Analysis of Students' Difficulties in Solving Probability Questions: The Case of Advanced Certificate of Secondary Education Examination in Tanzania

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Abstract

In this study, we investigated the performance of students in Mathematics, specifically focusing on their ability to solve probability-related questions within the context of the Advanced Certificate of Secondary Education Examination (ACSEE) in Tanzania. Our objective was to identify and highlight the challenges encountered by students when tackling probability problems. Our analysis revealed that a significant number of candidates faced difficulties in correctly solving probability-related questions. The reasons behind these difficulties are multifaceted. They may stem from failure to understand the concepts of probability distributions, especially Binomial and Poison distributions, and the inability to apply the techniques of probability, the inherent complexity of probability tasks, and a tendency to apply generalized problem-solving methods even when they are not suitable for probability questions. To address these challenges effectively, we recommend conducting a protocol analysis to gain deeper insights into the cognitive processes of students when attempting probability questions. By understanding the underlying reasoning and thought processes, teachers can tailor their instructional approaches to better assist students in comprehending and solving probability problems. This tailored approach can facilitate more meaningful learning experiences for students, ultimately improving their performance in probability and mathematics as a whole. The findings of this analysis have the potential to inform educational policies and instructional practices aimed at improving students' performance in probability-related questions in the ACSEE and beyond. By addressing students' difficulties comprehensively and equipping them with the necessary skills and conceptual understanding, we can foster a generation of mathematically literate individuals capable of confidently navigating probabilistic scenarios in various contexts.

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1. Introduction

Probability is an important topic in the secondary school mathematics curriculum, which introduces the student to the mathematical behaviour of uncertainty (Moore, 1990). It has become increasingly evident that probability is being treated as an important topic in standard level mathematics in many countries and is included in school final examinations (Shaughnessy, 2006). In recent years, there has been an increasing trend to introduce probability at all levels of the school curriculum, in particular, using an experimental approach with a view to developing probabilistic thinking as a tool for decision making (Batanero et al., 2016). With the mandatory introduction of Modern Mathematics into the syllabus, there is increased emphasis on the topic of probability, as it encompasses a wide range of areas such as permutations and combinations, data analysis, and inferential statistics. Difficulty with understanding probability has been recognized as a key mathematical learning difficulty for many school and college students (Wilensky, 1995).

Despite its importance, there has been little research to examine misconceptions associated with probability and to investigate the implications of these misconceptions for teaching and learning (Paul & Hlanganipai, 2014). At a purely anecdotal level, however, probability is increasingly being singled out as an area of difficulty for many students and teachers, and as a topic that is often not well understood by students coming out of the school system (Shaughnessy, 2006). This view is supported by the declining performance of Australian students in the International Assessments for School Students, which showed that year 9 students had performed worse, relatively to other countries, over the 10-year period, in comparison with year 5 student performance (Anderson et al., 2024).

In Tanzania, probability has been taught in the Advanced Certificate of Secondary Education (Kibona et al., 2020). Because of its abstractness, teaching and learning of probability has been a great challenge to both teachers and students (Nilsson & Li, 2015). Probability has become one of the hard topics for form six students in their ACSEE (Bartlett & Vavrus, 2013). It has been noted that students have had low performance in this topic, and the success rate of students on the probability topic in comparison to the rest of the topics is still very low. Teachers have observed that even students with high cumulative grades at the end of the course have still performed poorly in the probability topic. The low performance of the topic is also evident through the performance of schools on the national level. Measures need to be taken to ensure the success of students in ACSEE probability topic and increase the probability knowledge of students. This study proposes to identify the difficulties of the probability topic at ACSEE by assessing questions in the national exams and also by informal interviews with teachers.

The poor performance of students on the probability topic can also be seen in Tanzania CSEE (Limbe, 2017). An analysis conducted by NECTA on the performance of students in Basic Mathematics in 2023, particularly focusing on the topic of probability, reveals that out of the 521,886 (100%) candidates who attempted the question, only 65,498 (12.6%) candidates passed. This indicates that the candidates' performance on this question was weak, as they were unable to present the information in a tree diagram, thus failing to determine the sample space and the outcomes of the events. This suggests that the probability topic is less favoured by students in Tanzania.

This study analyses the difficulties encountered by students in Tanzania when solving probability questions from the Advanced Certificate of Secondary Education Examination (ACSEE).

