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Performance Evaluation of a Hybrid Fractional Carrier Frequency Offset Estimator in OFDM

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Abstract

The major drawback of the orthogonal frequency division multiplexing (OFDM) system is high sensitivity to synchronization errors caused by carrier frequency offsets (CFOs), which result in degradation in the bit error rate (BER) performance. This paper investigates the performance of a hybrid fractional carrier frequency offset estimator (FCFOE) for frequency synchronization in the OFDM system. The hybrid FCFOE exploits the pilots inserted within the OFDM symbol for channel estimation together with the information inherent in the cyclic prefix (CP), with a view to improving the estimation of the CP-based FCFOE. The performance of the developed hybrid FCFOE was evaluated in terms of the mean squared error (MSE) and bit error rate (BER) using OFDM-QPSK and OFDM-16QAM schemes it turn. The simulation results show that the hybrid FCFOE only gives slightly better performance over the CP-based FCFOE; but the performance enhancement of the hybrid FCFOE is noticeable in OFDM-16QAM.

Keywords: Carrier frequency offset estimation, Hybrid, Orthogonal frequency division multiplexing, Synchronization, Maximum likelihood, Cross-correlation.

1. Introduction

OFDM is a multicarrier transmission technique which has received a lot of attention in recent years due to its robustness in multipath frequency-selective fading channels; and it has been widely implemented in high speed digital wireless networks and applications such as the IEEE802.11a standards to provide WLAN connections (Kim & Park, 2007), digital video broadcasting satellite services to handheld devices (DVB-SH) (Yang *et al.*, 2008; Awoseyila *et al.,* 2009) and for high speed microwave connections such as multichannel multipoint distribution services (MMDS) (Rappaport, 2002). OFDM is, however, very sensitive to carrier frequency offsets (CFOs) caused by Doppler frequency shifts and oscillator instabilities (Morelli & Mengali, 1999; Grigoriadis & Kamath, 2008; Ghogho & Swami, 2008). These imperfections destroy the orthogonality among the subcarriers, and the signals of adjacent carriers will interfere with each other thereby introducing intercarrier interference (ICI) in addition to attenuation and rotation of each of the subcarriers' phase, which invariably results in performance degradation (Huang & Letaief, 2006; Ruan *et al*., 2010; Adeyemo & Ajayi, 2011). This makes carrier frequency synchronization very important in order to achieve quality of service (QoS) delivery. Frequency Synchronization of an OFDM signal requires estimating the CFO and compensating it at the receiver (Schmidl & Cox, 1997). A number of carrier frequency offset estimation techniques have been proposed in the literature ranging from channel group delay estimation to blind methods using the received signal's autocorrelation (Wyglinski, 2004). The CFO estimators fall under two main categories namely data-aided and non-data-aided (or blind). The data-aided method involves the use of either training symbol(s) or pilots for CFO estimation, while the blind method utilizes cyclic prefix (CP) or null subcarriers (NSCs) (Van de Beek *et al.*, 1997; Ghogho *et al.*, 2001). A training symbol is a preamble which is transmitted together with the data signal. Pilots represent a predefined sequence of symbols which are inserted within the OFDM symbol and are known by both the transmitter and the receiver. CFO normalized by subcarrier spacing is categorized into two namely 1) integer CFO, which is a multiple of the subcarrier spacing and 2) fractional CFO, which is less than half of the subcarrier spacing (Sameer & Kumar, 2007; Ruan *et al*., 2010). Schmidl & Cox (1997) developed a timing and frequency synchronization method which utilizes the autocorrelation of a training symbol with two identical parts to estimate timing and fractional frequency offset in the time domain; but it suffers training overhead and the estimation range is low. Morelli & Mengali (1999) proposed a method that uses just one training symbol that composed of L > 2 identical parts, where L is the number of the complex samples in one-half of the training symbol, with the aim of improving on the algorithm of (Schmidl & Cox, 1997) in terms of estimation range and accuracy. The algorithm gave an estimation range of ±L/2 the subcarrier spacing but at the cost of computational complexity. Ren *et al.* (2005) proposed a data-aided CFO estimation method based on Constant Amplitude Zero Autocorrelation (CAZAC). The method utilizes a constant preamble weighted by a binary pseudo-noise (PN) sequence. The constant envelope preamble contains two identical halves and the estimation is based on finding the highest correlation between two repeated sample sequences. The fractional CFO estimation accuracy is poor due to the distortion of its preamble structure by the weighting PN sequence applied.

