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Bicriteria in Constrained n x 3 Flow Shop to Minimize the Rental Cost, Setup Time, Processing Time Each Associated with Probabilities Including Transportation Time and Job Block Criteria

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Abstract

This paper is an attempt to obtain an optimal solution for minimizing the bicriteria taken as minimizing the total rental cost of the machines subject to obtain the minimum makespan for constrained n jobs 3 machines flowshop scheduling problem in which the processing times, independent set up times each associated with probabilities including transportation time and job block concept. The two criteria of minimizing the maximum utilization of the machines or rental cost of machines and minimizing the maximum makespan are one of the combinations of our objective function reflecting the performance measure. The proposed method is very simple and easy to understand. A computer programme followed by a numerical illustration is given to clarify the algorithm.

Keywords: Flowshop Scheduling, Heuristic, Processing Time, Set Up Time, Transportation Time, Rental Cost , Job Block.

1. Introduction

In flowshop scheduling, n jobs are to be processed on m machines in order to optimize some measures of performance. All jobs have the same processing requirements as they need to be processed on all machines in the same order. Three machine flowshop scheduling problem has been considered as a major sub problem due to its application in real life. Classical flow shop scheduling problems are mainly concerned with completion time related objectives, however, in modern manufacturing and operations management, the minimization of rental cost of machines and mean flow time are the significant factors as for the reason of upward stress of competition on the markets. . Industry has to offer a great variety of different and individual products while customers are expecting ordered goods to be delivered well in time with minimum possible cost. Hence, there is a requirement of bicriteria a subset of multicriteria through which the various objectives can be achieved simultaneously. The scheduling problem practically depends upon the important factors namely, Job Transportation which includes loading time, moving time and unloading time, Job block criteria which is due to priority of one job over the another, Setup time which includes work to prepare the machine, process or bench for product parts or the cycles so they are needed to be considered separately. These concepts are separately studied by various researchers Johnson (1954), Smith (1956), Maggu & Das (1977), Singh (1985), Bagga & Bhambani (1997), Brucker and Knust (2004), Chandramouli (2005), Singh and Kumar(2005), Chikhi (2008), Khodadadi,A (2008).

Gupta & Sharma (2011) studied bicriteria in n jobs three machines flowshop scheduling problem under specified rental policy, processing time associated with probabilities including job block and transportation time from one machine to the other machines is considered. The present paper is an attempt to extend the study made by Gupta & Sharma (2011) by introducing independent setup time with probabilities. The present paper differs with Gupta & Sharma (2011) in the sense that we have proposed heuristic algorithm for three machines based on Johnson's technique and the independent setup time required by jobs is considered. We have obtained an algorithm which gives minimum rental cost of machines while minimizing total elapsed time simultaneously. Thus the problem discussed here is wider and practically more applicable and has significant results in the process / production industry.

2. Practical Situation

Many applied and experimental situations exist in our day-to-day working in factories and industrial production concerns etc. In many manufacturing companies different jobs are processed on various machines. These jobs are required to process in a machine shop A, B, C, ---- in a specified order. When the machines on which jobs are to be processed are planted at different places the transportation time (which include loading time, moving time and unloading time etc.) has a significant role in production concern. Setup includes work to prepare the machine, .i.e. obtaining tools, positioning work-in-process material, return tooling, cleaning up, setting the required jigs and fixtures, adjusting tools and inspecting material and hence significant. Various practical situations occur in real life when one has got the assignments but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machine. Under such circumstances, the machine has to be taken on rent in order to complete the assignments. For example, In his starting career, we find a medical practitioner does not buy expensive machines say X-ray machine, the Ultra Sound Machine, Rotating Triple Head Single Positron Emission Computed Tomography Scanner, Patient Monitoring Equipment, and Laboratory Equipment etc., but instead takes on rent. Rental of medical equipment is an affordable and quick solution for hospitals, nursing homes, physicians, which are presently constrained by the availability of limited funds due to the recent global economic recession. Renting enables saving working capital, gives option for having the equipment, and allows upgradation to new technology. Further the priority of one job over the other may be significant due to the relative importance of the jobs. To be occurred as a block. It may be because of urgency or demand of that particular job. Hence, the job block criteria become important.

