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Application of Branch and Bound Method for Optimal Two Stage Flow Shop Scheduling Problem with Group Job-Restrictions

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Abstract:

This paper studies two stage flow shop scheduling problem in which equivalent job for group job, processing times on each machine are given. The objective of the study is to get optimal sequence of jobs in order to minimize the total elapsed time. The given problem is solved with branch and bound method.The method is illustrated by numerical example.

Keywords: Equivalent job, processing time, elapsed time, Branch and Bound.

1. Introduction:

A flowshop scheduling problem has been one of classical problems in production scheduling since Johnson [6] proposed the well known Johnson's rule in the two-stage flowshop makespan scheduling problem. Yoshida and Hitomi [11] further considered the problem with setup times. Yang and Chern [10] extended the problem to a two-machine flowshop group scheduling problem. Maggu and Dass[9] introduced the concept of equivalent job for a job-block when the situations of giving priority of one job over another arise. Kim, et al.[7] considered a batch scheduling problem for a two-stage flowshop with identical parallel machines at each stage. Brah and Loo [1] studied a flow shop scheduling problem with multiple processors. Futatsuishi, et al. [4] further studied a multi-stage flowshop scheduling problem with alternative operation assignments.

Lomnicki [8] introduced the concept of flow shop scheduling with the help of branch and bound method. Further the work was developed by Ignall and Scharge [5], Chandrasekharan [3] , Brown and Lomnicki [2], with the branch and bound technique to the machine scheduling problem by introducing different parameters In this paper we consider a two-stage flowshop scheduling problem with job-block with the help of branch and bound method. The given method is very simple and easy to understand. Thus, the problem discussed here has significant use of theoretical results in process industries.

2. Assumptions:

i. No passing is allowed.

ii. Each operation once started must performed till completion.

iii. Jobs are independent to each other.

iv. A job is entity, i.e. no job may be processed by more than one machine at a time.

3.Notations:

We are given n jobs to be processed on three stage flowshop scheduling problem and we have used the following notations:

Ai : Processing time for job ith on machine A
Bi : Processing time for job ith on machine B
Cij : Completion time for job i^{th} on machines A and B
S_0 : Optimal sequence

J_r : Partial schedule of r scheduled jobs

$J_{r'}$: The set of remaining (n-r) free jobs

4. Mathematical Development:

Consider n jobs say i=1, 2, 3 … n are processed on two machines A & B in the order AB. A job i (i=1,2,3…n) has processing time A_i & B_i on each machine respectively. . Let an equivalent job β is defined as (k, m) where k, m are any jobs among the given *n* jobs such that k occurs before job m in the order of job block (k , m).The mathematical model of the problem in matrix form can be stated as :

Jobs	Machine A	Machine B
i	A_i	B_i
1	A_1	B_1
2	A_2	B_2
3	A_3	B_3
4	A_4	B_4
-	----	---
-	--	--
n	A_n	B_n

Tableau – 1

Our objective is to obtain the optimal schedule of all jobs which minimize the total elapsed time, using branch and bound technique.

5. Algorithm:

Step 1: (i) $A_\beta = A_k + A_m - \min(A_k, B_m)$

(ii) $A_\beta = A_k + A_m - \min(A_k, B_m)$

Step 2: Calculate

(i) $l_1 = t(J_r, 1) + \sum_{i \in J_r'} A_i + \min_{i \in J_r'}(B_i)$

(ii) $l_1 = t(J_r, 2) + \sum_{i \in J_r'} B_i$

Step 3: Calculate

$$l = \max(l_1, l_2)$$

We evaluate l first for the n classes of permutations, i.e. for these starting with 1, 2, 3………n respectively, having labelled the appropriate vertices of the scheduling tree by these values.

Step 4: Now explore the vertex with lowest label. Evaluate l for the (n-1) subclasses starting with this vertex and again concentrate on the lowest label vertex. Continuing this way, until we reach at the end of the tree represented by two single permutations, for which we evaluate the total work duration. Thus we get the optimal schedule of the jobs.

Step 5: Prepare in-out table for the optimal sequence obtained in step 3 and get the minimum total elapsed time.

6.Numerical Example:

Consider 5 jobs 2 machine flow shop problem whose processing time of the jobs on each machine is given.

Job i	Machine A	Machine B
	A_i	B_i
1	11	17
2	15	21
3	22	25
4	34	19
5	27	38

Tableau – 2

Our objective is to obtain optimal schedule for above said problem in which jobs 2,4 are to be processed as a group job (2,4)

Solution:

Step1: Calculate

(i) $A_\beta = A_2 + A_4 - \min(A_4, B_2)$

= 28

(ii) $B_\beta = B_2 + B_4 - \min(A_4, B_2)$

= 21

The problem reduced as in tableau-3.

Step2: Calculate

(i) $l_1 = t(J_r,1) + \sum_{i \in J'_r} A_i + \min_{i \in J'_r}(B_i)$

(ii) $l_1 = t(J_r,2) + \sum_{i \in J'_r} B_i$

For J_1 = (1).Then J′(1) = {2,3,4}, we get l_1 = 43 , l_2 = 37

LB(1) $l = \max(l_1, l_2)$ =112 similarly,

we have LB(β)= 129, LB(3)= 123, LB(5)= 128

Step 3 & 4:

Now branch from J_1 = (1). Take J_2 =(1β). Then J'_2={3,4} and LB(1β) = 123

Proceeding in this way, we obtain lower bound values as shown in the tableau- 4

Step 5 :

Therefore the sequence S_1 is 1-3-5- β i.e. 1-3-5- 2 - 4 and the corresponding in-out table on sequence S_1 is as in tableau-5:

Hence the total elapsed time is 138 units.

7. Remarks:

The study may further be extended by considering various parameters such as transportation time, mean weightage time etc.

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Tables and Figures:

Table 3: The reduced as in tableau-3:

Job I	Machine A A_i	Machine B B_i
1	11	17
B	28	21
3	22	25
5	27	38

Tableau –3

Table 4: lower bounds for respective jobs are as in tableau-4

Tableau-4

Node Jr	LB (Jr)
(1)	112
(β)	129
(3)	123
(5)	128
(1β)	123
(13)	117
(15)	122
(13β)	126
(135)	119

Table 5: In-out table for S_1 and the minimum total elapsed time as in tableau-5.

Job I	Machine A In-out	Machine B In-out
1	0 - 11	11 – 28
3	11 – 33	33 – 58
5	33 – 60	60 - 98
3	60 -75	98 - 119
4	75 - 109	119 - 138

Tableau- 5