A joint maximum likelihood (ML) estimator of time and frequency offset which utilizes the redundant information contained within the CP in the OFDM symbol was developed by (Van de Beek *et al.*, 1997). The method gave a good performance over the additive white Gaussian noise (AWGN) channel but requires further modification to be effective over multipath fading channels. Luise *et al.* (2002) proposed a low-complexity blind CFO estimator scheme. The method exploits the time-frequency-domain exchange inherent to the modulation scheme together with virtual carriers. The estimation range is however low. Sameer & Kumar (2007) proposed the use of CP for fractional CFO estimation in the time domain, and observed that the CP-based method gives low computational complexity with low estimation range. However, the CP can be badly distorted in a very harsh multipath channel (Wu & Abu-Rgheff, 2010).

Hence, this paper attempts to combine both the information in the pilot subcarriers and the CP of the OFDM symbol so as to achieve better estimation accuracy as well as low computational cost.

2. The OFDM Signal Model

The samples of the OFDM symbol after the inverse discrete Fourier transform (IDFT) can be expressed as:

$$s(n) = \frac{1}{\sqrt{N}} \sum_{k \in \Gamma_n} S_k e^{j2\pi nk/N} , \qquad n = 0,1,\ldots,N-1 \tag{1}$$

where Γ_n is the set of indices of all the subcarriers; S_k denotes the symbol on the kth carrier and N is the IDFT size or points. After appending the CP, the transmitted OFDM symbol can be represented as $\{s(N-T_g),\ldots,s(0),\ldots,s(N-1)\}$ where T_g denotes the length of the CP. At the receiver, after removing the CP, the received OFDM symbol in time-domain can be expressed as:

$$y(n) = \frac{1}{\sqrt{N}} \sum_{k \in \Gamma_n} H_k S_k e^{j2\pi n(k+\varepsilon)/N} + w(n) , \quad n = 0,1,\ldots,N-1 \tag{2}$$

where H_k is the frequency response of the channel at the kth subcarrier frequency; S_k is the received symbol at the kth subcarrier, $w(n)$ is the AWGN and ε is the CFO to be estimated which results in phase rotation of $2\pi n\varepsilon/N$ in the received signal.

The mean squared error (MSE) of the CFO estimator is obtained by:

$$MSE = \frac{1}{I_t} \sum_{i=1}^{I_t} \left(\varepsilon - \bar{\varepsilon}(i)\right)^2 \tag{3}$$

where I_t is the number of Monte-Carlo simulations, ε is the true CFO and $\bar{\varepsilon}(k)$ is the estimated CFO at the i th iteration. The MSE is plotted together with the Cramer-Rao Bound (CRB), which is the benchmark, given as:

$$CRB(\hat{\varepsilon}) = \frac{1}{2\pi^2} \frac{3(SNR^{-1})}{N} \tag{4}$$

where SNR is the signal-to-noise ratio and N is the IDFT size, that is the number of OFDM subcarriers.

3. Fractional Carrier Frequency Offset Estimator

3.1 Cyclic Prefix-based FCFOE

Assuming perfect timing estimation at the OFDM receiver, by cross-correlating the cyclic prefix samples with the corresponding samples in the OFDM symbol a coarse (or fractional) CFO is estimated in the time-domain as:

$$R_c = \sum_{n=d}^{d+L-1} y^*(n)y(n+N) \tag{5}$$

where $(\cdot)^*$ denotes complex conjugate, d is the timing metric which is assumed to be zero because perfect timing is assumed and L is the length of the CP. Using the log-likelihood function, LLF (Van de Beek *et al.*, 1997), the maximum likelihood (ML) estimation of the fractional frequency offset is obtained by:

$$\hat{\varepsilon}_{f_1} = \frac{1}{2\pi}\arg[R_c] \tag{6}$$

where $\arg$ denotes the argument of a complex number.

3.2 Pilot-based FCFOE

This involves symbol by symbol multiplication of the complex conjugate of the received pilots with the expected pilot symbols. By cross-correlating the received pilot samples with the known samples at the receiver, a coarse CFO estimate can be obtained as:

$$R_p = \sum_{k \in \Gamma_p} y^*(k)x(k) \tag{5}$$

where Γ_p is the indices of the pilot subcarriers, $y(k)$ is the received pilot symbol at point k, $x(n)$ is the corresponding known pilot symbol. The FCFOE can be obtained by:

$$\hat{\varepsilon}_{f_2} = \frac{1}{2\pi}\left|\arg[R_p]\right| \tag{6}$$

The pilots are equallydistributed within the OFDM symbol.

3.3 Hybrid FCFOE

The resultant FCFOE in the time domain is the average of the fractional CFO estimates obtained from the CP and pilot sequence, and it is given as:

$$\hat{\varepsilon}_f = \frac{\left(\hat{\varepsilon}_{f_1} + \hat{\varepsilon}_{f_2}\right)}{2} \tag{7}$$

The received signal of (2) is then corrected using the value of $\hat{\varepsilon}_f$ as:

$$y_c(n) = y(n)e^{-j\frac{2\pi n \hat{\varepsilon}_f}{N}} \qquad n = 0,1,\ldots,N-1 \tag{8}$$

This is a time-domain carrier frequency synchronization, and gives the fractional part of the CFO estimate because the value of $\hat{\varepsilon}_f$ is a fractional part of the CFO, so the CFO synchronization performed in this stage is less accurate but of low complexity. The fractional CFO is corrected here before taken DFT of the received signal. Figure 1 shows the structure of the hybrid FCFOE.