3. Notations

S : Sequence of jobs 1, 2, 3,….,n

S_k : Sequence obtained by applying Johnson's procedure, k = 1, 2 , 3, -------

M_j : Machine j, j= 1, 2, 3

M : Minimum makespan

a_{ij} : Processing time of i^{th} job on machine M_j

p_{ij} : Probability associated to the processing time a_{ij}

s_{ij} : Set up time of i^{th} job on machine M_j

q_{ij} : Probability associated to the set up time s_{ij}

A_{ij} : Expected processing time of i^{th} job on machine M_j

S_{ij} : Expected set up time of i^{th} job on machine M_j

β : Equivalent job for job – block

C_i : Rental cost of i^{th} machine

$L_j(S_k)$: The latest time when machine M_j is taken on rent for sequence S_k

$t_{ij}(S_k)$: Completion time of i^{th} job of sequence S_k on machine M_j

$t'_{ij}(S_k)$: Completion time of i^{th} job of sequence S_k on machine M_j when machine M_j start processing jobs at time $L_j(S_k)$

$T_{i,j\rightarrow k}$: Transportation time of i^{th} job from j^{th} machine to k^{th} machine

$I_{ij}(S_k)$: Idle time of machine M_j for job i in the sequence S_k

$U_j(S_k)$:Utilization time for which machine M_j is required, when M_j starts processing jobs at time $L_j(S_k)$

$R(S_k)$: Total rental cost of the machines for the sequence S_k of all machine

4. Rental Policy (P)

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required i.e. the first machine will be taken on rent in the starting of the processing the jobs, 2nd machine will be taken on rent at time when 1st job is completed on 1st machine and transported to 2nd machine, 3rd machine will be taken on rent at time when 1st job is completed on the 2nd machine and transported.

4.1 Definition: Completion time of i^{th} job on machine M_j is denoted by t_{ij} and is defined as

$$t_{ij} = max\,(t_{i-1,j} + s_{(i-1),j} \times q_{(i-1),j}\ ,\ t_{i,j-1}) + T_{i,(j-1)\to j} + a_{ij} \times p_{ij} \text{ for } j \geq 2.$$

$$= max\,(t_{i-1,j} + S_{i-1,j}\ ,\ t_{i,j-1}) + T_{i,(j-1)\to j} + A_{i,j}$$

where $A_{i,j}$= expected processing time of i^{th} job on machine *j*.

$S_{i,j}$= expected set up time of i^{th} job on machine *j*.

4.2 Definition: Completion time of i^{th} job on machine M_j when M_j starts processing jobs at time L_j is denoted by $t'_{i,j}$ and is defined as

$$t'_{i,j} = L_j + \sum_{k=1}^{i} A_{k,j} + \sum_{k=1}^{i-1} S_{k,j} = \sum_{k=1}^{i} I_{k,j} + \sum_{k=1}^{i} A_{k,j} + \sum_{k=1}^{i-1} S_{k,j}$$

Also $t'_{i,j} = \max(t'_{i,j-1} + S_{i-1,j}, t'_{i-1,j}) + A_{i,j} + T_{i,(j-1)\to j}$.

Theorem 1: The processing of jobs on M_3 at time $L_3 = \sum_{i=1}^{n} I_{i,3}$ keeps $t_{n,3}$ unaltered.

Proof: Let $t'_{i,3}$ be the competition time of i^{th} job on machine M_3 when M_3 starts processing of jobs at time L_3. We shall prove the theorem with the help of Mathematical Induction.

Let $P(n): t'_{n,3} = t_{n,3}$

Basic Step: For n = 1

$$t'_{1,3} = L_3 + A_{1,3} = I_{1,3} + A_{1,3}$$

$$= (A_{1,1} + (T_{1,1\to 2} + A_{1,2}) + T_{1,2\to 3}) + A_{1,3} = t_{1,3}.$$

Therefore P (1) is true.

Induction Step: Let P (k) be true.

i.e. $t'_{k,3} = t_{k,3}$.

Now, we shall show that P(k+1) is also true.

i.e. $t'_{k+1,3} = t_{k+1,3}$

But $t'_{k+1,3} = \max(t'_{k+1,2}, t'_{k,3} + S_{k,3}) + T_{k,2\to 3} + A_{k+1,3}$ (As per Definition 2)

$\therefore t'_{k+1,3} = \max(t_{k+1,2}, t_{k,3} + S_{k,3}) + T_{k,2\to 3} + A_{k+1,3}$ ($\because t'_{k,3} = t_{k,3}$, By Assumption)

$= t_{k+1,3}$ (by Definition)

$\Rightarrow P(k+1)$ is true .

Hence by principle of mathematical induction P(n) is true for all n, .i.e. $t'_{n,3} = t_{n,3}$.

