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Branch and Bound Technique for three stage Flow Shop Scheduling Problem Including Breakdown Interval and Transportation Time

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Abstract

This paper deals with minimization of the total elapsed time for nx3 flow shop scheduling problem in which the effect of breakdown interval and the transportation time are considered. A Branch and Bound technique is given to optimize the objective of minimize the total elapsed time. The algorithm is very simple and easy to understand and, also provide an important tool for decision makers to design a schedule. A numerical illustration is given to clarify the algorithm.

Keywords: Flow shop scheduling, Processing time, Transportation time, Branch and Bound Technique, Optimal sequence.

1. Introduction:

Scheduling problems are common occurrence in our daily life e.g. ordering of jobs for processing in a manufacturing plant, programs to be run in a sequence at a computer center etc. Such problems exist whenever there is an alternative choice in which a number of jobs can be done. Now-a-days, the decision makers for the manufacturing plant have interest to find a way to successfully manage resources in order to produce products in the most efficient way. They need to design a production schedule to minimize the flow time of a product. The number of possible schedules in a flow shop scheduling problem involving n-jobs and m-machines is $(n!)^m$. The optimal solution for the problem is to find the optimal or near optimal sequence of jobs on each machine in order to minimize the total elapsed time.Johnson (1954) first of all gave a method to minimise the makespan for n-job, two-machine scheduling problems. The scheduling problem practically depends upon the important factors namely, Transportation time, break down effect, Relative importance of a job over another job etc. These concepts were separately studied by Ignall and Scharge (1965), Maggu and Dass (1981), Temiz and Erol(2004),Yoshida and Hitomi (1979), Lomnicki (1965), Palmer (1965) , Bestwick and Hastings (1976), Nawaz et al. (1983) , Sarin and Lefoka (1993) , Koulamas (1998) , Dannenbring (1977) , etc.

Singh T.P. and Gupta Deepak (2005)studied the optimal two stage production schedule in which processing time and set up time both were associated with probabilities including job block criteria. Heydari (2003)dealt with a flow shop scheduling problem where n jobs are processed in two disjoint job blocks in a string consists of one job block in which order of jobs is fixed and other job block in which order of jobs is arbitrary. Lomnicki (1965) introduced the concept of flow shop scheduling with the help of branch and bound method. Further the work was developed by Ignall and Scharge (1965), Chandrasekharan (1992), Brown and Lomnicki(1966) , with the branch and bound technique to the machine scheduling problem by introducing different parameters. The concept of transportation time is very important in scheduling when the machines are distantly situated. The break down of the machines have significant role in the production concern. The effect of break down interval is important as there are feasible situations where machine during process may get sudden break down due to either failure of any component of machine or the machines are supposed to stop their working for a certain interval of time due to some external imposed policy such as electric cut/shortage due to government policy. The working of machine no longer remains continuous and is subject to break-down for a certain interval of time. This paper extends the study made by Ignall and Scharge (1965) by introducing the concept of transportation time and break down interval. Hence the problem discussed here is wider and has significant use of theoretical results in process industries.

2 Practical Situation

Many applied and experimental situations exist in our day-to-day working in factories and industrial production concerns etc. In many manufacturing companies different jobs are processed on various machines. These jobs are required to process in a machine shop A, B, C, ---- in a specified order. When the machines on which jobs are to be processed are planted at different places, the transportation time (which includes loading time, moving time and unloading time etc.) has a significant role in production concern. The break down of the machines (due to delay in material, changes in release and tails date, tool unavailability, failure of electric current, the shift pattern of the facility, fluctuation in processing times, some technical interruption etc.) have significant role in the production concern.

3 Notations:

We are given n jobs to be processed on three stage flowshop scheduling problem and we have used the following notations:

A_i	:	Processing time for job i on machine A
B_i	:	Processing time for job i on machine B
C_i	:	Processing time for job i on machine C
C_{ij}	:	Completion time for job i on machines A, B and C
t_i	:	Transportation time of i^{th} job from machine A to machine B.
g_i	:	Transportation time of i^{th} job from machine B to machine C.
S_k	:	Sequence using johnson's algorithm
L	:	Length of break down interval.
J_r	:	Partial schedule of r scheduled jobs
$J_{r'}$	:	The set of remaining (n-r) free jobs