Simulation parameter	**Specification**
Number of channel realizations	100
Path delays (samples)	[0, 10, 20, 30, 40, 50]
Average path gains (dB)	[-4.8434,-6.2911,-7.7387,-9.1864,-10.6340,-12.0817]
Mobile speed	120 km/h
Carrier frequency	2 GHz
IDFT size	1024
System bandwidth	5 MHz
Sampling frequency	4.44 Msps
Length of Cyclic Prefix	128 samples

4. Results and Discussion

The performance of the developed hybrid fractional CFO estimator was investigated by computer simulation using MATLAB software package. Two OFDM symbols were transmitted and the primary modulation schemes used are quaternary phase-shift keying (QPSK) and sixteen quadrature amplitude modulation (16-QAM) in turn. Multipath Rician fading channel with a Rician factor of three is used, and the fading taps gains follow an exponential power delay profile. At the receiver after the DFT the pilot samples are extracted and used to estimate the channel based on Least Square (LS) estimation. The pilot samples are chosen from a subset of a 64-QAM constellation. The number of pilots used is 128 samples except stated otherwise; and the frequency offsets created are 0.125 and 0.25 in turn. The mean MSE and mean BER results taking the SNR from 0 to 20 dB are presented. The OFDM-QPSK signal constellation before and after the effect of the frequency offset are shown in Figure 2(a) and Figure 2(b) respectively; the application of CFO results in scattered constellation.

Figure 3 shows the comparison of the CP-based FCFOE and hybrid FCFOE for CFO of 0.125 in terms of MSE. The mean MSE results for the CP-based and the hybrid FCFOEs in OFDM-QPSK are 4.9881e-004 and 0.0017 respectively; while in OFDM-16QAM, the CP-based gives 6.1716e-004 and the hybrid method gives 9.8675e-004 as against the CRB mean MSE of 3.6331e-005. The results for CFO of 0.25 are presented in Figure 4. In the OFDM-QPSK scheme, the CP-based gives 4.1378e-004 and the hybrid method gives 0.0012; while in the OFDM-16QAM, the CP-based and the hybrid methods give 3.9267e-004 and 9.3640e-004, respectively, as against 3.6331e-005 for the CRB.

Figure 5 presents the effect of the CFO value on the estimators' performance in OFDM-QPSK. For the CFOs of 0.125 and 0.25, the CP-based FCFOE gives the mean MSE of 4.9881e-004 and 4.1378e-004 respectively. On the other hand, the hybrid FCFOE gives the mean MSE of 0.0017 and 0.0012 for CFO of 0.125 and 0.25, respectively. The results for the OFDM-16QAM scheme are shown in Figure 6. For the CFOs of 0.125 and 0.25, the CP-based FCFOE gives the mean MSE of 6.1716e-004 and 3.9267e-004 respectively; while the hybrid FCFOE gives the mean MSE of 9.8675e-004 and 9.3640e-004 for CFO of 0.125 and 0.25, respectively.

The effect of the number of pilots used on the performance of the hybrid FCFOE in OFDM-QPSK is presented in Figure 7. For CFO of 0.125, the mean MSE are 8.9272e-004 and 0.0017 for 64 and 128 pilots, respectively; while for CFO of 0.25, the mean MSE are 0.0021 and 0.0012 for 64 and 128 pilots, respectively. Figure 8 shows the results for the OFDM-16QAM scheme. For CFO of 0.125, the mean MSE are 0.0014 and 9.8675e-004 for 64 and 128 pilots, respectively; while for CFO of 0.25, the mean MSE are 0.0013 and 9.3640e-004 for 64 and 128 pilots, respectively.

The BER performances of the FCFOEs for CFO of 0.125 are presented in Figure 9. In the OFDM-QPSK scheme, the CP-based and the hybrid FCFOEs give mean BER of 0.0776 and 0.0771, respectively; while in the OFDM-16QAM, the CP-based gives 0.0939 and the hybrid gives 0.0937. Figure 10 shows the BER results for CFO of 0.25. In the

OFDM-QPSK scheme, the CP-based and the hybrid FCFOEs give mean BER of 0.0787 and 0.0789, respectively; while in the OFDM-16QAM, the CP-based gives 0.0939 and the hybrid gives 0.0916.