Remarks: If M_3 starts processing jobs for minimum $L_3(S_r) = t_{n3}(S_r) - \sum_{i=1}^{n} A_{i,3} - \sum_{i=1}^{n-1} S_{i,3}$ then the total elapsed time $t_{n3}(S_r) = L_3(S_r) + \sum_{i=1}^{n} A_{i3} + \sum_{i=1}^{n-1} S_{i3}$ is not altered and M_3 is engaged for minimum time.

Lemma 1.1: If M_3 starts processing jobs at $L_3 = \sum_{i=1}^{n} I_{i,3}$ then

(i). $L_3 > t_{1,2}$

(ii). $t'_{k+1,3} \geq t_{k,2}$, $k > 1$.

Theorem 2: The processing of jobs on M_2 at time $L_2 = \min_{1\leq k\leq n}\{Y_k\}$ keeps total elapsed time unaltered where $Y_1 = L_3 - A_{1,2} - T_{1,1\to 2}$ and $Y_k = t'_{k-1,3} - \sum_{i=1}^{k} A_{i,2} - \sum_{i=1}^{k} T_{i,2\to 3} - \sum_{i=1}^{k-1} S_{i,2}; k > 1.$

Proof: We have

$$L_2 = \min_{i\le k\le n}\{Y_k\} = Y_r \text{ (say)}$$

In particular for k =1

$$Y_r \le Y_1$$

$$\Rightarrow Y_r + A_{1,2} + T_{1,1\to 2} \le Y_1 + A_{1,2} + T_{1,1\to 2}$$

$$\Rightarrow Y_r + A_{1,2} + T_{1,1\to 2} \le L_3 \qquad \text{---- (1)} \qquad \left(\because Y_1 = L_3 - A_{1,2} - T_{1,1\to 2}\right)$$

By Lemma 1; we have

$$t_{1,2} \le L_3 \qquad \text{---- (2)}$$

Also, $t'_{1,2} = \max\left(Y_r + A_{1,2} + T_{1,1\to 2}, t_{1,2}\right)$

On combining, we get

$$t'_{1,2} \le L_3$$

For k >1, As $Y_r = \min_{i\le k\le n}\{Y_k\}$

$$\Rightarrow Y_r \le Y_k; \qquad k = 2,3.........,n$$

$$\Rightarrow Y_r + \sum_{i=1}^{k} A_{i,2} + \sum_{i=1}^{k} T_{i,2\to 3} + \sum_{i=1}^{k-1} S_{i,3} \le Y_k + \sum_{i=1}^{k} A_{i,2} + \sum_{i=1}^{k} T_{i,2\to 3} + \sum_{i=1}^{k-1} S_{i,3}$$

$$\Rightarrow Y_r + \sum_{i=1}^{k} A_{i,2} + \sum_{i=1}^{k} T_{i,2\to 3} + \sum_{i=1}^{k-1} S_{i,3} \le t'_{k-1,3} \qquad \text{---- (3)}$$

By Lemma 1; we have

$$t_{k,2} \le t'_{k-1,3} \qquad \text{---- (4)}$$

Also, $t'_{k,2} = \max\left(Y_r + \sum_{i=1}^{k} A_{i,2} + \sum_{i=1}^{k} T_{i,2\to 3} + \sum_{i=1}^{k-1} S_{i,3}, t_{k,2}\right)$

Using (3) and (4) , we get

$$t'_{k,2} \le t'_{k-1,3}$$

Taking $k = n$, we have

$$t'_{n,2} \le t'_{n-1,3} \qquad \text{---- (5)}$$

Total time elapsed = $t_{n,3} = \max\left(t'_{n,2}, t'_{n-1,3} + S_{n-1,3}\right) + A_{n,3} + T_{n,2\to 3}$

$$= t'_{n-1,3} + S_{n-1,3} + A_{n,3} + T_{n,2\to 3} = t'_{n,3}.$$

Hence, the total time elapsed remains unaltered if M_2 starts processing jobs at time $L_2 = \min_{i\le k\le n}\{Y_k\}$.

By Theorem 1, if M_3 starts processing jobs at time $L_3(S_r) = t_{n3}(S_r) - \sum_{i=1}^{n} A_{i,3} - \sum_{i=1}^{n-1} S_{i,3}$ then the total elapsed time $t_{n,3}$ is not altered and M_3 is engaged for minimum time equal to utilization time of M_3 Moreover total elapsed time/rental cost of M_1 is always least as utilization time of M_1 is always minimum. Therefore the objective remains to minimize the elapsed time and hence the rental cost of M_2.

The following algorithm provides the procedure to determine the times at which machines should be taken on rent to minimize the total rental cost without altering the total elapsed time in three machine flow shop problem under rental policy (P).

