

Development of Mathematical Models for Calculating Capacity Network Links, and Minimizing the Cost of the Structure and $\overline{\mathrm{T}}_{\mathrm{gc}} \leq \mathrm{T}_{\mathrm{gc3}}$ for Arbitrary Laws of Distribution of Service Time

Khaled Batiha
Computer science department, Al al-Bayt University, Jordan

Abstract
This paper shows new mathematical models to eliminate the weaknesses in previous models to assess the capacity and reliability of the communication channel structures of networks. This paper solves the problem of determining the structure of rational connection. To obtain the necessary dependencies used, the method and apparatus of the theory of coincidences pulse flows.
Keywords: queuing theory, theory of mathematical modeling, network design theory, theory of mathematical programming.

1. Introduction

The Capacity Assignment problem was to minimizing the cost find the best possible set of capacities for the links that satisfy traffic requirements in a telecommunication network.

We should outline several ways to approach the capacity assignment using different performance criteria. Of specific interest are the works of Kleinrock [Kleinrock, 1970; Kleinrock, 1972], which reflected the problem of determining the capacity assignment of communication channels. In the first (and simplest) case, according to Kleinrock, he assumes that cost to capacity is linearly proportional. Additionally, Martin and Schwartz [Martin, 1972; Schwartz, 1977] give much consideration to the design of the network structure, while the advantages and disadvantages of the already-developed algorithms to optimize structures (such as Prim's algorithm, Kruskal's algorithm etc.) [Fratta, 2014] review the basic flow deviation (FD) procedure of assigning flows within store-and-forward communication networks to minimize costs and/or delay a given topology. Reaz [Reaz, 2009] optimized cost/performance tradeoffs, proposing an access network that uses a combination of optical and radio links, and Chiaraviglio [Chiaraviglio, 2012] solved the problems of energy consumption and greenhouse effects.

The works of the above authors were studied with the same network for a long stream of messages, and the results are approximate. In our current research, we gave significant attention to dealing with the task of constructing a communication network of a minimum length.

Queuing theory plays a key role in the quantitate understanding of computer- communication networks. It is obvious from our discussion of these networks thus far that queues improve at each concentration point (node) in the network as a message arrives and waits for service. In fact, messages encountering when traversing a network is just the queueing delay due to waiting for service [Saaty, 1961].

Khaled [2015] discusses a model for the calculation of capacities of channels, assuming an exponential distribution of the lengths of the transmitted messages and discipline of their service in order of priority. The results obtained in Khaled's models allow us to determine the capacity of not only an arbitrary distribution of the lengths of the transmitted messages, but also for many different algorithms for their work protocols, the optimum capacity assignment of the ith channel network, has the form [Khaled, 2015]:

$$C_i^{opt.} = \frac{b + \sqrt{b^2 + 4.S.n}}{2S} \qquad (1)$$

Where $b = (\lambda_i . T_{gc3i} + 1). \sum_{l=1}^{k} \lambda_l \, \bar{\tau}_l^{(b)}, S = \lambda_i . T_{gc3i},$

$$n = \lambda_i . \sum_{l=1}^{k} \lambda_l (\bar{\tau}_l^{(b)})^2 - (\sum_{l=1}^{k} \lambda_l \, \bar{\tau}_l^{(b)})^2$$

$\lambda_i = \sum_{l=1}^{k} \lambda_l$- The intensity of the total flow in the i^{th} link.
T_{gc3}- Defined by the customer or user of the average message delivery time (or a package) in the network,
$\bar{\tau}_i^{(b)}$ - The average length of the message (packet) in bits.

Expression (1) represents the minimum required capacity assignment and, consequently, its minimum value depending on the capacity at which the following condition is satisfied $\bar{T}_{gci} = T_{gc3i}$ – this expression is appropriate to apply to the calculation of capacities of channels, as the network structure of reliability corresponds to the desired ratio of availability.

In addition, it is of interest to evaluate the model's optimal bandwidth with limited and known reliability. This issue was ignored by the authors of the structures of networks. Consequently, this problem must be solved, beginning with the assumptions of the exponential distribution law of message flows and the recovery time that is necessary for the comparative analysis of models with an arbitrary distribution.