1.1 *The System Of Education In Tanzania*

The education system in Tanzania follows a structured, pyramidal format encompassing various levels from primary to university (Malmberg & Hansen, 1996). This system is designed to provide students with a comprehensive educational journey that prepares them for higher academic pursuits and professional endeavors. The formal education structure in Tanzania is typically delineated as 2-7-4-2-3, representing the duration of each educational phase: 2 years of pre-primary education, 7 years of primary education, 4 years of ordinary secondary education, 2 years of advanced secondary education, and a minimum of 3 years of university education (Ndalichako, 2013). Figure 1 shows the Education System in Tanzania.

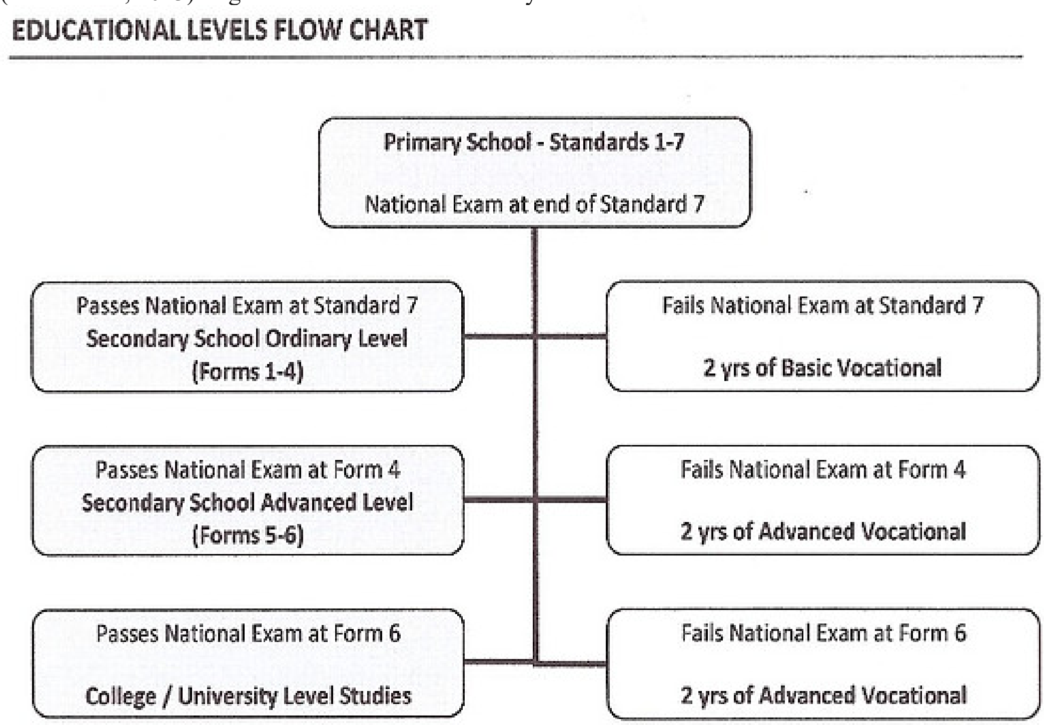

Figure 1: The Education System in Tanzania.

At the primary level, which forms the foundation of the education system, students undergo 7 years of structured learning aimed at imparting foundational knowledge and skills across various subjects (Yahl, 2015). Primary education serves as a crucial phase in students' academic development, laying the groundwork for their future educational achievements.

During this stage, students are introduced to core subjects such as mathematics, science, language arts, and social studies, among others, to foster holistic development. The culmination of primary education in Tanzania is marked by the Primary School Leaving Examination (PSLE), which serves as a pivotal assessment tool to evaluate students' academic proficiency and readiness for secondary education. The PSLE assesses students' mastery of key subject areas and their ability to apply acquired knowledge in problem-solving contexts. Performance in the PSLE plays a significant role in determining students' progression to the next level of education, particularly in gaining admission to secondary schools. Individuals who do not proceed to secondary education have the option to enrol in vocational training programs or explore alternative pathways toward entering the workforce.

Following successful completion of primary education and the PSLE with good performance, students transition to secondary education, where they undergo two distinct phases: ordinary level (O-level) and advanced level (A-level).

During the O-level phase, which spans 4 years, students delve deeper into a more specialized curriculum aimed at strengthening their comprehension and skills in core subjects. This phase is crucial for laying the foundation for further academic pursuits and professional development. The curriculum during this period is designed to provide students with a comprehensive understanding of subjects such as Mathematics, Science, English, Kiswahili and Social Studies, among others. Additionally, students may have the opportunity to explore elective subjects based on their interests and career aspirations, further enriching their educational experience.

Moreover, the O-level phase not only focuses on academic knowledge but also emphasizes the development of critical thinking, problem-solving, and practical skills. Students are encouraged to engage in hands-on activities, experiments, and projects to apply theoretical concepts to real-world scenarios. This approach not only enhances their understanding of the subjects but also fosters creativity, innovation, and independent learning.

