towed force (PT), torque (Q), sink age (z) and 13 independent variables tire diameter (d), section height (h), section width (w), deflection (δ), soil friction angle (φ), cohesion (c), specific weight (γ), spissitude (β), system load (W), translational velocity (v), slip (S), tire-soil friction (μ), acceleration (g), cone index (C).	Tire diameter (d),translational velocity (v) and system load (W)	$\frac{P}{W}, \frac{P_T}{W}, \frac{Q}{dW}, \frac{z}{d} = f(\mu, \varphi, S, \frac{b}{d}, \frac{h}{d} \frac{\Delta}{d}, \frac{Cd^2}{W}, \frac{\Gamma d^3}{W}, \frac{\beta v d}{W}, \frac{gd}{v^2})$ After a careful consideration he reduced the pi term and get: Clay: $\frac{P}{W}, \frac{P_T}{W}, \frac{Q}{dW}, \frac{z}{d} = f(\frac{Cd^2}{W}, \frac{b}{d}, \frac{\Delta}{h}$

(Monifar & Shahgholi, 2018)	Dimensional analysis tractor tractive efficiency parameters	Study applied force on tillage tool (P) and tire slip (S) using six and seven independent variables respectively as follows. Soil property (γ), cohesion (c), tool design parameter (w), rake angle (α), tool depth (d), operation speed (v) and dynamic rear wheel load (Wd)	Soil property (γ), operation speed (v) and tool design parameter/ width (w)	$\frac{P}{v^2\gamma w^2} = f(\frac{C}{v^2\gamma}, \frac{d}{w}, \sin\alpha)$ and $S = f(\frac{C}{v^2\gamma}, \frac{d}{w}, \frac{Wa}{v^2\gamma w^2}, \sin\alpha)$
(Garciasuarez et al., 2019)	Dimensional analysis: overview and applications to problems of soil structure interaction	They identified parameters considered in the SSI problems. It includes (T_b) fixed-base fundamental period of the building, (h_b) first-modal height of the fixed-base building, (B) half-width of the building foundation, (D) depth of the building foundation, (m_b) first modal mass of the fixed-base building, (ρ_f) foundation density, (ν_f) poison's ratio of foundation material, (ρ_s) soil density, (ν_s) Poisson's ratio of the soil, (V_s) Shear-wave velocity of the soil, (V_f) Shear wave velocity within the foundation, (a_p) Amplitude of the acceleration ground motion, (T_p) ground motion period with the highest energy and displacement (u)	Amplitude of the acceleration ground motion (a_p),ground motion period with the highest energy (T_p), half width of the building foundation (B),soil density (ρs)	$\frac{u}{a_{p}{T_p}^2} = f(\frac{T_b V_s}{B}, \frac{h_b}{B}, \frac{D}{B}, \frac{M_b}{P_s B^3}, \frac{P_f}{P_s}, vf, vs, \frac{V_f}{V_s}, \frac{A_{pB}}{{V_s}^2}, \frac{T_p}{B}$
(Nkakini et.al., 2019)	Modeling fuel consumption rate for Harrowing operations in loamy Sandy soil	Use fuel consumption(Fc) as dependent variable and independent variables speed (V), depth (d), width (W), cone Index (CI), draught (D), bulk density(ρ) and moisture Content (Mc)	Bulk Density(ρ), depth (d) and speed (V)	The model fuel Consumption $Fc = \varphi \frac{\text{Dvdmc}}{\text{CIW}} + C$ C, is constant φ, is the function

(Pandey &Sharma,2017)	Modeling power requirement for traction Tyres With zero Linkage	Use torque (T), rolling radius (r), normal load (W), tire Diameter (D), tire section (b),Tire deflection (δ), tire section, Height (h), velocity (v) and Gravity (g) variables	Normal Load (W). Rolling resistance (r) and velocity (v)	After rearrangement The model equation is: $\frac{T}{\mathrm{Wr}} = Cf(\frac{b}{\mathrm{D}}, \frac{\Delta}{\mathrm{H}}, \frac{\mathrm{R}}{\mathrm{H}}, \frac{\mathrm{D}}{\mathrm{R}}, \frac{\mathrm{Gh}}{v^2})$
(Carman & Tarhan, 2004)	Modelling the torque and power requirement of traction, tires of horticultural tractors, Using dimensional analysis	Use dependent variable of torque (T) and rolling radius (r) and independent variables normal load (W), tire diameter(D), tire section (b), tire, deflection (δ), tire section height (h), velocity (v) and Gravity (g)	Normal Load (W), Rolling Radius(r) and Velocity (v)	$\frac{T}{\mathrm{Wr}} = f(\frac{b}{\mathrm{D}}, \frac{\Delta}{\mathrm{H}}, \frac{\mathrm{R}}{\mathrm{H}}, \frac{\mathrm{R}}{\mathrm{D}}, \frac{\mathrm{Gd}}{v^2})$
(Wadhwa,1979)	Similitude Its Place in Tillage research	Use tool length (λ_1), width (λ_2),depth (λ_3), angle (α), soil, friction angle (ϕ), cohesion (c),specific weight (γ), force (F), adhesion (δ), velocity (v), gravity (g) and soil metal friction (ψ)	Tool depth (λ_3), gravitational acceleration (g) and specific weight (γ)	As a result of test equation developed the pi-terms to: $\frac{F}{\Gamma\lambda_3} = f(\frac{\lambda_1}{\Lambda 3}, \frac{\lambda_2}{\Lambda 3}, \frac{C}{\Gamma\lambda_3}, \frac{v^2}{\mathrm{G}\Lambda_3})$