5. Conclusion

5.1 Conclusion

A hybrid fractional carrier frequency offset estimation technique that combines the information in both the pilots and cyclic prefix in the OFDM signal has been developed and the performance compared with the CP-based technique for 128 pilots. Using the CFO estimates obtained by the estimators, the mean MSE and mean BER were computed. The results obtained from computer simulation reveal that with CFO of 0.125 the hybrid method gives about 0.32% and 0.11% lower BER compared with the CP-based method in the OFDM-QPSK and OFDM-16QAM, respectively; and about 1.24% BER advantage over the CP-based method for CFO of 0.25 in the OFDM-16QAM. This implies that the hybrid FCFOE gives a relative improvement over the CP-based FCFOE, which is noticeable in the OFDM-16QAM scheme. Generally, the estimation errors for the CFO of 0.25 are lower than those for CFO of 0.125. Also, increasing the number of pilots used in the hybrid FCFOE improves the performance of the estimator, which is revealed by the MSE results obtained for the 64 and 128 pilots in turn.

5.2 Recommendations

The developed hybrid FCFOE is still sub-optimal due to the estimation variance it suffers; hence, the technique can be improved upon by maximizing the LLF function. Also, by distinctively arranging the pilots within the OFDM symbol, better information about the CFO can be obtained.

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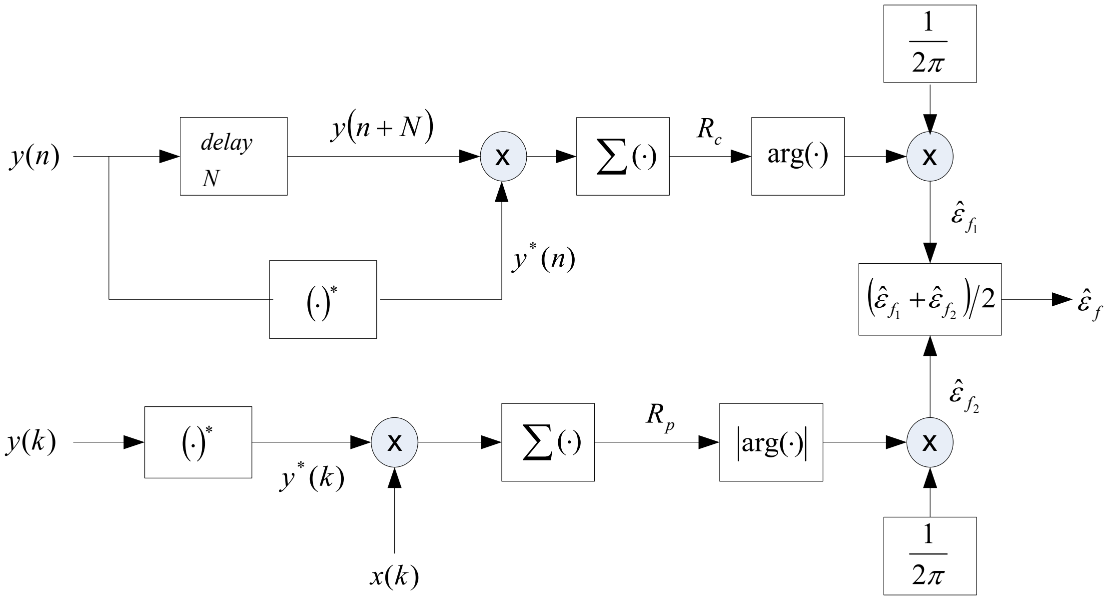
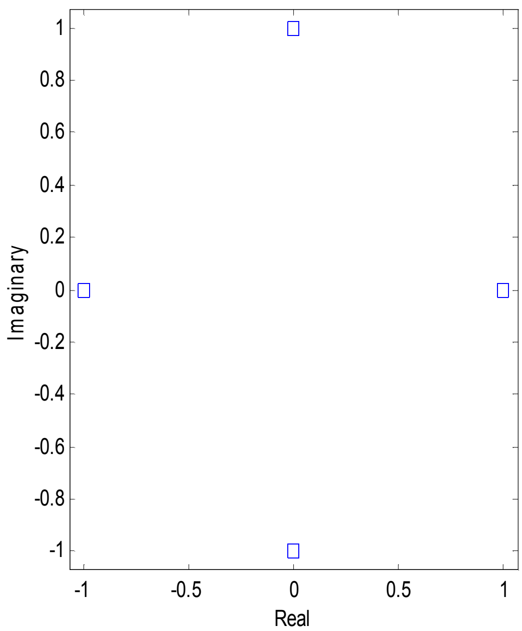