Furthermore, the O-level curriculum often includes extracurricular activities, such as sports, arts, and community service, to promote holistic development among students. Participation in these activities helps students develop essential life skills such as teamwork, leadership, and time management. Additionally, extracurricular activities provide students with opportunities to explore their interests, talents, and passions beyond the classroom, contributing to their personal growth and overall well-being. Given that the primary focus of this paper is on the advanced certificate of secondary education examination, the subsequent sections will illuminate the educational structure at the advanced secondary level and delve into the characteristics of its culminating examination.

1.1.1 **The Structure Of Advanced Secondary Education In Tanzania**

The A-level phase comprises of 2 years of advanced studies tailored to their chosen academic or career paths, marking a crucial stage in students' educational journey. During this phase, students have the opportunity to specialize in specific subjects based on their interests, aptitudes, and future aspirations. A-Level education in Tanzania is structured to offer students a specialized focus on subjects that are critical to their intended career paths. These combinations are grouped into three main streams: Natural Sciences, Arts, and Business. Each stream caters to different academic interests and career goals, with specific combinations designed to prepare students for higher education and professional careers in related fields. The curriculum is designed to provide in-depth knowledge and understanding of the selected subjects, preparing students for higher education, professional endeavors, or vocational training programs. In the Natural Sciences stream, combinations such as Physics, Chemistry, and Mathematics (PCM) are tailored for engineering and STEM fields, while Physics, Chemistry, and Biology (PCB) suit those interested in medicine and biological sciences. For students inclined towards humanities and social sciences, the Arts stream provides combinations like History, Geography, and English Language (HGL) for careers in education and law. In the Business stream, combinations like Economics, Commerce, and Accountancy (ECA) are designed for finance and business management careers. Additionally, new combinations such as Physics, Mathematics, Computer Studies (PMC) address the demands of the digital age, offering pathways to careers in information technology and cybersecurity.

1.1.2 **The Advanced Certificate Of Secondary Education Examination**

The Advanced Certificate of Secondary Education Examination (ACSEE) signifies the culmination of secondary education in Tanzania. ACSEE comprises multiple papers assessing various subjects from the secondary education curriculum. The examination is overseen by the National Examinations Council of Tanzania (NECTA) and is administered with strict supervision by regional and district authorities. This is offered to candidates who have completed two years of secondary education (advanced level) and have had three credits at CSEE level. The ACSEE is administered on the first week of May every year (NECTA).

The objectives of this examination are to assess the learner’s knowledge and ability to pursue with further education such as diploma and degree courses; to examine the extent to which the learners can use the skills gained to meet the social, political, economic and technological challenges for the individual and the national development at large. Therefore, candidates at this level are expected to possess the following skills in a broad range of activities: knowledge, comprehension, application, analysis, synthesis and evaluation (NECTA).

The examined subject sat ACSEE are as follow: General Studies which is compulsory subject; other subjects are grouped in combination, i.e., natural science which include Physics, Chemistry and Mathematics (PCM), Physics Chemistry and Biology (PCB), Physics Geography and Mathematics (PGM), Economics, Geography and Mathematics (EGM), Chemistry, Biology and Geography (CBG), Chemistry, Biology and Agriculture (CBA) and Chemistry, Biology and Food and Human Nutrition (CBN). Another category is Arts Combinations which Includes History, Geography and English Language (HGL), History, Geography and Kiswahili (HGK), History, Kiswahili and English Language (HKL), Kiswahili, English Language and French (KLF), Economics, Commerce and Accountancy (ECA) and History, Geography and Economics (HGE). Each subject has an examination format, which describes the structure of the examination paper, rubric and the content in which that particular examination covers (NECTA).
The Advanced Mathematics examination consists of two papers. Paper 1, coded as 142/1 Advanced Mathematics 1, featuring ten (10) compulsory questions, with each question carrying ten (10) marks. Paper 2, coded as 142/2 Advanced Mathematics 2, comprises sections A and B. Section A includes four (4) compulsory questions in which question number 1 is probability, each worth fifteen (15) marks, while Section B comprises four (4) optional questions, with each question carrying twenty (20) marks. Candidates are required to attempt only two (2) questions from Section B (NECTA).

