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Outage Probability of Decentralized Coded User Cooperation in Wireless Communications

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Abstract

In this article, the outage probability as a standard metric for evaluating the performance of a distributed (or decentralized) Reed-Solomon (R-S) coded cooperative communication system is presented. In this system, the users' codewords are split into two frames and are transmitted via two independent fading paths. The first is directly to the destination while the second frame is transmitted via a relaying partner, also to the destination. The outage probability expressions here derived further prove that the R-S coded scheme achieves full diversity. Moreover, the comparisons under different source-partner and source-destination channel conditions made with the centralized Rate Compatible Punctured Convolutional (RCPC) coded cooperative scheme show that the decentralized R-S coded scheme outperforms the centralized RCPC cooperative scheme.

Keywords: coded cooperation, cooperative diversity, outage probability, fading channel, source, partner

1. Introduction

Multipath propagation in wireless networks is highly susceptible to fading, which causes impairments on the channel. This however can be mitigated via a concept known as diversity [1]. And as such, the need for the performance evaluation of such a cooperative communication system operating in such fading environment cannot be overemphasized. One of such parameters for evaluating the performance is the probability of outage or outage probability. This refers to the probability that the received signal will fall below a threshold and cause an outage or a total loss.

In literature, there have been a few researches in computing the outage probability in wireless networks. For example, in [2], the authors derived a general outage probability for a coded cooperative communication scheme for a centralized system, using the rate compatible punctured code (RCPC), while in [3], an outage probability for a cooperative relay is derived. The authors of [4] derived the outage probability for a centralized coded cooperative communication system using the Reed-Solomon (R-S) codes. The coded scheme in [5] in which the R-S codes are used is reported to perform better than the RCPC in [2]. The details of this work are readily available in [5].

However, all the aforementioned researches are for a centralized system in which all the components in the wireless network are controlled by a base station. In this paper, we present a new derivation for the outage probability of a coded cooperative system in a distributed or decentralized network. A drawback of the centralized system is that it brings considerable overheads on the entire network, due to the fact that the base station needs to be in the know of the channel state information of each user in the network. A decentralized system however need not have the precise channel state information of its users.

In these derivations, arbitrarily chosen values for the signal-to-noise ratio (SNR) would be used, and the analytical comparisons under various user-to-user and user-to-destination are intended to showcase the relative preference of the R-S coded scheme for a distributed network over the RCPC system for a centralized configuration. Fig.1 shows a 3-node coded cooperative diversity scheme. The rest of this paper is organized as follows: Section 2 describes and derives the outage probability of the cooperative diversity scheme while simulation results are discussed in Section 3. Section 4 concludes the paper.

2. Outage Probability

A very standard performance evaluation parameter for cooperative diversity is the outage probability, P_o. Let us consider a source – destination direct transmission that is not cooperative and under a slow fading situation. This kind of system has a capacity that can be expressed by the popular Claude-Shanon formula as $C_{\Gamma_{s,d}} = \log_2\left(1+\Gamma_{s,d}\right)$ where C is the capacity of the channel, Γ is the signal-to-noise ratio, s denotes user and d denotes destination. The channel is said to experience outage if the capacity falls below a threshold transmission rate

R, or if $C_{u,d} < R$ [6]. In particular, the outage probability is obtained by solving the integral of the probability density function (pdf) over the interval of the region of the outage event [2].

$$P_o = pr\left[C_{\Gamma_{s,d}} < R\right] = pr\left[\Gamma_{s,d} < 2^R - 1\right] = \int_0^{2^R-1} pr\left(\Gamma_{s,d}\right) d\Gamma_{s,d}$$

$$= \int_0^{2^R-1} \frac{1}{\overline{\Gamma}_{s,d}} e^{-\frac{\Gamma_{s,d}}{\overline{\Gamma}_{s,d}}} d\Gamma_{s,d} = 1 - e^{\left(-\frac{2^R-1}{\overline{\Gamma}_{s,d}}\right)} \tag{1}$$

However, for the R-S coded diversity scheme, with the details in [5], the parity P is split into two parts, P_1 and P_2 where $P_1 = (1 - \beta)P$ and $P_2 = \beta P$ and $0 \leq \beta \leq 1$. The first frame transmission is indicated by $N_1 = k + P_1$ and the second frame by $k + P_2$ where *k* is the length of the original data of say, user 1 (source). The destination receives two data frames: one from the source (first frame) and the other from the partner (second frame) – these two frames each contain the original message *k* and part of the parity bits P_1 and P_2. These two copies (from two users) are combined by the maximal ratio combining (MRC) [7]. The ratio of data received from the partner in the 2nd frame to the total symbols at the destination is called the cooperation level, l_c, while that for the source in the 1st frame is given as $1 - l_c$