5. Problem Formulation

Let some job i (i = 1,2,……..,n) are to be processed on three machines M_j (j = 1,2,3) under the specified rental policy P. Let a_{ij} be the processing time of i^{th} job on j^{th} machine with probabilities p_{ij} and s_{ij} be the setup time of i^{th} job on j^{th} machine with probabilities q_{ij}. Let A_{ij} be the expected processing time and $S_{i,j}$ be the expected setup time of i^{th} job on j^{th} machine. Let $T_{i,j\to k}$ be the transportation time of i^{th} job from j^{th} machine to k^{th} machine. Our aim is to find the sequence $\{S_k\}$ of the jobs which minimize the rental cost of all the three machines while minimizing total elapsed time.

The mathematical model of the problem in matrix form can be stated as:

Jobs	Machine A				$T_{i,1\to 2}$	Machine B				$T_{i,2\to 3}$	Machine C			
i	a_{i1}	p_{i1}	s_{i1}	q_{i1}		a_{i2}	p_{i2}	s_{i2}	q_{i2}		a_{i3}	p_{i3}	s_{i3}	q_{i3}
1	a_{11}	p_{11}	s_{11}	q_{11}	$T_{1,1\to 2}$	a_{12}	p_{12}	s_{12}	q_{12}	$T_{1,2\to 3}$	a_{13}	p_{13}	s_{13}	q_{13}
2	a_{21}	p_{21}	s_{21}	q_{21}	$T_{2,1\to 2}$	a_{22}	p_{22}	s_{22}	q_{22}	$T_{2,2\to 3}$	a_{23}	p_{23}	s_{23}	q_{23}
3	a_{31}	p_{31}	s_{31}	q_{31}	$T_{3,1\to 2}$	a_{32}	p_{32}	s_{32}	q_{32}	$T_{3,2\to 3}$	a_{33}	p_{33}	s_{33}	q_{33}
4	a_{41}	p_{41}	s_{41}	q_{41}	$T_{4,1\to 2}$	a_{42}	p_{42}	s_{42}	q_{42}	$T_{4,2\to 3}$	a_{43}	p_{43}	s_{43}	q_{43}
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
n	a_{n1}	p_{n1}	s_{n1}	q_{n1}	$T_{n,1\to 2}$	a_{n2}	p_{n2}	s_{n2}	q_{n2}	$T_{n,2\to 3}$	a_{n3}	p_{n3}	s_{n3}	q_{n3}

Table 1

Minimize $U_j(S_k)$ and Minimize $R(S_k) = t_{n1}(S_k)\times C_1 + U_2(S_k)\times C_2 + U_3(S_k)\times C_3$

Subject to constraint: Rental Policy (P)

Our objective is to minimize rental cost of machines while minimizing total elapsed time.

6. Algorithm

Step 1: Calculate the expected processing times and expected set up times as follows

$$A_{ij} = a_{ij}\times p_{ij} \text{ and } S_{ij} = s_{ij}\times q_{ij} \qquad \forall i, j = 1,2,3$$

Step 2: Check the condition

either $\text{Min}\{A_{i1} + T_{i,1\to 2} - S_{i2}\} \geq \text{Max}\{A_{i2} + T_{i,1\to 2} - S_{i1}\}$

or $\text{Min}\{A_{i3} + T_{i,2\to 3} - S_{i2}\} \geq \text{Max}\{A_{i2} + T_{i,2\to 3} - S_{i3}\}$ or both for all i

If the conditions are satisfied then go to step 3, else the data is not in the standard form.

Step 3: Introduce the two fictitious machines G and H with processing times G_i and H_i as

$G_i = A_{i1} + A_{i2} + \max(S_{i1}, S_{i2}) + T_{i,1\to 2}$, $H_i = A_{i2} + A_{i3} - S_{i3} + T_{i,2\to 3}$

Step 4: Find the expected processing time of job block β = (k, m) on fictitious machines G & H using equivalent job block criterion given by Maggu & Das (1977). Find G_β and H_β using

$G_\beta = G_k + G_m - \min(G_m, H_k)$, $H_\beta = H_k + H_m - \min(G_m, H_k)$

Step 5: Define new reduced problem with processing time G_i & H_i as defined in step 3 and replace job block (k,m) by a single equivalent job β with processing times G_β & H_β as defined in step 4

Step 6: Using Johnson's procedure, obtain all sequences S_k having minimum elapsed time. Let these be $S_1, S_2, \ldots, S_r$

Step 7: Prepare In – Out tables for S_k and compute total elapsed time $t_{n3}(S_k)$