In general, there is so much to each of the laws of distribution that building a model for evaluating the characteristics of a particular network is not feasible. It should be noted that the distribution may be characterized by the value of the coefficient of variation which is one equal to the exponential law, for which regular is equal to zero, for normal - within 0.5 and so on. Moreover, in practice, it is unlikely in each case that the distribution law will be determined; the best case scenario, excluding the mathematical probability, is that the variance will be found for the investigated quantity, i.e., we found the coefficient of variation. This is sufficient for more precise assessment of the characteristics of the network structure than in Khaled's [2015]. In addition, the coefficient of variation will allow for comparative analysis influence on the distribution laws of the investigated processes.

We note that the development of proposed models to assess the optimum bandwidth will be use via the method discussed by Khaled [2015]. This is the standard notation to contribute to queuing systems, which are formalized channels of communication. So M / M / 1 - refers to a system with Poisson Arrivals and Exponential Service Time; M/D / 1 – a system with Poisson flow and regular (permanent) service time; M/G / 1 – a system with arbitrary servicing.

Everywhere, as noted earlier, the problem of minimizing the cost of the required network bandwidth (capacity assignment) network, which provides the condition $\bar{T}_{gc} \leq T_{gc3}$.

2. General requirements for proposed models, and assumptions

The general requirements for our proposed model include:

•The model must be appropriate to the test process; thus, it should be considered only the main parameters or factors of the investigated process;

• The model should serve as a basis for making decisions on the choice of the optimal structure of the network;

• The model should give a clear idea of the process or phenomenon under study;

• The model should be quite easy and convenient to engineering practice, and it should provide sufficient accuracy of calculations.

At the same time, we note that the general term "sufficient accuracy" is uncertain, when there is no complete certainty in the values of the original data and the laws of the studied phenomena.

The rate of the exact model is not high, if as a result of its use, mathematical relationships are not so complex that it is not possible to obtain a numerical solution. However, the model cannot be applied, and that does not reflect the essential features of the process under investigation.

For our ongoing studies such necessary features are:

♣ random arrival of user messages;

♣ random and arbitrary law of distribution of the lengths of messages transmitted;

♣ a set of possible protocols that can be used in various purposes and structures of the network, and which differ in the number and parameter of service messages. These parameters are generally different from the parameters of data messages bearing the transferred data;

♣ unreliable communication channels and devices that lead to their failure and the need for recovery;

♣ Transmitted information is exposed to disturbances that cause its distortion and the need for retransmission.

All these and other features should be reflected in the proposed models.

In the development of mathematical models, we will use the following assumptions:

All streams transmitted through the communication channels are described by a Poisson distribution law. The validity of this assumption is explained in Khaled, [2015].

Capacity nodes significantly greater than the performance of communication channels (on the order of or greater), which corresponds to reality;

Before each communication channel allowed being a queue of messages (packet), the length of which is not limited. The latter assumption needs to be justified, as in real systems; the buffer memory on the nodes is limited. However, as shown in [55], if the memory unit is designed to be at least 15, then for analysis and synthesis of such systems, we can apply the queuing theory model with unlimited queue. The error of such formalization is sufficiently small.

3. Evaluation of the optimal network bandwidth for the M / G / 1 system with absolutely reliable communication channels

In this case, to determine the optimal of bandwidth i-th channel- C_i^{opt} the Pollaczek- Khinchine formula for the delay of messages on the channel will be in the form

$$\bar{T}_i = \frac{\sum_{l=1}^{k} \lambda_l (\bar{\tau}_l^{(b)})^2 . (1 + k_l^{2)}}{2. C_i (C_i - \sum_{l=1}^{k} \lambda_l \bar{\tau}_l^{(b)})} + \frac{\rho_i^{(b)}}{C_i \lambda_i}, \qquad (2)$$

Where, k_l- is the coefficient of variation equal to the ratio of the mean-square deviation to the mathematical expectation of the message length l -th inhomogeneous flow.

d_i– The cost incurred for each unit of capacity built into the i^{th} channel.
C_i-The channel capacity assignment of the i^{th} channel (bps),
$\overline{T_i}$ −The average time delay for messages in the network
λ_i- The i^{th} link message rate (message / sec)
γ- The total external message arrival rate
$\rho_l = \lambda_l \overline{\tau}_l = \frac{\lambda_i}{\mu_l}$- loading i^{th} channel l^{th} message flow, the average length, which equals $\overline{\tau}_l(\mathrm{c})$;
μ_l- The intensity of the message service l^{th} stream;
$\rho_i = \sum_{l=1}^{k} \rho_l$ - Total loading all the i^{th} channel stream of messages;
k - The amount of different (non-uniform) message flows;
$\rho_i^{(b)} = \sum_{l=1}^{k} \lambda_l \, \overline{\tau}_l{}^{(b)}$ - total load of the i^{th} channel-uniform flow, as measured in bits/sec.
$\overline{\tau}_i{}^{(s)}$ - The average length of the message (packet) in seconds.