2.1 Methods of Model Development for Fuel consumption Equation for Traction using Dimensional Analysis

Dimensional analysis used to develop a fuel consumption equation estimation and parameter factor for traction operation for different machineries. In order to apply dimensional analysis there are two common methods of solving the variables according to (Bansal, 2010, Kundu et al., 2012). These methods include Rayleigh's and Buckingham pi theorem using mass, length time or force, length and time combinations. Due to its complexity for large number of variables Rayleigh's method is not common and also, we use Buckingham pi theorem used for our analysis. In order to get the final relating equation follows as step by steps application of the Buckingham pi theorem using mass, length and time basic dimensions

Step 1: List the parameters in the problem and count their total number (n). There are nine variables required to determine the fuel consumption of tractors. These variables Include dependent variable (fuel consumption (FC)) and independent variables (tire diameter (d), tire width (w), cone index (CI), wheel dynamic load (W), rolling resistance (R), and wheel slip (S), forward speed (V) and bulk density (ρ)). Since the performance of tractor fuel consumption without attaching implements is analyzed we do not consider the implement working the mechanical strength of the soil to reduce the number of variables. Similar trends are observed and width and depth. From the soil properties we just only consider the cone index since it determines analyzed in (Karparvarfard & Koushkaki, 2015; Nkakini et al., 2019).

Step 2: List the primary dimensions of each of the 'n' parameters the primary dimensions of the eight identified variables are tabulated as follows based on the three basic dimensions mass, length and time.

Table 2: Effective variables affecting fuel consumption

Variable	Symbol	Dimension	Unit
		Dependent variable	
Fuel consumption	FC	L^3T^{-1}	$m^3.s^{-1}$
		Independent variable	
Tire diameter	d	L	m
Tire width	w	L	m
Cone index	CI	$ML^{-1}T^{-2}$	$N.m^{-2}$
Wheel dynamic load	W	MLT^{-2}	N
Rolling resistance	R	MLT^{-2}	N
Wheel slip	S	-	-
Forward speed	V	LT^{-1}	$m.s^{-1}$
Bulk density	ρ	ML^{-3}	$Kg.m^{-3}$

Step 3: Set the reduction 'm' as the number of primary dimensions. Calculate k, the expected number of II's k=n-m where n= total number of variables, m=number of fundamental dimensions (mass (M), length (L) and time (T)) and k=9-3= 6 so 6 pi are required to develop the fuel consumption equation.

Step 4: Choose j repeating parameters. In order to choose the repeating variables there are a number of guidelines proposed by (Bansal, 2010; Cengel & Cimbala, 2006; Kundu et al., 2012). These guide lines are never pick the dependent variable. Otherwise, it appears in the entire pi's which is undesirable. The choice of repeating variable must not by them to be able to form a dimensionless group. The chosen repeating variables must represent the entire primary dimension in the entire problem. Never pick parameters that are already dimensionless these are pi terms already themselves. Never pick two variables with the same dimension or with dimension that differ by only an exponent. Whenever possible choose dimensional constants over dimensional variable so that only one pi contains the dimensional value. Pick common parameters since they may appear in each of the pi. Pick simple variables over complex variables whenever possible. Based on the above eight guidelines tire diameter (d), forward speed (V) and bulk density (ρ). Are selected for repeating variables and the same trend is observed in

(Karparvarfard & Koushkaki, 2015; A. M. Moeenifar et al., 2013; Moinfar & Shahgholi, 2018; Nkakini et al., 2019).

Step 5: Construct the 'k' II's, and manipulate as necessary. The determined six pi terms are manipulated and constructed as follows applied by (Moeenifar et al., 2013).

$$FC=f\,(d, w, W, R, CI, S, V, \rho) \quad (1)$$

$$\pi_1 = (\pi_2, \pi_3, \pi_4, \pi_5, \pi_6) \quad (2)$$

$\pi_1 = (FC, d, V, \rho)$, $\pi_2 = (w, d, V, \rho)$, $\pi_3 = (W, d, V, \rho)$, $\pi_4 = (CI, d, V, \rho)$, $\pi_5 = (R, d, V, \rho)$ and $\pi_6 = (S, d, V, \rho)$. Manipulation of π_1 and the formulation $\pi_1 = FC.\ d^a.\ V^b.\ \rho^c$