Figure 1: Structure of the Hybrid Fractional CFO Estimator



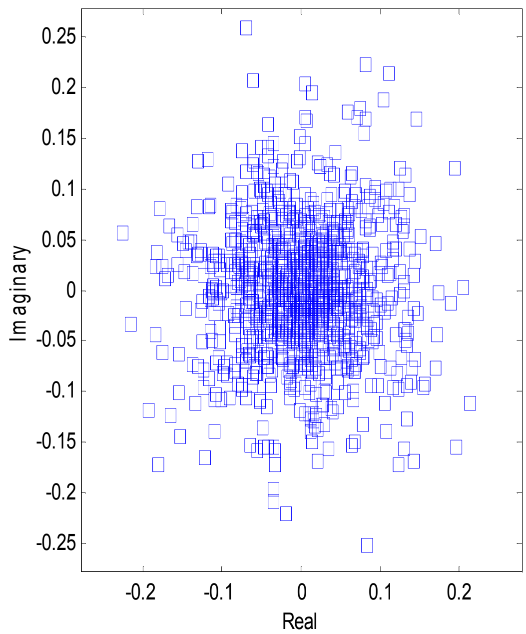


Figure 2(a): Constellation diagram of OFDM-QPSK before the frequency offset

Figure 2(b): Constellation diagram of OFDM-QPSK after spinning with frequency offset of 0.125