The solution of this equation with respect to T_{gc3i} is the expression

$$C_i^{opt.} = \frac{b + \sqrt{b^2 + 4.S.\acute{n}}}{2S}, \qquad (3)$$

Where,

$$\acute{n} = \lambda_i. \sum_{l=1}^{k} \lambda_l (\, \overline{\tau}_l{}^{(b)})^2 . (1 + k_l^2) - 2. \left(\sum_{l=1}^{k} \lambda_l \, \tau_l{}^{(b)} \right)^2 ;$$

$$b = \left(\lambda_i . T_{gc3i} + 1 \right) . \sum_{l=1}^{k} \lambda_l \, \tau_i{}^{(b)}, \quad S = \lambda_i . T_{gc3i},$$

Analysis (3.13) shows that, when $k_l = 1$, it is identical to expression (1), i.e., the general case can be reduced to private. When $k_l = 0$, expression (1) allows us to calculate the capacity assignment for regular-length message inhomogeneous flows, which are transmitted over the communication channel.

4. Selection of the optimal capacity assignment with limited reliability of communication channels

Prior to that, the problem of choosing the capacity assignment and reliability was addressed separately, independently of each other. When selecting a capacity assignment, it was assumed that the channels are absolutely reliable, having the required reliability. However, if we expand the understanding of capacity assignment in the physical sense and present it not as a transmission speed of electrical signals in a communication channel (as well as the average capacity to transmit a certain amount of information per unit of time), then the practice shows that damage to the channels and hardware failures lead to a significant reduction in real capacity assignment for channels due to downtime at fault; we need to restore communication links, repeated the previously transmitted packets, etc. Therefore, the joint decision of both problems becomes relevant. It is necessary to determine the optimum value of capacity assignment to ensure the functioning of the network with a specified delivery time information of users with limited reliability of communication channels and steady-state operation, i.e., subject to condition

$$\sum_{l=1}^{k-1} \frac{\lambda_l}{\mu_l C_i} < 1 \qquad (4)$$

Where, $(k-1)$ - number of information and service flow in the channel.

From (4), which is considered a queuing system with heterogeneous flows, one of which is a stream of failures, it is thus necessary to express the loading system of the flow of failure through the reliability of the communication channel, which will be characterized by the availability coefficient K_i

(28) shows that the relationship with a load factor of availability described by the relation

$$\rho_{oT} = 1 - K_i \qquad (5)$$

Where ρ_{oT} - the download link failure flow.

For ease of analysis, we assume the discipline of service is fair for all streams. As will be shown below, this assumption would lead to a minor error in the calculation of the characteristics of the network structure.

To obtain the settlement of dependencies, we use formula (3), taking into account the notation, the characteristics of the model and the valueT_{gc3i}. After simple transformations, we have

$$T_i = \frac{\sum_{l=1}^{k-1} \lambda_l * \frac{\left(\tau_l^{(\delta)}\right)^2}{C_i^2} + (1 - K_i) * \bar{t}_b}{1 - \sum_{l=1}^{k-1} \lambda_l * \frac{\tau_l^{(\delta)}}{C_i} - (1 - K_i)} + \frac{\rho_i}{C_i \lambda_l}, \qquad (6)$$

Wherein $\rho_i = \sum_{l=1}^{k-1} \lambda_l \tau_j^{(b)}$- the total loading in) bit / s) i-th channel stream of information and service messages;
λ_i - total intensity of information and service messages sent by the i-th channel; .
$\bar{t}_b$- mean time to failure (recovery).
Once again, we note that the difference between the models considered here from the previous models is that, during the waiting time in the queue, there is allocated a separate term time due to downtime and information service messages for the elimination of the failures occurred in the i-th channel. This conventional device, however, is under a clear justification of the physical processes occurring in the channel when sending messages.
Solving (6) with respect to T_{gc3i}, we have:

$$C_i^{opt} = \frac{-A \pm \sqrt{A^2 - 4\lambda_i.B.E}}{2.\lambda_i.B}, \qquad (7)$$