4 Mathematical Development:

Consider n jobs say i=1, 2, 3 … n are processed on three machines A, B & C in the order ABC. A job i (i=1,2,3…n) has processing time A_i , B_i & C_i on each machine respectively, assuming their respective probabilities p_i , q_i & r_i such that $0 \le p_i \le 1$, $\Sigma p_i = 1$, $0 \le q_i \le 1$, $\Sigma q_i = 1$, $0 \le r_i \le 1$, $\Sigma r_i = 1$. Let t_i and g_i be the transportation time of machine A to machine B and machine B to machine C respectively. The mathematical model of the problem in matrix form can be stated as :

Jobs	Machine A		Machine B		Machine C
i	A_i	t_i	B_i	g_i	C_i
1	A_1	t_1	B_1	g_1	C_1
2	A_2	t_2	B_2	g_2	C_2
3	A_3	t_3	B_3	g_3	C_3
4	A_4	t_4	B_4	g_4	C_4
-	----	---	---	---	----
-	--	--	--	---	--
n	A_n	t_n	B_n	g_n	C_n

Tableau – 1

Our objective is to obtain the optimal schedule of all jobs which minimize the total elapsed time whenever the effect of break down interval (a, b) is given, using branch and bound technique.

5 Algorithm:

Step1: Calculate

(i) $g_1 = t(J_r,1) + \sum_{i \in J_r'} A_i + \min_{i \in J_r'}(B_i + C_i)$

(ii) $g_2 = t(J_r,2) + \sum_{i \in j_r'} B_i + \min_{i \in J_r'}(C_i)$

(iii) $g_3 = t(J_r,3) + \sum_{i \in j_r'} C_i$

Step 2: Calculate g = max [g_1, g_2, g_3] We evaluate g first for the n classes of permutations, i.e. for these starting with 1, 2, 3………n respectively, having labelled the appropriate vertices of the scheduling tree by these values.

Step 3: Now explore the vertex with lowest label. Evaluate g for the (n-1) subclasses starting with this vertex and again concentrate on the lowest label vertex. Continuing this way, until we reach at the end of the tree represented by two single permutations, for which we evaluate the total work duration. Thus we get the optimal schedule of the jobs.

Step 4: Prepare in-out table for the optimal sequence obtained in step 4 and read the effect of break down interval (a, b) on different jobs.

Step 5: Form a modified problem with processing times p'_{i1}, p'_{i2} & p'_{i3} on machines A, B & C respectively. If the break down interval (a, b) has effect on job i then $p'_{i1} = p_{i1} + L$, $p'_{i2} = p_{i2} + L$ and $p'_{i3} = p_{i3} + L$ where L = b – a, the length of the break down interval.

If the break down interval (a, b) has no effect on job i then $p'_{i1} = p_{i1}$, $p'_{i2} = p_{i2}$ and $p'_{i3} = p_{i3}$.

Step 6: Repeat the procedure to get the optimal sequence for the modified scheduling problem using step1 to step 3. Compute the in-out table and get the minimum total elapsed time.

6 Numerical Example:

Consider 5 jobs 3 machine flow shop problem. processing time of the jobs on each machine is given. Our objective is to obtain the optimal schedule of all jobs which minimize the total elapsed time whenever the effect of break down interval (25, 35) is given.

Jobs	Machine A		Machine B		Machine C
I	A_i	t_i	B_i	g_i	C_i
1	15	4	20	5	16
2	25	7	10	8	5
3	10	6	12	3	12
4	18	9	15	7	18
5	16	2	25	6	3

Tableau – 2

Solution:Step1: Calculate

(i) $g_1 = t(J_r,1) + \sum_{i\in J'_r} A_i + \min_{i\in J'_r}(B_i + C_i)$

(ii) $g_2 = t(J_r,2) + \sum_{i\in j'_r} B_i + \min_{i\in J'_r}(C_i)$

(iii) $g_3 = t(J_r,3) + \sum_{i\in j'_r} C_i$

For J_1 = (1).Then J′(1) = {2,3,4}, we get g_1 = 43 , g_2 = 37 & g_3 = 43

g = max(g_1, g_2, g_3) = 43 similarly, we have LB(2)= 51 , LB(3)= 52 and LB(4)= 58

Step 2 & 3: Now branch from J_1 = (1). Take J_2 =(12). Then J'_2={3,4} and LB(12) = 51

Proceeding in this way, we obtain lower bound values on the completion time on machine C as shown in the tableau- 3

*Step 4 :*Therefore the sequence S_1 is 1-3-4-2 and the corresponding in-out table and checking the effect of break down interval (25, 35) on sequence S_1 is as in tableau4:

Step 5: The modified problem after the effect of break down interval (25,35) with processing times A'_i, B'_i and C'_i on machines A, B & C respectively is as in tableau-5:

Step 6: Now, on repeating the procedure to get the optimal sequence for the modified scheduling problem using step 1 to step 3, we obtain lower bound values on the

completion time on machine C as shown in the tableau- 6 we have get the sequence S_2 : 3-1-4-5-2. Compute the in-out table for S_2 and get the minimum total elapsed time as in tableau-7.
Hence the total elapsed time is 131 units.