2. Performance Of Candidates In Advanced Mathematics In Acsee 2023

The candidates' performance in the Advanced Mathematics paper was good, with 96.86 percent of those who sat for the examination passing. The analysis indicated that candidates performed well in 13 out of the 18 topics tested. Notably, candidates excelled in the topics of Functions, Logic, Statistics, Sets, Trigonometry, Differential Equations, Coordinate Geometry I, Algebra, Hyperbolic Functions, Linear Programming, Calculating Devices, Complex Numbers, and Vectors. However, candidates showed average performance in the topics of Integration, Coordinate Geometry II, and Differentiation, while their performance was weak in the areas of Probability and Numerical Methods. Weaknesses were observed in understanding the concepts of probability distributions, particularly Binomial and Poisson distributions (NECTA,2024). The summary of candidates' performance in this question is presented in Figure 3.

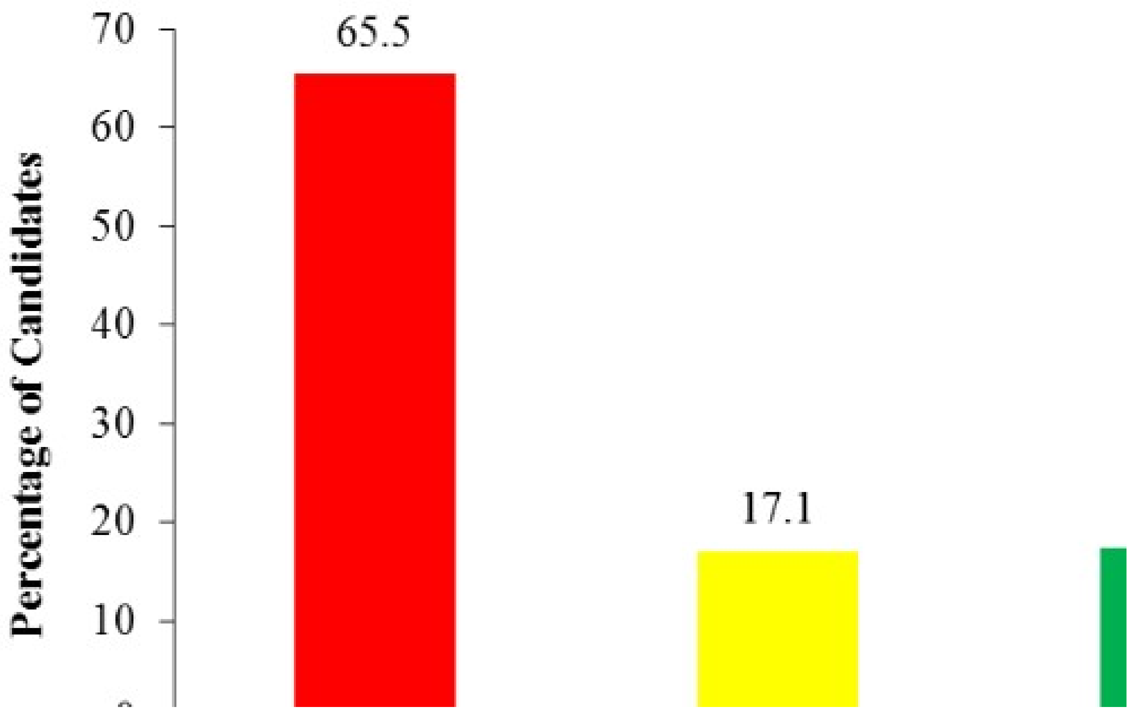

Figure 2:Candidates' Performance in probability question in 2023 ACSEE.

OVERVIEW OF CHALLENGES IN LEARNING PROBABILITY

Learning probability poses various challenges for students, including misconceptions, difficulty in grasping abstract concepts, and applying appropriate problem-solving strategies (Paul & Hlanganipai, 2014). Many learners struggle with understanding the fundamental principles of probability, often confusing them with other mathematical concepts. Additionally, overgeneralization of mathematical rules and procedures can lead to errors in probabilistic reasoning (Bartlett & Vavrus, 2013). Factors such as inadequate teaching methods and insufficient teacher training further exacerbate these challenges (Kibona et al., 2020).

2.1 Insufficient knowledge of basic probability principles

Students learn probability at many levels of education, from junior high school to university (Batanero & Sánchez, 2005). In learning probability, students are expected to be able to apply the probability rules to count the probabilities in certain problems. However, in reality, there are many students who cannot apply the probability rules they have understood to solve problems. This will hinder their understanding of probability itself. This problem will affect their ability to apply probability in real-life problems and explain certain events from a probability perspective.

For instance, Probability question in ACSEE 2017 in part (b) (i), the candidates were asked to *"find the number of ways in which a team of 5 members can be selected from a group of students consisting of 4 girls and 7 boys if the team has at least a boy and a girl."(NECTA, 2017)*

The analysis shows that the candidates were unable to apply the combination formula in order to find the number of ways a team of 5 members would be selected.