Where,

$$A = \rho_i(K_i + \lambda_i.T_{gc3i}); \; B = \bar{t}_b(1 - K_i) - T_{gc3i}.K_i;$$

$$E = \lambda_i \sum_{l=1}^{k-1} \lambda_l * \left(\tau_l^{(b)}\right)^2 - (\rho_i)^2$$

Let us analyze (7) to determine the permissible values of the roots of (6) with the equality $\overline{T_\iota} = \overline{T_{gc3\iota}}$
First, we note that when $B \neq 0$, then

$$\bar{t}_b.(1 - K_i) - T_{gc3i.}K_i \neq 0$$

Or,

$$K_i \neq \frac{\bar{t}_b}{t_b + T_{gc3i}} = 1 - \frac{T_{gc3i}}{T_{gc3i} + t_b} \qquad (8)$$

what should be the availability factor (greater than or less than the right side of the formula [8]), we consider two cases

1. $B > 0$

We define the signs of all the components of the formula (7). The value of E is always positive, as

$$\lambda_i \sum_{l=1}^{k-1} \lambda_l.\left(\tau_l^{(b)}\right)^2 \geq (\rho_i)^2$$

Let us prove this. Assume (for simplicity consideration) are only messages with parameters $\lambda_1, \bar{\tau}_1$ *and* $\lambda_2, \bar{\tau}_2$.
Then for E we can write

$$\lambda_i \sum_{l=1}^{k-1} \lambda_l * \left(\tau_l^{(b)}\right)^2 - (\rho_i)^2 = (\lambda_1 + \lambda_2)\left[\lambda_1\left(\bar{\tau}_1^{(b)}\right)^2 + \lambda_2.\left(\bar{\tau}_2^{(b)}\right)^2\right] - (\lambda_1.\bar{\tau}_1^{(b)} + \lambda_2.\left(\bar{\tau}_2^{(b)}\right)^2 = \lambda_1.\lambda_2\left[\bar{\tau}_1^{(b)} - \bar{\tau}_2^{(b)}\right]^2$$

If $E > 0$, then the value $4.\lambda_1.B.E$ is greater than zero, and therefore the value of the discriminant modulo less than A. Then

$$C_i^{opt} = \frac{-A + \sqrt{A^2 - 4.\lambda_i.B.E}}{2.\lambda_i.B} < 0$$

and especially

$$C_i^{opt} = \frac{-A - \sqrt{A^2 - 4.\lambda_i.B.E}}{2.\lambda_i.B} < 0$$

then $B > 0$ OR $K_i < \frac{\bar{t}_b}{\bar{t}_b + T_{gc3i}}$ existence cannot be.

2. B < 0

The value of E is still positive, then $-4.\lambda_1.B.E > 0$ and value of discriminant

$$\sqrt{A^2 - 4.\lambda_i.B.E} > A$$

Consequently,

$$C_i^{opt} = \frac{-A + \sqrt{A^2 - 4.\lambda_i.B.E}}{2.\lambda_i.B} < 0$$

$$C_i^{opt} = \frac{-A - \sqrt{A^2 - 4.\lambda_i.B.E}}{2.\lambda_i.B} > 0$$

And so, when ***B<0,*** the value of the bandwidth has a positive value determined by the expression (7). To this channel availability factor one used in formula (7) to be

$$K_i > \frac{\bar{t}_b}{\bar{t}_b + T_{gc3i}} \qquad (9)$$

Thus, the analysis of formula (7) has shown that in order to change the value of the SS within the permissible range, it is necessary that the value of availability of the channel satisfies (3.19).

We perform validation of the resulting expression for determining C_i^{opt}, supplied by the formula (7). To do this, compare it with the expression (1) of the previous model in the formalization of the channel system M / M / 1. Let the equipment absolutely reliably, then $K_i = 1$ and expression (7) reduces to (1), i.e.,

$$A = (1 + \lambda_i . T_{gc3i}), \quad i.e. \quad A = B; B = -T_{gc3i}, E = \acute{n}$$

We say that the value of the availability that inequality (9) becomes an equality, the limit or boundary, as this value bandwidth takes an infinite value, which is unacceptable. Let it K_i^b.