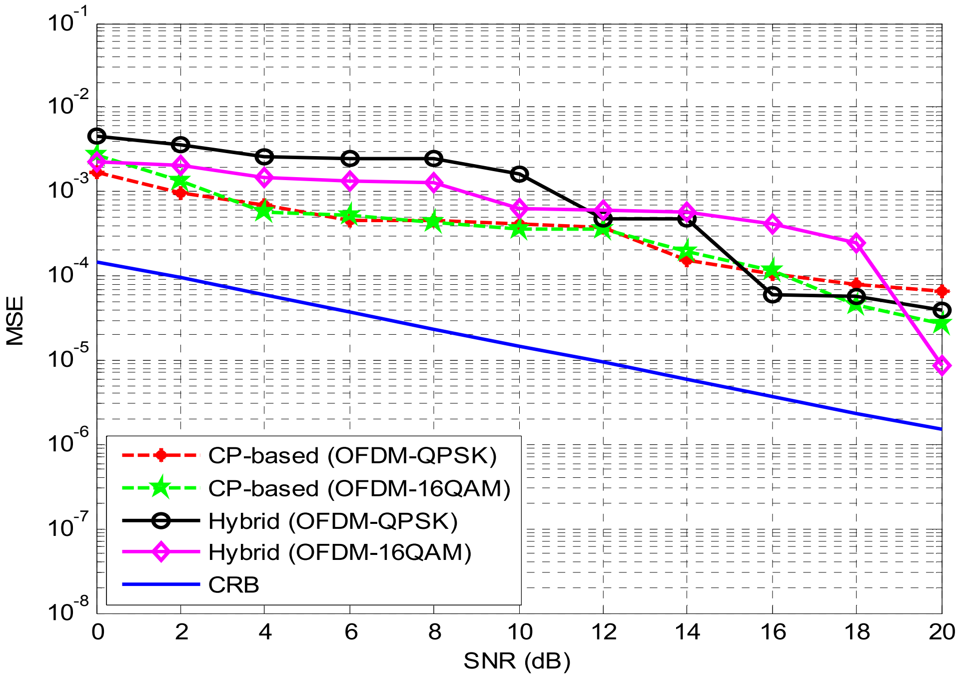


Figure 3: Mean-Squared Error of Fractional CFO Estimators for CFO of 0.125

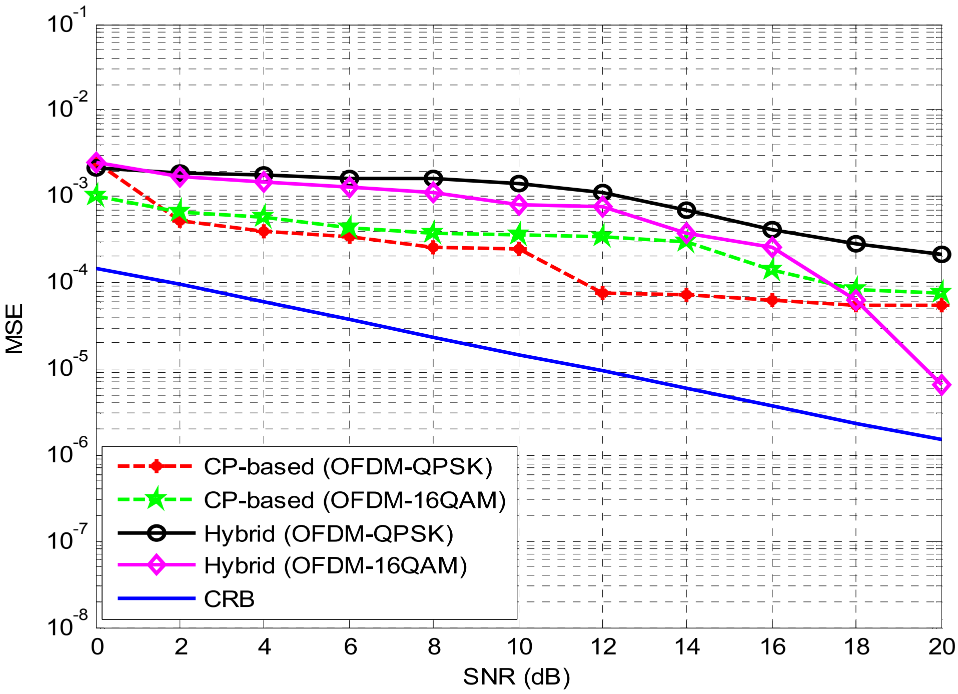


Figure 4: Mean-Squared Error of Fractional CFO Estimators for CFO of 0.25

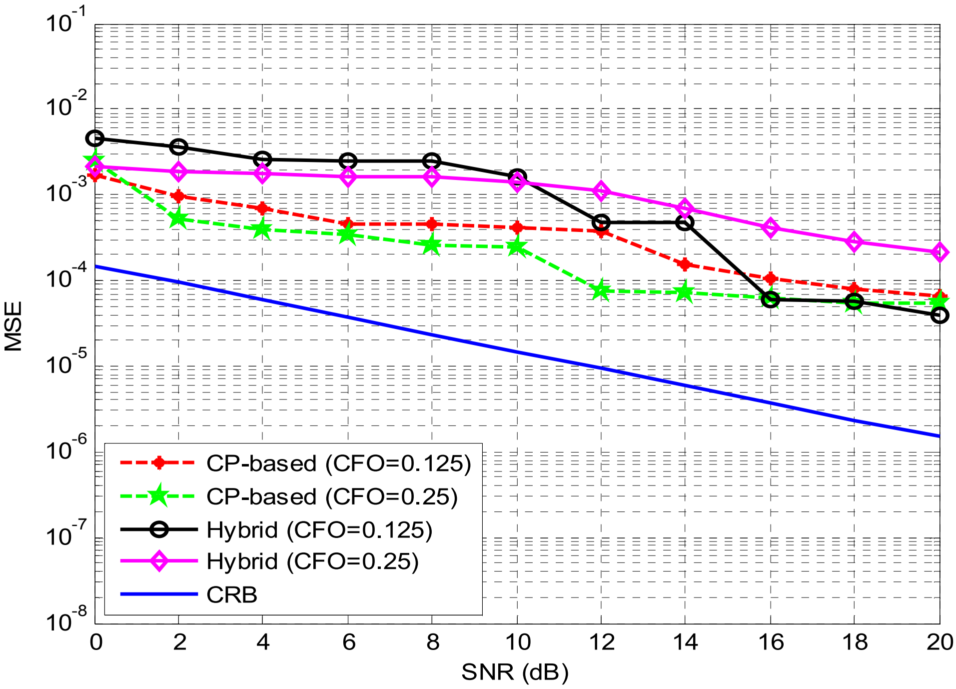


Figure 5: Mean-Squared Error of Fractional CFO Estimator for OFDM-QPSK