Students' probability knowledge consists of six basic concepts: sample space, event (A and B), probability, permutations, and combinations. Since probability is a difficult subject to understand, students' prior knowledge on the basic concepts is important. This is due to their inability to differentiate the meaning of sample space and event. Mostly, they think of sample space as an event and have a problem understanding permutations and combinations. For instance, the poor performance of students in the 2017 ACSEE probability questions, based on the analysis conducted by NECTA, reveals that candidates lacked knowledge and skills in the topic of probability, leading to 31.7 percent scoring a zero mark (NECTA, 2017).

2.2 Difficulty in comprehending complex probability concepts

This difficulty arises when students struggle to understand advanced probability concepts, such as conditional probability, Bayes' theorem, or probability distributions. For instance, students may find it challenging to comprehend the concept of independence between events or the application of probability rules in complex scenarios. This difficulty may stem from a lack of foundational knowledge or inadequate exposure to higher-level probability concepts, making it difficult for students to engage with and apply these concepts effectively in problem-solving situations.

For instance, Probability question in ACSEE 2019 in part (a)(ii), the question stated that, *"A machine produces a total of 10,000 nails a day which on average 5% are defective. Find the probability that out of 500 nails chosen at random 10 will be defective".* The analysis shows that a significant number of candidates did not realize that the problem could be solved by applying binomial distribution formula. Some candidates used the formula $P(E) = \frac{n(E)}{n(S)}$ while the question required them to use the formula for binomial distribution (NECTA,2019).

Furthermore, students may encounter difficulties in understanding the mathematical notation and terminology associated with complex probability concepts. For example, symbols such as $P(A \cap B)$ for the intersection of events or $P(A|B)$ for conditional probability may be confusing for students, particularly if they have not been adequately introduced to these symbols and their meanings. Additionally, the abstract nature of probability concepts, such as random variables and probability distributions, may pose challenges for students in visualizing and conceptualizing these concepts. Without a solid understanding of the underlying principles, students may struggle to apply these concepts accurately in solving probability questions on assessments like the ACSEE.

2.3 Inability to apply probability rules correctly

When working on probability problems, it is not simply enough to stick the different pieces of information into a form of the type P(A|B) and hope that the correct answer will pop out the other end. To envision a problem, recognize the given results, and apply the correct formula, one needs a strong foundation and precise interpretations of the probability rules. A student must have a procedural understanding of the rules, backed up by a strong understanding of the theory. What we frequently find in students who have only been briefly exposed to probability is that they have merely memorized formulas, but don't really understand what's going on. This is not a criticism of students, it is just that probability is typically only a small portion of a student's course, and since the calculations are often straightforward, not much time is spent on the concepts. This makes probability misleading, because the short and easy calculations can be quite deceptive in terms of the logic and concepts behind them.

For instance, *the poor performance of students in the 2022 ACSEE probability questions, based on the analysis conducted by NECTA, reveals that candidates lacked knowledge and skills in the topic of probability particularly on using conditional probability formula, leading to (45.0%) candidates scored 0 to 5 marks (NECTA, 2023). It was further observed that a few candidates used the poison distribution formula. Moreover, it was observed that a few candidates used the variance formula which was not the equation as per the correct procedure*. Many errors in probability can be attributed to the fact that students attempt to use methods of computation from other areas of mathematics, and have trouble distinguishing between different types of problems. Errors can also arise from students' haste, as they are often reluctant to spend time thinking about a probability problem since they know that the calculations are not very complex.

2.4 Lack of systematic approach to solving probability problems

It is apparent that this inability to understand probability is due to an inability to associate probability with real-life situations. Students struggle more to apply a consistent and organized approach when solving probability problems. For example, instead of systematically applying probability rules or techniques such as the multiplication rule or Bayes' theorem, students may resort to trial and error or rely on intuition, leading to inconsistent results and potential errors in their solutions.

For instance, Probability question in ACSEE 2022 in part (a), the question stated that, ***"****The time taken by John to deliver milk to the High Street is normally distributed with mean 12 minutes and standard deviation 2 minutes. Candidates were required to estimate the number of days during the year when he takes longer than 17 minutes if he delivers milk every day (1 year = 365 days)"*, the analysis shows that some candidates were able to write $Z = \frac{X-\mu}{\sigma}$ correctly but wrongly substituted $\mu = 17, \sigma = 2 \text{ } and \text{ } X = 12$ to obtain $P(z > -2.5)$ instead of $P(z > 2.5)$ (NECTA, 2023). This shows that the students lacked consistency in solving the problem accurately as some of students substituted $\mu = 17, \sigma = 2 \text{ } and \text{ } X = 365$. Probability scales provide a step-by-step method to solve a probability problem and are an effective tool for understanding the concept of probability.