2.1 Outage Probability in case of Error – Free Inter – User Channel

Assuming that the partner's channel involved in the cooperative scheme is error-free, then the destination would receive the sent data from the source and partner without any error. It is also noteworthy that these two channels $\Gamma_{s,d}$ and $\Gamma_{r,d}$ are independent. As mentioned earlier, the channel capacity is very important in determining outage events and always used as the upper limit in its approximation. Thus we express the outage probability P_o for source in the following manner:

$$C_{s,d}\left(\Gamma_{s,d}, \Gamma_{r,d}\right) < R \tag{2}$$

where *R* = information rate, and

$$C_{s,d}\left(\Gamma_{s,d}, \Gamma_{r,d}\right) = \left(1 - l_c\right)\log_2\left(1 + \Gamma_{s,d}\right) + l_c \log_2\left(1 + \Gamma_{r,d}\right) = \log_2\left(1 + \Gamma_{s,d}\right)^{(1-l_c)} + \log_2\left(1 + \Gamma_{r,d}\right)^{l_c} \tag{3}$$

The expression in (3) is the outage event. So, for the event, the outage probability P_o is given by:

$$P_{o,errorfree} = pr\left[C_{s,d}\left(\Gamma_{s,d}, \Gamma_{r,d}\right) < R\right] \tag{4}$$

Finding the $\log_2$ of each term in the expression in (4), yields

$$P_{o,errorfree} = pr\left[\left(\left(1 + \Gamma_{s,d}\right)^{(1-l_c)} . \left(1 + \Gamma_{r,d}\right)^{l_c}\right) < 2^R\right] \tag{5}$$

Rewriting (5), we have

$$\left[\left(1 + \Gamma_{s,d}\right)^{(1-l_c)} . \left(1 + \Gamma_{r,d}\right)^{l_c}\right] < 2^R \tag{6}$$

Solving for $\Gamma_{s,d}$, (5) becomes

$$\left(1 + \Gamma_{s,d}\right)^{(1-l_c)} < \frac{2^R}{\left(1 + \Gamma_{r,d}\right)^{l_c}}$$

$$\Rightarrow \Gamma_{s,d} < \frac{2^{R/(1-l_c)}}{\left(1 + \Gamma_{r,d}\right)^{l_c/(1-l_c)}} - 1 \tag{7}$$

Let expression in (7) be x_1;

It is obvious in (7) that $\Gamma_{s,d} > 0$, and as such,

$$\frac{2^{R/(1-l_c)}}{\left(1+\Gamma_{r,d}\right)^{l_c/(1-l_c)}} > 1 \tag{8}$$

$$2^{R/1-l_c} > \left(1+\Gamma_{r,d}\right)^{l_c/1-l_c}$$

This also implies that $\left(1+\Gamma_{r,d}\right)^{l_c/1-l_c} < 2^{R/1-l_c}$

$$\left(\Gamma_{r,d}\right)^{l_c/1-l_c} < 2^{R/1-l_c} - 1$$

Therefore,

$$\Gamma_{r,d} < 2^{R/l_c} - 1 \tag{9}$$

Let the expression in (9) be x_2

Now, for slow fading Rayleigh channel, the outage probability is given as in (10),

$$P_{o,errorfree} = \int_0^{x_1}\int_0^{x_2} \frac{1}{\overline{\Gamma}_{s,d}} e^{-\frac{\Gamma_{s,d}}{\overline{\Gamma}_{s,d}}} \cdot \frac{1}{\overline{\Gamma}_{r,d}} e^{-\frac{\Gamma_{r,d}}{\overline{\Gamma}_{r,d}}} d\Gamma_{s,d}\, d\Gamma_{r,d}$$

$$= \frac{1}{\overline{\Gamma}_{r,d}} \int_0^{x_2} \left(1 - e^{-\frac{x_2}{\overline{\Gamma}_{s,d}}}\right) e^{-\frac{\Gamma_{r,d}}{\overline{\Gamma}_{r,d}}} d\Gamma_{r,d} \tag{10}$$

2.2 *Reciprocal Source – Partner Channel with Erroneous Inter-User Channel*

In coded cooperative diversity, four distinct cooperative scenarios are usually considered. These cases are detailed in [5]. In the case of inter-user channels that are reciprocal, that is, when $\Gamma_{s,r} = \Gamma_{r,s}$, only cooperative scenarios1 and 2 are actually concerned.

Scenario 1: when none of the users successfully decodes its partner. So in the 2nd frame, each user transmits additional parity with its own data. Outage event in such a case is as follows:

$$\begin{aligned} C_{s,r}\left(\Gamma_{s,r}\right) &= \log_2\left(1+\Gamma_{s,r}\right) < R_2 \\ C_{r,s}\left(\Gamma_{r,s}\right) &= \log_2\left(1+\Gamma_{r,s}\right) < R_2 \end{aligned} \tag{11}$$

where $R_2 = R/l_c$.