Step 8: Compute latest time L_3 for machine M_3 for sequence S_k as

$$L_3(S_k) = t_{n3}(S_k) - \sum_{i=1}^{n} A_{i3} - \sum_{i=1}^{n-1} S_{i,3}(S_k)$$

Step 9: For the sequence S_k (k = 1,2,…………...,r), compute

I. $t_{n2}(S_k)$

II. $Y_1(S_k) = L_3(S_k) - A_{1,2}(S_k) - T_{1,1\to 2}$

III. $Y_q(S_k) = L_3(S_k) - \sum_{i=1}^{q} A_{i2}(S_k) - \sum_{i=1}^{q} T_{i,2\to 3} - \sum_{i=1}^{q-1} S_{i,2}(S_k) + \sum_{i=1}^{q-1} A_{i,3} + \sum_{i=1}^{q-1} T_{i,1\to 2} + \sum_{i=1}^{q-2} S_{i3}(S_k); q = 2,3,\ldots\ldots,n$

IV. $L_2(S_k) = \min_{1 \le q \le n} \{Y_q(S_k)\}$

V. $U_2(S_k) = t_{n2}(S_k) - L_2(S_k), U_3(S_k) = t_{n3}(S_k) - L_3(S_k)$.

Step 10: Find min $\{U_2(S_k)\}; k = 1, 2, ..., r$

Let it be for the sequence S_p, and then sequence S_p will be the optimal sequence.

Step 11: Compute total rental cost of all the three machines for sequence S_p as:

$$R(S_p) = t_{n1}(S_p) \times C_1 + U_2(S_p) \times C_2 + U_3(S_p) \times C_3$$

8. Programme

```
#include<iostream.h>
#include<stdio.h>
#include<conio.h>
#include<process.h>
int n,j;
float a1[16],b1[16],c1[16],g[16],h[16],T12[16],T23[16],s11[16],s22[16],s33[16];
float macha[16],machb[16],machc[16];float cost_a,cost_b,cost_c,cost;int f=1;
int group[2];//variables to store two job blocks
float minval,minv,maxv1[16],maxv2[16];float gbeta=0.0,hbeta=0.0;
void main()
{
	clrscr();
	int a[16],b[16],c[16],j[16],s1[16],s2[16],s3[16];float p[16],q[16],r[16],u[16],v[16],w[16];
	cout<<"How many Jobs (<=15) : ";cin>>n;
	if(n<1||n>15)
	{cout<<endl<<"Wrong input, No. of jobs should be less than 15..\n Exitting";getch();exit(0);}
	for(int i=1;i<=n;i++)
		{j[i]=i;
cout<<"\nEnter the processing time ,setup time and their probability of "<<i<<" job for machine A and Transportation time from Machine A to B : ";cin>>a[i]>>p[i]>>s1[i]>>u[i]>>T12[i];
cout<<"\nEnter the processing time ,set up time and their probability of "<<i<<" job for machine B and Transportation time from Machine B to C : ";cin>>b[i]>>q[i]>>s2[i]>>v[i]>>T23[i];
cout<<"\nEnter the processing timesetup time and their probability of "<<i<<"job for machine C: ";
cin>>c[i]>>r[i]>>s3[i]>>w[i];}
for(i=1;i<=n;i++)
{//Calculate the expected processing times of the jobs for the machines:
a1[i] = a[i]*p[i];b1[i] = b[i]*q[i];c1[i] = c[i]*r[i];s11[i]= s1[i]*u[i];s22[i]= s2[i]*v[i];s33[i]= s3[i]*w[i];}
cout<<"\nEnter the rental cost of Machine M1:";cin>>cost_a;
cout<<"\nEnter the rental cost of Machine M2:";cin>>cost_b;
cout<<"\nEnter the rental cost of Machine M3:";cin>>cost_c;
cout<<endl<<"Expected processing and setup time of machine A, B and C: \n";
for(i=1;i<=n;i++)
{cout<<j[i]<<"\t"<<a1[i]<<"\t"<<s11[i]<<"\t"<<b1[i]<<"\t"<<s22[i]<<"\t"<<c1[i]<<"\t"<<s33[i]<<"\t"
;
cout<<endl;}
//Function for two ficticious machine G and H//Finding smallest in a1
float mina1;mina1=a1[1]+T12[1]-s22[1];
```



```
for(i=2;i<n;i++)
{if(a1[i]+T12[i]-s22[i]<mina1)mina1=a1[i]+T12[i]-s22[i];}
//For finding largest in b1
float maxb1;maxb1=b1[1]+T23[1]-s11[1];
for(i=2;i<n;i++)
{if(b1[i]+T23[i]-s11[i]>maxb1)maxb1=b1[i]+T23[i]-s11[i];}
float maxb2;maxb2=b1[1]+T23[1]-s33[1];
for(i=2;i<n;i++)
{if(b1[i]+T23[i]-s33[i]>maxb2)maxb2=b1[i]+T23[i]-s33[i];}
//Finding smallest in c1
float minc1;minc1=c1[1]+T23[1]-s22[1];
for(i=2;i<n;i++)
{if(c1[i]+T23[i]-s22[1]<minc1)minc1=c1[i]+T23[i]-s22[1];}
float maxs;
        if(mina1>=maxb1||minc1>=maxb2)
        {for(i=1;i<=n;i++)
        {if(s11[i]<s22[i])
        {maxs=s22[i];}
else    {maxs=s11[i];}
         g[i]= a1[i]+b1[i]+T12[i]+maxs; h[i]=b1[i]+c1[i]+T23[i]-s33[i];}}
else    {cout<<"\n data is not in Standard Form...\nExitting";getch();exit(0);}
cout<<endl<<"Expected processing time for two fictious machines G and H: \n";
 for(i=1;i<=n;i++)
  {cout<<endl;cout<<j[i]<<"\t"<<g[i]<<"\t"<<h[i]; cout<<endl;  }
  cout<<"\nEnter the two job blocks(two numbers from 1 to "<<n<<"):"; cin>>group[0]>>group[1];
  //calculate G_Beta and H_Beta
if(g[group[1]]<h[group[0]])
        {minv=g[group[1]];}
else    {minv=h[group[0]];}
gbeta=g[group[0]]+g[group[1]]-minv;hbeta=h[group[0]]+h[group[1]]-minv;
cout<<endl<<endl<<"G_Beta="<<gbeta;cout<<endl<<"H_Beta="<<hbeta;
int j1[16];float g1[16],h1[16];
        for(i=1;i<=n;i++)
        {if(j[i]==group[0]||j[i]==group[1])
        {f--;}
else    {j1[f]=j[i];}f++;}j1[n-1]=17;
for(i=1;i<=n-2;i++)
{g1[i]=g[j1[i]];h1[i]=h[j1[i]];}
g1[n-1]=gbeta;h1[n-1]=hbeta;
cout<<endl<<endl<<"displaying original scheduling table"<<endl;
for(i=1;i<=n-1;i++)
{cout<<j1[i]<<"\t"<<g1[i]<<"\t"<<h1[i]<<endl;}
float mingh[16];char ch[16];
   for(i=1;i<=n-1;i++)
   {if(g1[i]<h1[i])
                 {mingh[i]=g1[i];ch[i]='g'; }
        else     {mingh[i]=h1[i];ch[i]='h'; }}
for(i=1;i<=n-1;i++)
        {for(int j=1;j<=n-1;j++)
```



```
if(mingh[i]<mingh[j])
    {float temp=mingh[i]; int temp1=j1[i]; char d=ch[i];mingh[i]=mingh[j]; j1[i]=j1[j];
ch[i]=ch[j];
              mingh[j]=temp; j1[j]=temp1; ch[j]=d;}  }
// calculate beta scheduling
float sbeta[16];int t=1,s=0;
for(i=1;i<=n-1;i++)
    {if(ch[i]=='h')
    { sbeta[(n-s-1)]=j1[i]; s++;}
else    if(ch[i]=='g')
    {sbeta[t]=j1[i];t++;}}
int arr1[16], m=1;
    cout<<endl<<endl<<"Job Scheduling:"<<"\t";
for(i=1;i<=n-1;i++)
    {if(sbeta[i]==17)
    {      arr1[m]=group[0];arr1[m+1]=group[1];cout<<group[0]<<"       "<<group[1]<<"
";m=m+2;continue;}
else    {cout<<sbeta[i]<<" ";arr1[m]=sbeta[i];m++;}}
//calculating total computation sequence
float time=0.0;macha[1]=time+a1[arr1[1]];
for(i=2;i<=n;i++)
    {macha[i]=macha[i-1]+a1[arr1[i]]+s11[arr1[i-
1]];}machb[1]=macha[1]+b1[arr1[1]]+T12[arr1[1]];
for(i=2;i<=n;i++)
    {if((machb[i-1]+s22[arr1[i]])>(macha[i]+T12[arr1[i]]))maxv1[i]=machb[i-1]+s22[arr1[i]];
else    maxv1[i]=macha[i]+T12[arr1[i]];machb[i]=maxv1[i]+b1[arr1[i]];}
    machc[1]=machb[1]+c1[arr1[1]]+T23[arr1[1]];
for(i=2;i<=n;i++)
    {if((machc[i-1]+s33[arr1[i-1]])>(machb[i]+T23[arr1[i]]))maxv2[i]=machc[i-1]+s33[arr1[i-
1]];
else    maxv2[i]=machb[i]+T23[arr1[i]];machc[i]=maxv2[i]+c1[arr1[i]];}
//displaying solution
cout<<"\n\n\n\n\n\t\t\t   #####THE SOLUTION##### ";
cout<<"\n\n\t***************************************************************";
cout<<"\n\n\n\t   Optimal Sequence is : ";
for(i=1;i<=n;i++)
    {cout<<" "<<arr1[i];}
    cout<<endl<<endl<<"In-Out Table is:"<<endl<<endl;
cout<<"Jobs"<<"\t"<<"Machine   M1"<<"\t"<<"\t"<<"Machine   M2"   <<"\t"<<"\t"<<"Machine
M3"<<endl;
cout<<arr1[1]<<"\t"<<time<<"--"<<macha[1]<<"          \t"<<"\t"<<macha[1]+T12[arr1[1]]<<"--
"<<machb[1]<<" \t"<<"\t"<<machb[1]+T23[arr1[1]]<<"--"<<machc[1]<<endl;
for(i=2;i<=n;i++)
{cout<<arr1[i]<<"\t"<<macha[i-1]+s11[arr1[i]]<<"--"<<macha[i]<<"       "<<"\t"<<maxv1[i]<<"--
"<<machb[i]<<" "<<"\t"<<maxv2[i]<<"--"<<machc[i]<<endl;}
cout<<"\n\n\nTotal Elapsed Time (T) = "<<machc[n];float L3,Y[16],min,u2,u3;
float sum1=0.0,sum2=0.0,sum3=0.0;
for(i=1;i<=n;i++)
{sum1=sum1+a1[i];sum2=sum2+b1[i];sum3=sum3+c1[i]; }float sum_1;
for(i=1;i<=n;i++)
{sum_1=0.0,s33[0]=0.0;
```