$M^0L^0T^0 = L^3T^{-1}\,(L)^a\,(LT^{-1})^b\,(ML^{-3})^c$ apply basic dimensions

For M, 0= c; c=0 For L, 0=3+a+b-3c solve for T and insert b value in this equation gives; a= -2, and For T, 0 = -1-b; b=-1, finally, $\pi_1 = FCd^{-2}\,V^{-1}\rho^0 = \frac{FC}{d^2\mathrm{V}}$ dimensionless

Manipulation of π_2 and the value of $\pi_2 = w.\ d^a.\ V^b.\ \rho^c$

$M^0L^0T^0 = (L)^a\,(LT{-}1)^b\,(ML{-}3)^c$ apply basic dimensions and For M, 0=c; c=0, For L, 0=1+a+b-3c solve for T and insert b value in this equation gives; a= -1. For T, 0= - b; b=0 finally, $\pi_2 = wd^{-1}V^0\rho^0 = \frac{W}{\mathrm{d}}$ dimensionless

Manipulation of $\pi_3$andthe value of $\pi_3 = W.\ d^a.\ V^b.\ \rho^c$

$M^0L^0T^0 = MLT^{-2}\,(L)^{\mathrm{a}}\,(LT^{-1})^b\,(ML^{-3})^c$ apply basic dimensions

For M, 0=1+c; c=-1 and For L, 0=1+a+b-3c solve for T and insert b value in this equation gives; a= -2and For T, 0=-2-b; b=-2 and finally, $\pi_3 = Wd^{-2}V^{-2}\rho^{-1} = \frac{W}{d^2v^2\rho}$ dimensionless

Manipulation of π_4 and $\pi_4 = CI.\ d^a.\ V^b.\ \rho^c$

$M^0L^0T^0 = ML^{-1}T^{-2}\,(L)\,(LT^{-1})^b\,(ML^{-3})^c$ apply basic dimensions
For M, 0=1+c; c=-1 then For L, 0=-1+a+b-3c solve for T and insert b value in this equation gives; a=0 and For T, 0=-2-b; b = -2 then finally, $\pi_4 = CId^0\,V^{-2}\rho^{-1} = \frac{CI}{d^2\rho}$ dimensionless

Manipulation of π_5 and $\pi_5 = R.\ d^a.\ V^b.\ \rho^c$

$M^0L^0T^0 = MLT^{-2}\,(L)^a\,(LT^{-1})^b\,(ML^{-3})$ apply basic dimensions For M, 0=1+c; c = -1. For L, 0=1+a+b-3c solve for T and insert b value in this equation gives; a= -2 and For T, 0=-2-b; b=-2 and finally, $\pi_5 = Rd^{-2}V^{-2}\rho^{-1} = \frac{R}{d^2V^2\rho}$ dimensionless

Manipulation of π_6 and $\pi_6 = S.\ d^a.\ V^b.\ \rho^c$

$M^0L^0T^0 = (L)\,(LT^{-1})^{\,b}\,(ML^{-3})^{\,c}$ apply basic dimensions

For M, 0=c; c=0 and For L, 0=a+b-3c solve for T and insert b value in this equation gives; a=0

For T, 0=-b; b=0 and finally $\pi_6 = Sd^0V^0\rho^0 = S$ dimensionless. The required π groups are determined successfully and they are checked and dimensionless.

Step 6: Write the final functional relationship and check your algebra. Since fuel consumption is a function of other variables equation 1 and 2, we can express it as: $\pi_1 = (\pi_2, \pi_3, \pi_4, \pi_5, \pi_6)$ then the equation developed was $\frac{FC}{d^2\mathrm{v}} = (\frac{w}{\mathrm{d}}, \frac{W}{d^2v^2\rho}, \frac{CI}{v^2\rho}, \frac{R}{d^2v^2\rho}) * S))$ this is the developed fuel consumption model for traction.

3. Conclusion and Recommendation

The study had developed a model for fuel consumption equation using dimensional analysis with Buckingham pi theorem for traction. Eight dependent variables tire diameter, tire width, cone index, wheel dynamic load, rolling resistance, slip, bulk density and forward speed of the tractor are considered to determine the fuel consumption equation. The developed fuel Consumption model is: $\mathrm{FC} = d^2\mathrm{V} * (\frac{w}{\mathrm{d}}, \frac{W}{d^2v^2\rho}, \frac{CI}{v^2\rho}, \frac{R}{d^2v^2\rho} * S)$ fuel consumption model equation. In order to fully define the predicted model equation, we need to have implemented an experiment and study the effect of each non-dimensional pi groups ($\frac{w}{\mathrm{d}}, \frac{W}{d^2v^2\rho}, \frac{CI}{v^2\rho}, \frac{R}{d^2v^2\rho}$ and S and it needs comparison of the predicted equation and experimental equations to validate the developed fuel consumption model equation.

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