And so, in this case, the outage event for the source is

$$C_{s,d}\left(\Gamma_{s,d}\right) = \log_2\left(1+\Gamma_{s,d}\right) < R \tag{12}$$

It can be observed here in both (11) and (12) that the outage event in this case is akin to a non-cooperative situation.

Scenario 2: This is when both users (source and partner) successfully decode each other, corresponding to the events as follows:

$$\begin{aligned} C_{s,r}\left(\Gamma_{s,r}\right) &= \log_2\left(1+\Gamma_{s,r}\right) > R_2 \\ C_{r,s}\left(\Gamma_{r,s}\right) &= \log_2\left(1+\Gamma_{r,s}\right) > R_2 \end{aligned} \tag{13}$$

So for the source in this case, the outage event is as follows:

$$C_{s,d}\left(\Gamma_{s,d},\Gamma_{r,d}\right) = \left[\left(1-l_c\right)\log_2\left(1+\Gamma_{s,d}\right) + l_c\left(1+\Gamma_{r,d}\right)\right] < R \tag{14}$$

Then, the outage probability P_o for the source with an erroneous partner is given as

$$P_{o,error} = pr\left[\Gamma_{s,r} > 2^{R_2} - 1\right]\cdot pr\left[\Gamma_{r,s} > 2^{R_2} - 1\right]\cdot pr\left[\left(1+\Gamma_{s,d}\right)^{(1-l_c)}\cdot\left(1+\Gamma_{r,d}\right)^{l_c}\right] < 2^{R}$$
$$+ pr\left[\Gamma_{s,r} < 2^{R_2} - 1\right]\cdot pr\left[\Gamma_{r,s} < 2^{R_2} - 1\right]\cdot pr\left[\Gamma_{s,d} < 2^{R} - 1\right] \qquad (15)$$

$$P_{o,error} = e^{\left(\frac{1-2^{R_2}}{\overline{\Gamma}_{s,r}}\right)} e^{\left(\frac{1-2^{R_2}}{\overline{\Gamma}_{r,s}}\right)} P_{o,errorfree} + \left(1 - e^{\left(\frac{1-2^{R_2}}{\overline{\Gamma}_{s,r}}\right)}\right)\left(1 - e^{\left(\frac{1-2^{R_2}}{\overline{\Gamma}_{r,s}}\right)}\right)\left(1 - e^{\left(\frac{1-2^{R}}{\overline{\Gamma}_{s,d}}\right)}\right) \qquad (16)$$

For reciprocity of the two channels, (16) becomes

$$P_{o,error} = e^{\left(\frac{1-2^{R_2}}{\overline{\Gamma}_{s,r}}\right)} P_{o,errorfree} + \left(1 - e^{\left(\frac{1-2^{R_2}}{\overline{\Gamma}_{s,r}}\right)}\right)\left(1 - e^{\left(\frac{1-2^{R_2}}{\overline{\Gamma}_{s,d}}\right)}\right) \qquad (17)$$

3. Results and Discussions

In Fig. 2, plots showing the SNR vs. BER for a R-S coded cooperative diversity are depicted alongside the non-cooperative case. The 50% and 30% cooperative cases are compared. 50% cooperation is seen to perform better because that is the case when both the source and partner always cooperate and are able to successfully decode each other's information which provides a better result in terms of transmit diversity. For instance, using an inter-user channel SNR of 20 dB, the 50% cooperation performs better than the 30% cooperation. On the other hand, in case of a worsening inter-user channel, say, <<30%, the 50% cooperation performance would drop.

Fig. 3 gives a comparison between the rate-compatible punctured coded (RCPC) cooperative scheme and the R-S coded scheme on one hand and the non-cooperative case on the other. For either the RCPC or the R-S coded scheme, the cooperative case performs much better than the non-cooperative situation. Also, the R-S coded cooperative diversity is seen to outperform the RCPC system.

In Fig. 4, the plots of the SNR vs. outage probability for a distributed system are shown. Using the R-S coded cooperative scheme, the probability of outage when there is no cooperation is much higher than when cooperation is involved. This is because multiple copies of the same information are received at the destination, which ultimately reduces the possibility of having an outage, unlike in a non-cooperative case when only one copy of the signal is received at the destination, which makes it susceptible to degradation and ultimately outage, because of the effect of multipath fading.

However in Fig. 5, a comparison of the theoretical and simulated outage probability for a distributed R-S coded cooperative diversity is carried out. The plots show that there is a close agreement between the theoretical or analytical values of outage probability and the simulated values.