Consider the inequality (9) and continue the analysis to determine the boundaries of permissible change K_i. As we know, for stationary operation, SPI is necessary to download the total flow of messages and failures ψ_0 was less than one, i.e. $\psi_0 + 1 - K_i < 1$ or

$\psi_0 < K_i, \quad where\ \psi_0 = \frac{\rho_i^{(b)}}{C_i}$- Network load information and service messages (dimensionless).

Consequently, margins are determined by the availability:

$$\psi_0 < K_i > \frac{\bar{t}_b}{\bar{t}_b + T_{gc3i}} \qquad (10)$$

Analyzing inequality (3.20), we can make one conclusion concerning the permissible values of network bandwidth. Substituting the left side of inequality (10) value for ψ_0, then

$$\frac{\sum_{l=1}^{k} \lambda_l^i (\bar{\tau}_l^b)}{C_i} < K_i, \qquad (11)$$

Or

$$\frac{\rho_{0i}}{K_i} < C_i,$$

Where ρ_{0i}- channel load information and service flow, measured in bit / s.

In the origin of the formula for determining capacity assignment by the rule "square root" Kleinrock implied, the lower limit for C_i value ρ_{0i} [Khaled, 2015]. The resulting model allows us to clarify here that value as the reliability factor increased requirements to capacity assignment. Consequently, inequality (11) determines the minimum allowable value is the capacity of i-channel having availability K_i.

5. Results

Using programs in C++ the main characteristics of capacity assignment is analyzed, in figure (1) shows that with increasing the channel load ρ_{0i} requires more bandwidth value

In addition, this figure shows a significant dependence C_i^{opt} on the magnitude of the coefficient of channel availability. Decreasing K_i from 1 to $\frac{\bar{t}_b}{\bar{t}_b + T_{gc3i}}$ leads to a sharp increasing.

An important characteristic for the assessment of the network structure is the boundary coefficients availability K_i, whose dependence on the parameters contained in it, is shown in Fig (2). Value K_i for a given set of T_{gc3i} and $\bar{t}_b$ channel is independent of the load and service flow information and is related to the intrinsic properties of the communication channel.

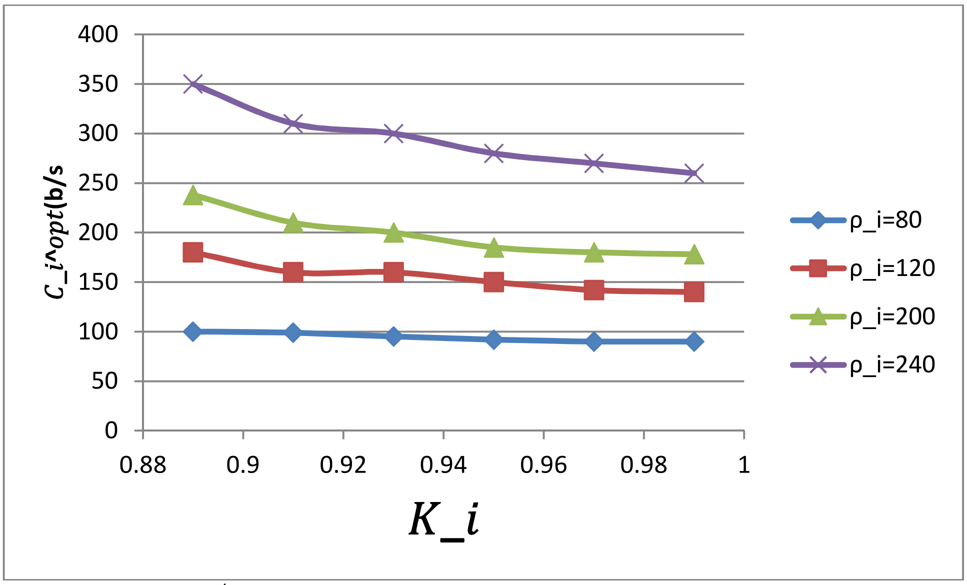

Fig.1 dependenceC_i^{opt} on the value of K_i to $\bar{t}_b = 300\ s$ and $T_{gc3i} = 180s$ for different values of load