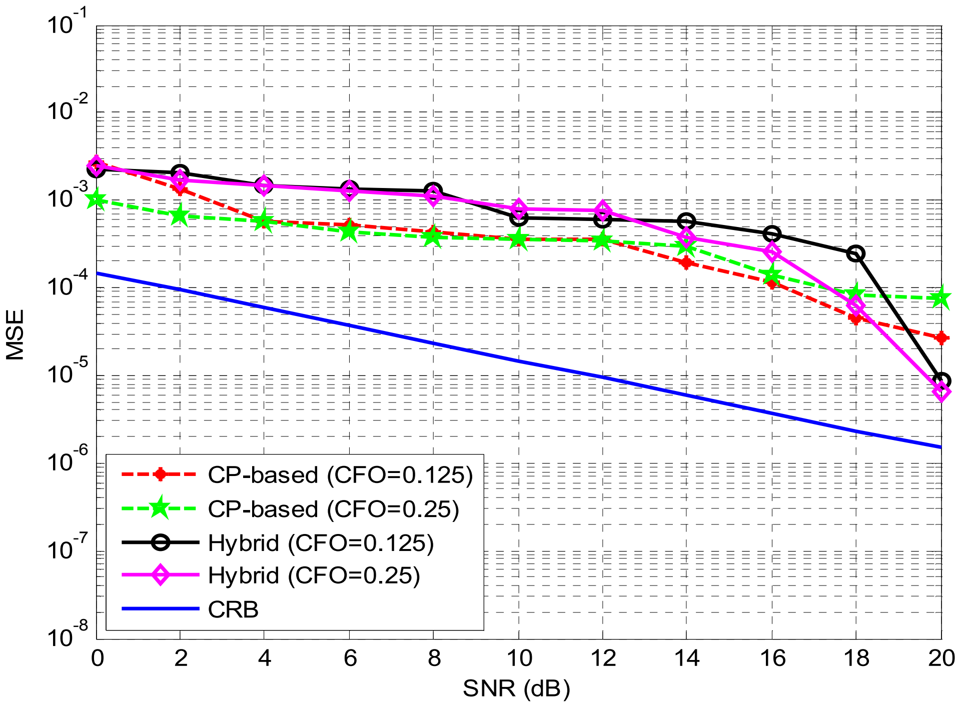


Figure 6: Mean-Squared Error of Fractional CFO Estimator for OFDM-16QAM

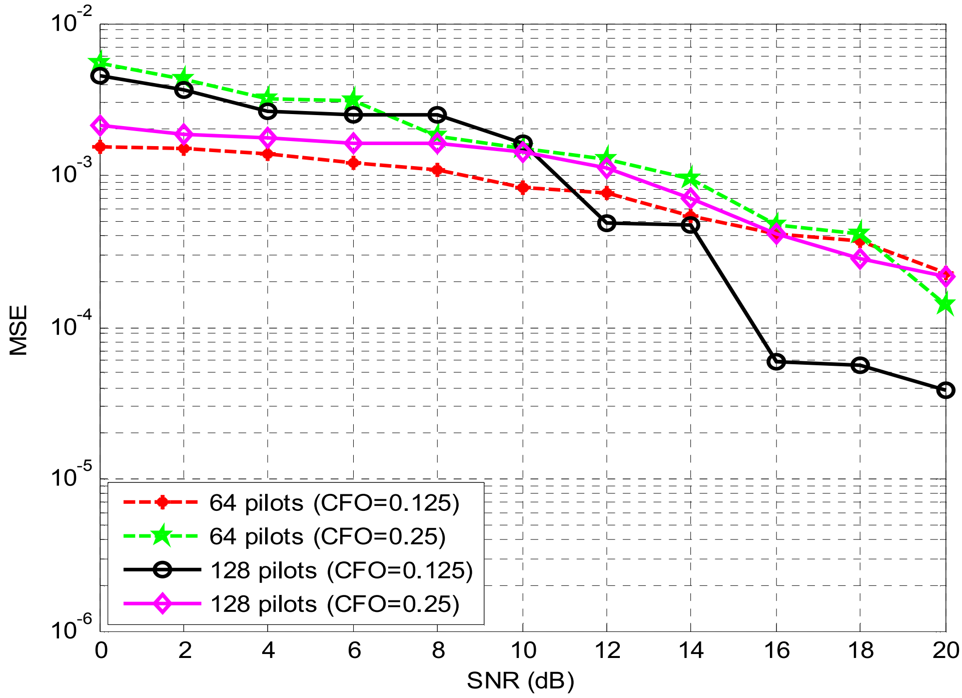


Figure 7: Effect of the number of pilots used for the hybrid FCFOE in OFDM-QPSK

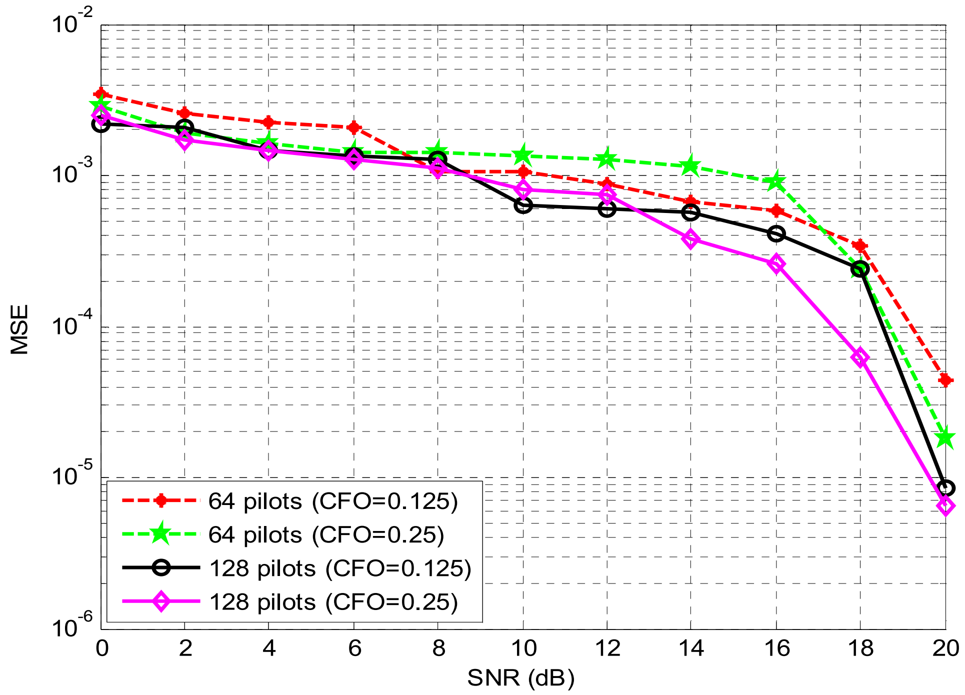