4. Conclusion

In this paper we have been able to derive and analyze the outage probability for a distributed coded cooperative communication system, as a veritable means of evaluating its performance. Simulations have also been carried out to validate our derivations.

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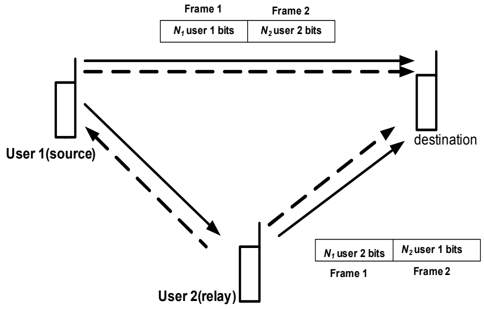

Fig. 1 *3*-node coded cooperative diversity scheme

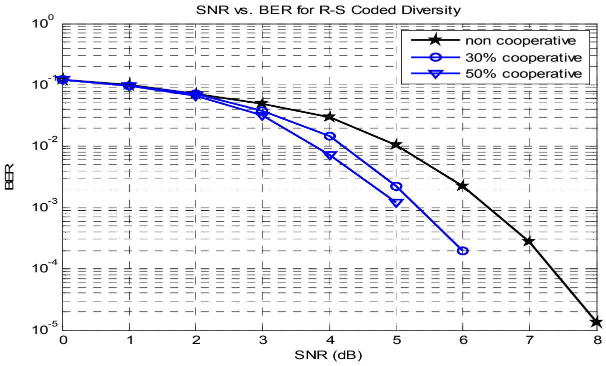

Fig.2. Plots showing the SNR vs. BER for non-cooperative and cooperative systems

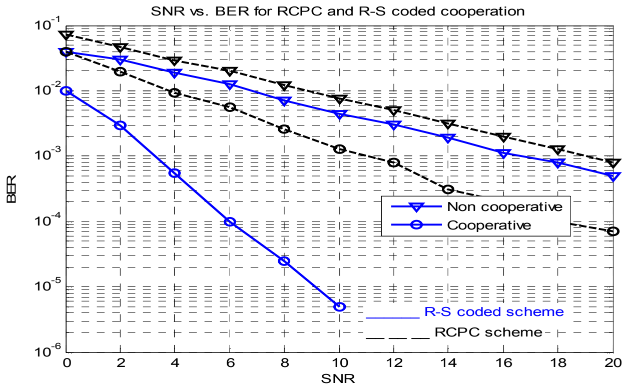

Fig. 3. Plots showing the SNR vs. BER for RCPC and R-S coded cooperative scheme

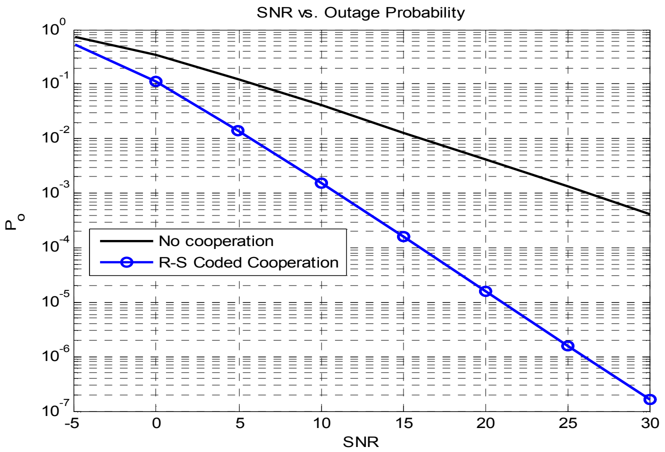

Fig.4 Plots showing the SNR vs. Outage Probability for both non-cooperative and cooperative systems

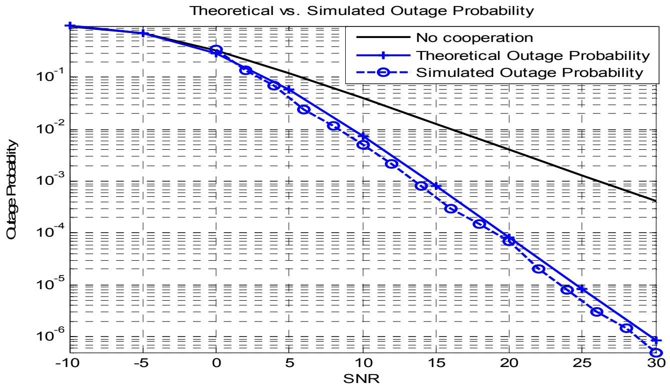

Fig.5 Plots showing the theoretical vs. simulated outage probability

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