So by not being able to understand this type of problem, students are failing to use a simple and effective method to solve probability problems. The lack of a systematic approach to solving probability problems is a common occurrence among elementary school students (Smith, 2019). Although students are able to compute correct answers, they often do not understand and are unable to explain the concept of probability (Salido & Dasari, 2019).

2.5 Inability to identify relevant information

This challenge emerges when students face probability questions that contain extraneous or non-essential information alongside critical data needed to solve the problem. For instance, students may encounter difficulty in distinguishing between relevant information, such as given probabilities or event outcomes, and irrelevant distractions, such as superfluous numerical values or background context. This difficulty may stem from a lack of discernment or critical thinking skills, making it challenging for students to isolate and focus on the essential components of the problem.

For example, a probability question asked in ACSEE 2018 In part (b), the question stated that, *"Kalihose's family consists of mother, father and their ten children and it is invited to send a group of 4 representatives to a wedding". They were required to find the number of the ways in which the group could be formed so that it could contain (i) both parents, (ii) one parent only and (iii) none of the parents"*. The analysis shows that several candidates could not comprehend the given word problem so as to realize that they should use the principle of combination in answering this question. Some of them employed the principle of permutations, while others interpreted the given word problem in the perception of the Sets' concept and hence used Venn diagrams contrary to the requirements of the question (NECTA, 2018).

Moreover, the inability to identify relevant information may result from a lack of familiarity with probability problem-solving strategies or techniques. For example, students may struggle to recognize key probability concepts or formulas applicable to the given scenario, leading to uncertainty about which information is essential for solving the problem. Additionally, students may encounter difficulty in interpreting complex probability questions that contain multiple variables or conditions, making it challenging to discern which data points are crucial for formulating a solution strategy. Without a clear understanding of the problem requirements or the underlying probability concepts involved, students may become overwhelmed or confused by the abundance of information presented in the question.

Furthermore, the inability to identify relevant information can hinder students' ability to formulate an effective problem-solving approach and may lead to errors or inaccuracies in their solutions. For instance, if students overlook critical details or misinterpret the given information, they may apply incorrect probability rules or make faulty assumptions, resulting in flawed solutions. Addressing this difficulty requires providing students with explicit instruction in identifying and prioritizing relevant information within probability problems, as well as opportunities for guided practice in applying problem-solving strategies to extract essential data and formulate accurate solutions. By developing students' proficiency in discerning relevant information, educators can empower them to approach probability problems with clarity and confidence, thereby enhancing their overall problem-solving skills.

2.6 Failure to create a step-by-step plan

This challenge arises when students struggle to outline a systematic strategy or roadmap for solving probability questions effectively. For example, when faced with complex probability scenarios, such as those encountered in the ACSEE examination, students may find it challenging to break down the problem into manageable steps and identify the appropriate techniques or formulas to apply at each stage. Without a clear plan of action, students may become overwhelmed or lost in the problem-solving process, leading to confusion and inefficiency in their approach.

For example, the probability question asked in ACSEE 2016 in part (a), the candidates were required to use the given information that *"the probability that a keyboard picked at random from the assembly line in a factory to be defective is 0.01, whereas a sample of three is to be selected" and then asked to (i) construct the probability distribution of the defective keyboards and in (ii) find the mean and standard deviation (leaving the answers correct to 2 decimal places).* The analysis shows that in part a(i) there were candidates who used the tree diagram method which was wrong because that method could not account the number of trials $X = 0, 1, 2$ or 3. At the same time, even the computation for $P(X)$ did not involve the binomial distribution as the necessary strategy in solving part (a) (i). Also, in part (a) (ii) they used wrong formulae to find the mean and standard deviation (NECTA,2016).

Furthermore, the failure to create a step-by-step plan may result from a lack of metacognitive awareness or problem-solving skills among students. For instance, students may not recognize the importance of devising a structured strategy for approaching probability problems or may struggle to anticipate the necessary steps required to reach a solution. Additionally, students may lack experience in applying problem-solving heuristics or algorithms, such as working backwards or breaking the problem into smaller subproblems, which can help facilitate the creation of a systematic plan. Without guidance or practice in developing these problem-solving skills, students may resort to trial-and-error methods or rely on intuition, leading to inefficient and ineffective problem-solving strategies.