Figure 8: Effect of the number of pilots used for the hybrid FCFOE in OFDM-16QAM

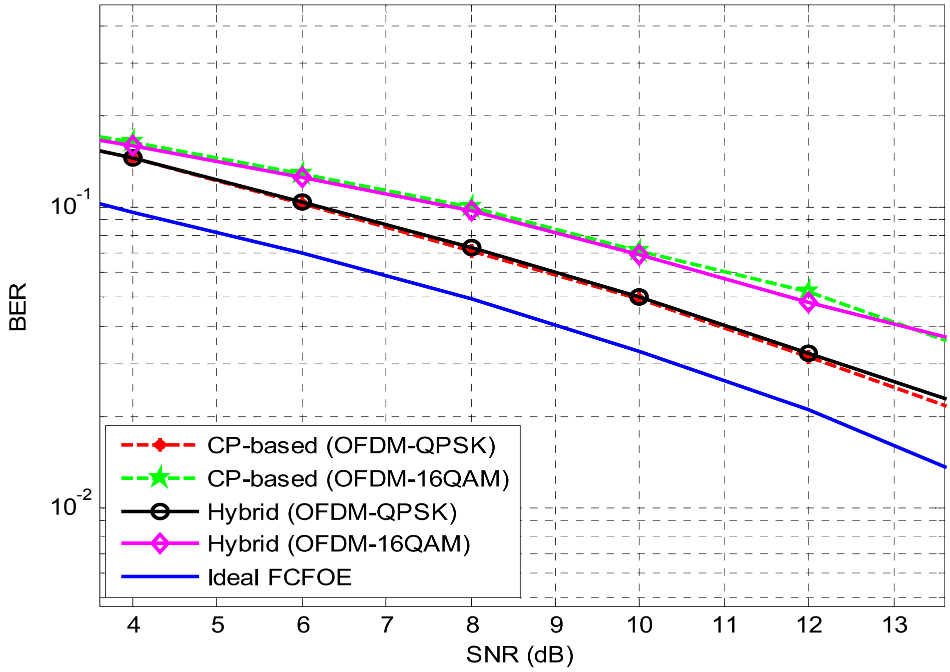


Figure 9: Bit Error Rate performance of the fractional CFO estimators for CFO of 0.125

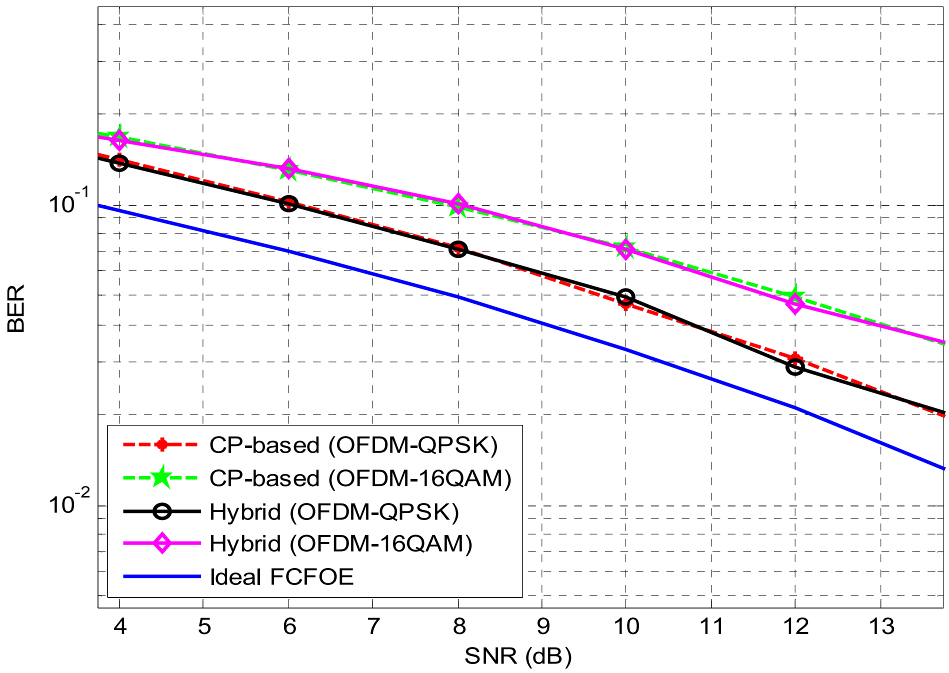


Figure 10: Bit Error Rate performance of the fractional CFO estimators for CFO of 0.25