References :

[1] Brown, A.P.G. and Lomnicki, Z.A. (1966), "Some applications of the branch and bound algorithm to the machine scheduling problem", Operational Research Quarterly, Vol. 17, pp.173-182.

[2] Bestwick, P.F. and Hastings, N.A.J. (1976), "A new bound for machine scheduling", Operational Research Quarterly, Vol. 27, pp.479-490.

[3] Cormen, T.H., Leiserson, C.E. and Rivest, R.L. (1990), "Introduction to Algorithms", Cambridge, MA: MIT Press.

[4] Chandramouli, A.B.(2005), "Heuristic approach for N job 3 machine flow shop scheduling problem involving transportation time, break-down time and weights of jobs", Mathematical and Computational Application, Vol.10 (No.2), pp 301-305.

[5] Chander Shekharan, K, Rajendra, Deepak Chanderi (1992), "An efficient heuristic approach to the scheduling of jobs in a flow shop", European Journal of Operation Research 61, 318-325.

[6] Dannenbring, D.G. (1977) ,"An evaluation of flowshop sequencing heuristics", Management Science, Vol. 23, No. 11, pp.1174-1182.

[7] Heydari (2003), "On flow shop scheduling problem with processing of jobs in a string of disjoint job blocks: fixed order jobs and arbitrary order jobs", JISSOR , Vol. XXIV , pp 1- 4.

[8] Ignall, E. and Schrage, L. (1965), "Application of the branch-and-bound technique to some flowshop scheduling problems", Operations Research, Vol. 13, pp.400-412.

[9] Johnson S. M. (1954), "Optimal two and three stage production schedule with set up times included". Nay Res Log Quart Vol 1, pp 61-68

[10] Koulamas, C. (1998), "A new constructive heuristic for the flowshop scheduling problem", European Journal of Operations Research', Vol. 105, pp.66-71.

[11] Lomnicki, Z.A. (1965), "A branch-and-bound algorithm for the exact solution of the three-machine scheduling problem", Operational Research Quarterly, Vol. 16, pp.89-100.

[12] Maggu & Das (1981), "On n x 2 sequencing problem with transportation time of jobs", Pure and Applied Mathematika Sciences, 12-16.

[13] Nawaz M., Enscore Jr., E.E. and Ham, I. (1983) , "A heuristic algorithm for the m-machine n-job flowshop sequencing problem", OMEGA International Journal of Management Science, Vol. 11, pp.91-95.

[14] Palmer, D.S.(1965), "Sequencing jobs through a multi-stage process in the minimum total time - a quick method of obtaining a near-optimum", Operational Research Quarterly, Vol. 16,No. 1, pp.101-107.

[15] Park, Y.B. (1981), "A simulation study and an analysis for evaluation of performance-effectiveness of flowshop sequencing heuristics: a static and dynamic flowshop model", Master's Thesis, Pennsylvania State University.

[16] Singh, T.P., K, Rajindra & Gupta Deepak (2005**)**, "Optimal three stage production schedule the processing time and set up times associated with probabilities including job block criteria", Proceeding of National Conference FACM- (2005), pp 463-470.

[17] Sarin, S. and Lefoka, M. (1993**)**, "Scheduling heuristics for the n-job, m-machine flowshop", OMEGA, Vol. 21, pp.229-234.

[18] Turner S. and Booth D. (1987**)**, "Comparison of heuristics for flowshop sequencing", OMEGA,Vol.15, pp.75-78.

[19] Temiz Izzettin and Serpil Erol(2004), "Fuzzy branch and bound algorithm for flow shop scheduling", Journal of Intelligent Manufacturing, Vol.15, pp.449-454.

[20] Yoshida and Hitomi (1979), "Optimal two stage production scheduling with set up times separated",AIIETransactions. Vol. II.pp 261-263**.**