Moreover, the failure to create a step-by-step plan can impede students' ability to monitor their progress and make adjustments as needed during the problem-solving process. For example, without a clear roadmap, students may encounter difficulties in identifying errors or misconceptions in their approach and may struggle to backtrack or revise their strategies accordingly. Additionally, the absence of a structured plan may hinder students' ability to manage their time effectively during examinations, leading to rushed or incomplete solutions. Addressing this difficulty requires fostering students' metacognitive skills and providing explicit instruction in problem-solving strategies, such as breaking down problems into manageable steps, identifying relevant information, and selecting appropriate solution techniques. By equipping students with the tools and techniques needed to create a step-by-step plan, educators can empower them to approach probability problems with confidence and proficiency.

2.7 Neglecting to evaluate the reasonableness of the solution

This challenge arises when students overlook the importance of verifying whether their obtained solutions align with the context of the problem and meet logical expectations. For instance, students may arrive at numerical answers without considering whether the results make sense given the given scenario or if they fall within a plausible range of values. This neglect of evaluating solution reasonableness can lead to the acceptance of erroneous answers that do not accurately represent the real-world situation presented in the problem.

Furthermore, this difficulty may manifest when students lack the necessary critical thinking skills to assess the coherence and accuracy of their solutions. For example, when solving probability questions involving real-life scenarios, such as those encountered in the ACSEE examination, students may overlook inconsistencies or illogical conclusions in their solutions.

For example, a probability question asked in ACSEE 2021 In part (b), the question stated that, *"(b) Two dice are thrown simultaneously, (i) list the sample space for the events. (ii) find the probability that the sum of the numbers obtained on the dice is neither a multiple of 2 nor a multiple of 3".* The analysis shows that some of candidates failed to differentiate between a dice and a coin. There were candidates for example who used a tree diagram to list the elements in the sample space as {HH, HT, TH and TT}. Other candidates quoted the elements in the first and second die as {1, 2, 3, 4, 5, 6} and {1, 2, 3, 4, 5, 6} which was contrary to the demand of the question. Hence, they could not provide the correct answers in (ii) (NECTA, 2021). This actually shows that they lacked to have a correct reasoning of the reality of the scenarios of die and coin.

Without engaging in a reflective process to evaluate the reasonableness of their answers, students may inadvertently accept incorrect solutions that deviate from expected outcomes. Consequently, this neglect can lead to misconceptions about probability concepts and undermine students' ability to apply mathematical reasoning effectively in problem-solving contexts. Addressing this difficulty requires fostering students' metacognitive skills, encouraging them to reflect on their problem-solving processes, and promoting a mindset of critical evaluation of solution outcomes to enhance their overall mathematical proficiency

2.8 Assessment Procedures

This difficulty manifests when students struggle to comprehend the specific requirements and expectations outlined in the assessment procedures. For instance, students may find it challenging to interpret the instructions provided in probability questions, leading to misinterpretation of problem requirements and subsequently incorrect solutions. Additionally, variations in the format and structure of assessment tasks, such as multiple-choice questions, short-answer questions, or problem-solving tasks, can pose difficulties for students in selecting appropriate solution strategies and demonstrating their understanding effectively. Consequently, students may experience difficulty in showcasing their true knowledge and proficiency in probability, impacting their performance in examinations like the ACSEE.

Furthermore, inconsistencies or lack of clarity in assessment procedures can exacerbate students' difficulties and hinder their ability to accurately gauge their progress and identify areas for improvement. For example, ambiguous or poorly worded questions may lead to confusion among students, making it challenging for them to discern the intended mathematical concepts being assessed. For example, the probability question in ACSEE 2023 in part (c), they were required to find the number of students who scored between 30% and 70% inclusive of 100 students in a senior mathematics contest examination for the year 2014, with the mean and standard deviation of 64 and 16, respectively. One of the challenges faced by students is the failure to understand the need of the question, recall it, and write the correct formulae as most of the candidates applied the wrong approach to compute the probability $P(30\% \leq X \leq 70\%)$ *by using an improper formula as they wrote the formula for the probability of non-mutually exclusive events (NECTA,2024).*

Similarly, discrepancies between the content covered in instructional materials and the topics included in assessment tasks can contribute to students' confusion and uncertainty regarding the relevance of certain mathematical concepts. As a result, students may experience frustration and anxiety during assessments, potentially compromising their performance and hindering their overall learning experience. Addressing these challenges in assessment procedures is essential for promoting fair and equitable evaluations of students' understanding of probability concepts and fostering a supportive learning environment conducive to their academic success.

