Journal of Information Engineering and Applications
ISSN 2224-5782 (print) ISSN 2225-0506 (online)
Vol.4, No.11, 2014

Fixed Point Theoremsrelated To Compact Metric Spaces

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Abstract
In the present paper we established a fixed point theorem in compact metric space and another result is proved for pseudo compact tichnov space. Our results are generalization form of many known results.
Keywords: Compact metric spaces, Pseudo compact tichnov spaces, fixed point
AMS subject classification: 47H10, 54H25

2. Introduction & Preliminaries:

There are several genralizations of classical contraction mapping theroem of Banach [1].In 1961 Edelstein [4] established the existence of a unique fixed point of a self map T of a compact metric space satisfying the inequality d(T(x), T(y)) <d (x, y)
which is genralization of Banach. In the past few years a number of authers such as Fisher [5], Soni [11,12] have stablished a number of intresting results on compact metric spaces. More recently Fisher and Namdeo [6], Popa and Telci [10], Sahu [13] described some valuble results in compact metric spaces.
Jain and Dixit [7], Pathak [9], Khan, S. and Sharma [8] worked on pseudo-compact Tichonov spaces. Recently Bhardwaj etl.[2,3] also worked for thease spaces.

3. Main Results

Theorem 3.1:
Let F be a continuous mappings of a compact metric space X into itself satisfying the condition; (3.1)

$$d(F(x), F(y)) < a_1 d(x, F_x) + a_2 d(y, F_y) + a_3 d(x, y) + a_4 \max\{d(x, F_x), d(y, F_y), d(x, y)\} \quad (3.1)$$

For all x, y ϵ X, x≠y and $a_1 + a_2 + a_3 + 2a_4 \leq 2$, where a_1, a_2, a_3, a_4 are non negative real numbers, then F has a unique fixed point.

PROOF:
First we define a function T as follows:
T(x) =d(x,y(x)), for all x ϵ X. Since d and F are continuous on X, T is also continuous X. From compactness of X, there exists a point P∈X, such that
(3.1.1) T(P) =inf{T(x): x ∈X}
If T(P) ≠ 0, it follows that P≠F(p)
And so $T(F(P))=d(F(P),F^2(p))$

$$d(F(P)),F(F(P))<a_1 d(P,F_p)+a_2 d(P, F_p) + a_3 d(P, P) + a_4 \max\{d(P, F_p), d(P, P)\}$$

therefore,

$$d(F(P)),F^2(P)) < \frac{a_1}{2} d(F(P),F^2(P)) + \frac{a_2}{2} d(F(P), F^2(P)) + (\frac{a_3}{2} + a_4) d(P,F(P))$$

That is, $d(F(P),F^2(P)) [1-\frac{a_1}{2} - \frac{a_2}{2}] < (\frac{a_3}{2} + a_4) d(P,F(P))$

$$d(F(P),F^2(P)) < (\frac{a_3}{2} + a_4)/ [1-\frac{a_1}{2} - \frac{a_2}{2}] d(P,F(P))$$

That is, T(F(P)) < ST(P)

$$\text{Where } S= (\frac{a_3}{2} + a_4)/ [1-\frac{a_1}{2} - \frac{a_2}{2}] \leq 1$$

$$a_1 + a_2 + a_3 + 2a_4 \leq 2$$

Which is a contradiction to the condition (1.2) and hence P=F(P),consequently P is a fixed point of F.

Uniqueness:
Now we shall prove the uniqueness of P. Let if possible Q≠P be another fixed point of F.
Now d(P,Q)= d(F(P)),F(Q))
$d(F(P),F(Q)) < a_1 d(P, F_p)+ a_2 d(P, F_p)+a_3 d(P, Q)+a_4 \max[d(P, F_p), d(Q, F_Q)]$+d(P,Q)
That is $d(P,Q)< (\frac{a_3}{2} + a_4)d(P,Q)$
Which is a contradiction because $a_2 + 2a_4 \leq 2$
Hence P is a unique point of F.

Theorem 3.2:
Let P be a pseudo compact Tichonov space and μ be a non-negative real number valued continuous function over (P×P) satisfying.
[3.2.1] μ(x, x) = 0, for all x ∈ P and
μ(x, y) = μ(x, z)+ μ(z, y) for all x,y and z ∈ P
Let T:P → P is a continuous map satisfying;
[3.2.2]

$\mu(T_x, T_y) = \mu(x,y)[1+ \mu(x, T_x)+ \mu(y, T_y)+ \mu(x,y)+\min\{ \mu(x, T_x), \mu(y, T_y), \mu(x,y)\}]$
for all distinct x,y ∈ P, then T has a unique fixed point in P.
Proof: We define d: P→ R by
$\phi(P) = \mu(Tp, p)$
for all p ∈ P, when R is a set of real numbers clearly ϕ is continuous, being the composite of two functions T and μ, since P is pseudo compact Tichonov space; every real valued continuous function over P is bounded and attains its bounds. Thus there exists a point say V∈ P, such that
$\phi(V) = \inf\{ \phi(P) : p \in P\}$
It is clear that $\phi(P) \subset R$. We now affirm that v is a fixed point for T. If not, let us that $T_v \neq v$.
so by (2.2)
$\phi(T_v)= \mu(T^2v, T_v)$
$= \mu(T(TV), TV)$
$\phi(T(v)) < (T_v, v)[1+ \mu(T_v,v)]$
$= \mu(T_v,v)$
This implies $\phi(T(v))= \mu(T^2v, T_v) < \mu(T_v, v)$
A contradiction, so T(v) = v
i.e. v ∈ P is a fixed point for T.
To prove the uniqueness of v, if possible, let w∈ P be another fixed point for T,i.e
$T_w = w$ and $w \neq v$
So by, [3.2.2]
$\mu(v, w) = \mu(T_v, T_w)$
$< \mu(v, w)[1+ \mu(v, T_v)+ \mu(w, T_w)+ \mu(v, w) + \min\{ \mu(v, T_v), \mu(w, T_w), \mu(v, w)\}]$
$\mu(v, w) < \mu(v, w)$
Again it is a contradiction. Hence v ∈ P is a unique fixed point for T in P.
This completes the proof.

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