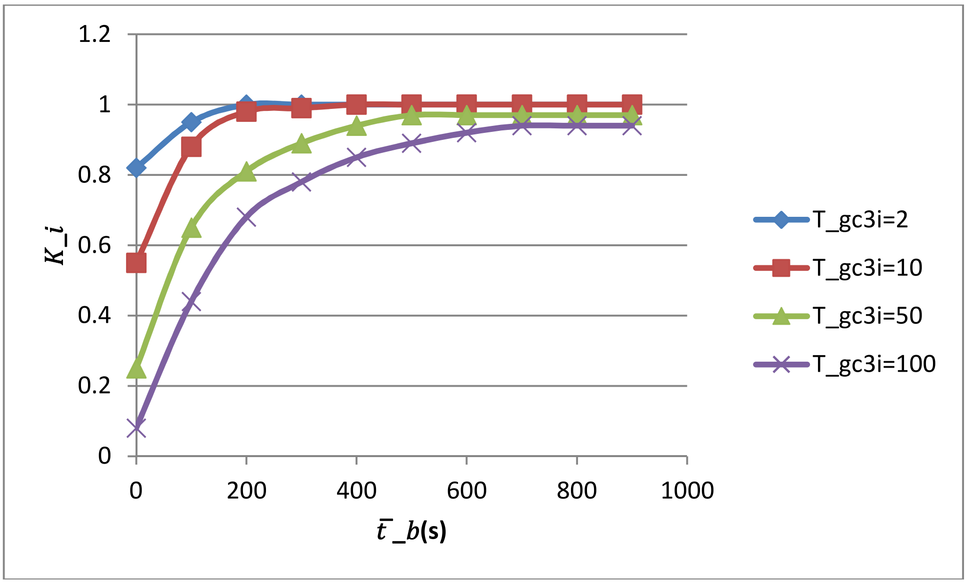

Figure 2 shows the dependence of the boundary value of availability factor of the recovery time for different values of T_{gc3i}

6. Conclusion

In this paper, we have studied the Capacity Assignment (CA) problem. This problem focuses on finding the lowest cost link capacity assignments that satisfy certain delay constraints for several distinct classes of packets that traverse the network. The resulting model allows us to clarify that the value as the reliability factor increased requirements to capacity assignment.

References

1. [Saaty, 1961]T.L. Saaty, Elements of Queuenig Theory, McGraw-Hill, New York, 1961
2. [Chiaraviglio , 2012]L. Chiaraviglio, M. Mellia, F. Neri, Minimizing isp network energy cost: formulation and solutions, IEEE/ACM Trans Netw (TON) 20 (2) (2012) 463–476.
3. [Frank 1971] Frank H. and Frish T., Communication, Transmission, and Transportation Networks, Addison- Wesley, Reading, Mass., 1971.
4. [Fratta , 197319A] L. Fratta, M. Gerla, L. Kleinrock, The flow deviation method: an approach to store-and-forward communication network design, Networks 3 (2) (1973) 97–133.
5. [Fratta , 2014] Fratta L. , Gerla M. , Kleinrock L. , Flow Deviation: 40 years of incremental flows for packets, waves, cars and tunnels, Computer Networks, Volume 66, June 2014, Pages 18-31

6. [Kleinrock , 1972] L. Kleinrock, Communication Nets: Stochastic Message Flow and Delay, McGraw-Hill Book Company, 1964 (Out of Print) Reprinted by Dover Publications, 1972 and in 2007.
7. [Kleinrock, 1970] L. Kleinrock, Analytic and simulation methods in computer network design, in: Proceedings of the Spring Joint Computer Conference, Atlantic City, New Jersey, May 5–7, 1970, pp. 569–579.
8. [Martin 1972] Martin J., systems Analysis for Data Transmission, Prentice-Hall, Englewood Cliffs, N.J., 1972
9. [Reaz, 2009] A.S. Reaz, V. Ramamurthi, S. Sarkar, D. Ghosal, S. Dixit, B. Mukherjee, CaDAR: an efficient routing algorithm for a wireless-optical broadband access network (WOBAN), J Opt Commun Netw 1 (5) (2009) 392–403.
10. [Schwartz 1977] M. Schwartz, Computer- Communication Network Design and Analysis, Prentice-Hall, Englewood Cliffs, N.J., 1977.
11. [Zaichenko 2003] Zaichenko Y.P. Operations Research, Publishing House "Word", Kiev, 2003.
12. [Khaled, 2015]Khaled Batiha, Development of Mathematical Models to Estimate Optimum Capacity Assignment Network Connections, International Journal of Computer Applications (0975 – 8887) Volume 129 – No.16, November2015.