3. Statement Of The Problem

The pivotal role of mathematics in fostering logical reasoning skills among students is widely recognized by educators and scholars. In Tanzania, the education system underscores this importance by placing significant emphasis on mathematics across primary and secondary levels. Mathematics receives a higher allocation of teaching periods compared to other subjects starting from Grade I, underscoring its significance. Within the Certificate for Secondary School Examination, failure in mathematics carries substantial consequences; candidates who do not meet the required standard in mathematics cannot achieve first or second division, regardless of their performance in other subjects. This measure aims to incentivize students to devote more effort to mathematics, laying a robust groundwork for their proficiency in related subjects and nurturing their logical thinking abilities. However, despite these efforts, the performance in mathematics at both primary and secondary levels has persistently fallen short of desired standards over the years.

4. Objectives Of The Study

To assess the difficulties encountered by students when answering questions that test their understanding of probability in the ACSEE, and to find out what were the most common misconceptions that students faced when dealing with probability questions, this is a case study of students in selected schools in the Coast region of Tanzania. This is a qualitative study that analysed students' answers to probability questions in a test and one-on-one interview of students was carried out to further understand how the test questions were interpreted by the students. This study looks at students from the best schools. Best, in this case, is defined by the schools with the best pass rate in the ACSEE, in the region and compares them to students in ordinary schools. The findings and analysis will show that students faced several difficulties in probability questions. These difficulties were not only limited to students from ordinary schools who are generally considered by teachers to be weak in the subject. It was surprising to see that even students from the best schools faced similar difficulties and made similar mistakes when answering the probability questions. This points to a deeper problem in the way probability is taught by teachers and understood by students at the secondary level in Tanzania.

5. Analysis of Candidates' Responses to Probability questions.

In this section, we present the analysis of candidates' responses to probability questions. These questions encompass various scenarios and levels of complexity within the field of probability. For each question, we examine the patterns and trends in candidates' responses, aiming to identify common errors and challenges encountered by students. The analysis provided here offers insights into the performance of candidates on specific probability questions:

Question 1: Probability

The question comprised three parts: (a), (b), and (c). Part (a) required the candidates to prepare the probability distribution table of obtaining 0, 1, 2, 3, 4, and 5 defective tomatoes in a random batch of 20 tomatoes for which, on average, 15 percent of tomatoes were defective using: (i) Binomial distribution and (ii) Poison distribution. In part (b), the candidates were required to compute the mean and standard deviation of the two cases, in part (a) and in part (c), they were required to find the number of students who scored between 30% and 70% inclusive of 100 students in a senior mathematics contest examination for the year 2014, with the mean and standard deviation of 64 and 16, respectively.

The question was attempted by 13,754 (99.99%) candidates. The analysis shows that among the 13,754 candidates who attempted the question, 9,011 (65.5%) scored 0 to 5 marks. While 2,355 (17.1%) candidates scored 5.5 to 8.5 marks and 2,388 (17.4%) scored 9 to 15 marks. Therefore, the performance of the candidates in this question was weak since 34.5 per cent of the candidates who attempted this question scored from 5.5 to 15 marks (NECTA,2024). The majority of candidates who tackled the question experienced difficulties, resulting in a subpar performance. Among the challenges they encountered was a lack of understanding regarding the question's requirements, leading to difficulty in recalling it and applying the correct formulas.

6.0 Conclusion

In this investigation, we examined students' difficulties in solving probability questions, focusing on the Advanced Certificate of Secondary Education Examination in Tanzania. We found that many students struggled with understanding and applying probability concepts correctly. They often mixed up probability ideas with other math concepts. For instance, when faced with probability questions, they sometimes approached them as they would with regular math problems, rather than using probability principles. This confusion may stem from students' struggles in grasping the appropriate procedures for solving probability problems and the overall complexity of these tasks.

Moreover, students tended to over-generalize mathematical procedures, leading them to apply inappropriate methods to probability questions. For example, some students believed that multiplying always results in larger numbers. However, this rule doesn't hold true when dealing with probabilities, especially when multiplying by fractions or decimals less than one. This misunderstanding could pose significant challenges for students who rely on such generalizations when tackling probability problems.

Additionally, the educational context and resources available may also contribute to students' difficulties in learning probability. Inadequate teaching methods, limited access to quality instructional materials, and insufficient teacher training can hinder students' ability to understand and apply probability concepts effectively. Furthermore, cultural factors and societal expectations may impact students' attitudes towards mathematics and probability, influencing their motivation and engagement in learning. Addressing these systemic challenges requires collaborative efforts from educators, policymakers, and stakeholders to improve the quality of mathematics education and provide students with the necessary support and resources to succeed in learning probability. By addressing these factors, we can help students overcome their difficulties and develop a solid foundation in probability, empowering them to apply mathematical reasoning and problem-solving skills in various contexts.

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