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Vol 2, No.1, 2012

```
for(int d=0;d<=n-1;d++)
{sum_1=sum_1+s33[arr1[d]];}}L3=machc[n]-sum3-sum_1;
cout<<"\n\nLatest Time When Machine M3 is Taken on Rent:"<<L3;
cout<<"\n\nTotal Completion Time of Jobs on Machine M2:"<<machb[n];
Y[1]=L3-b1[arr1[1]]-T12[arr1[1]];cout<<"\n\n\tY[1]\t="<<Y[1];float sum_2,sum_3;
for(i=2;i<=n;i++)
{sum_2=0.0,sum_3=0.0;for(int j=1;j<=i-1;j++)
{sum_3=sum_3+c1[arr1[j]]+T12[arr1[j]]+s33[arr1[j-1]];}
for(int k=1;k<=i;k++)
{sum_2=sum_2+b1[arr1[k]]+T23[arr1[k]]+s22[arr1[k-1]];}
Y[i]=L3+sum_3-sum_2;cout<<"\n\n\tY["<<i<<"]\t="<<Y[i];}
min=Y[1];
for(i=2;i<n;i++)
{if(Y[i]<min)min=Y[i];}
cout<<"\n\nMinimum of Y[i]="<<min;u2=machb[n]-min;u3=machc[n]-L3;
cout<<"\n\n Utiliztaion time of machine M1 = "<<macha[n];cout<<"\n\nUtilization Time of Machine M2="<<u2;
cout<<"\n\n Utiliztion time of machine M3 = "<<u3;cost=(macha[n]*cost_a)+(u2*cost_b)+(u3*cost_c);
cout<<"\n\nThe Minimum Possible Rental Cost is="<<cost;
cout<<"\n\n\t***************************************************************";
getch();}
```

7. Numerical Illustration

Consider 5 jobs, 3 machine flow shop problem with processing time , setup time associated with their respective probabilities and transportation time as given in table and jobs 2 and 5 are processed as a group job (2, 5). The rental cost per unit time for machines M_1, M_2 and M_3 are 6 units, 7 units and 8 units respectively, under the rental policy P.

Jobs	Machine M_1				$T_{i,1\to 2}$	Machine M_2				$T_{i,2\to 3}$	Machine M_3			
i	a_{i1}	p_{i1}	s_{i1}	q_{i1}		a_{i2}	p_{i2}	s_{i2}	q_{i2}		a_{i3}	p_{i3}	s_{i3}	q_{i3}
1	16	0.2	6	0.1	2	4	0.2	7	0.1	2	12	0.1	3	0.2
2	12	0.3	7	0.2	1	6	0.2	6	0.3	1	8	0.2	4	0.3
3	13	0.2	4	0.3	2	5	0.2	3	0.4	2	15	0.2	6	0.2
4	15	0.2	7	0.3	3	4	0.2	3	0.1	3	4	0.2	5	0.1
5	14	0.1	4	0.1	4	6	0.2	6	0.1	1	6	0.3	4	0.2

Table - 2

Our objective is to obtain an optimal schedule for above said problem to minimize the total production time / total elapsed time subject to minimization of the total rental cost of the machines.

Solution: As per Step 1; The expected processing times and expected setup times for machines M_1, M_2 and M_3 are as shown in table 3.

Using steps 2, 3, 4, 5 and 6 and Johnson's method, the optimal sequence is

$$S = 3 - 4 - 1 - \beta \text{ , .i.e. } S = 3 - 4 - 1 - 2 - 5$$

As per Step 7: The In – Out table for the optimal sequence S is as shown in table 4.

Total elapsed time $t_{n3}(S)$ = 27.1 units

As per Step 8: $L_3(S) = t_{n3}(S) - \sum_{i=1}^{n} A_{i,3} - \sum_{i=1}^{n-1} S_{i,3}(S)$ = 27.1 – 8.4 – 3.5 = 15.2 units

As per Step 9: For sequence S, we have

$$t_{n,1}(S) = 19.1\,,\ t_{n,2}(s) = 24.3\,,\ t_{n,3}(s) = 27.1$$

$$Y_1 = 15.2 - 1.0 - 2 = 12.2, Y_2 = 15.2 - 8 + 5 = 12.2, Y_3 = 15.2 - 11.1 + 10 = 14.1$$

$$Y_4 = 15.2 - 14 + 13.7 = 14.9, Y_5 = 15.2 - 18 + 16.9 = 14.1$$

$$L_2(S) = Min\{Y_k\} = 12.2, U_2(S) = t_{n2}(S) - L_2(S) = 24.3 - 12.2 = 12.1$$

$$U_3(S) = t_{n,3}(S) - L_3(S) = 27.1 - 15.2 = 11.9.$$

The new reduced Bi-objective In – Out table is as shown in table 5.

The latest possible time at which machine M_2 should be taken on rent = L_2(S) = 12.2 units.

Also, utilization time of machine M_2 = U_2(S) = 12.1 units.

Total Minimum rental cost of machines = $R(S_p) = t_{n1}(S_p) \times C_1 + U_2(S_p) \times C_2 + U_3(S_p) \times C_3$

= 19.1 × 6 + 12.1 × 7 +11.9 = 294.5 units.

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Tables:

Table 3: The expected processing times and expected setup times for machines M_1, M_2 and M_3 are

Jobs	A_{i1}	S_{i1}	$T_{i,1\to2}$	A_{i2}	S_{i2}	$T_{i,2\to3}$	A_{i3}	S_{i3}
1	3.2	0.6	2	0.8	0.7	2	1.2	0.6
2	3.6	1.4	1	1.2	1.8	1	1.6	1.2
3	2.6	1.2	2	1.0	1.2	2	3.0	1.2
4	3.0	2.1	3	0.8	0.3	3	0.8	0.5
5	1.4	0.4	4	1.2	0.6	1	1.8	0.8

Table 4: The In – Out table for the optimal sequence S is

Jobs	Machine M_1	$T_{i,1\to2}$	Machine M_2	$T_{i,2\to3}$	Machine M_3
i	In – Out		In – Out		In - Out
3	0.0 – 2.6	2	4.6 – 5.6	2	7.6 – 10.6
4	3.8 – 6.8	3	9.8 – 10.6	3	13.6 – 14.4
1	8.9 – 12.1	2	14.1 – 14.9	2	16.9 – 18.1
2	12.7 – 16.3	1	17.3 – 18.5	1	19.5 – 21.1
5	16.7 – 19.1	4	23.1 – 24.3	1	25.3 – 27.1

Table 5: The new reduced Bi-objective In – Out table is

Jobs	Machine M_1	Machine M_2	Machine M_3
i	In – Out	In – Out	In - Out
3	0 – 2.6	12.2 – 13.2	15.2 – 18.2
4	3.8 – 6.8	14.4 – 15.2	19.4 – 20.2
1	8.9 – 12.1	15.5 – 16.3	20.7 – 21.9
2	12.7 – 16.3	17.3 – 18.5	22.5 – 24.1
5	16.7 – 19.1	23.1 – 24.3	25.3 – 27.1