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Effect of Reinforcement Ratio on Damage in Reinforced Concrete Beams- A Damage Mechanics Approach

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ABSTRACT

The principles of damage mechanics are used to study the effect of reinforcement ratio on the total damage of reinforced concrete beams. The definition of the damage variable in terms of the damaged and effective cross-sectional areas is adopted. A consistent and simple mathematical derivation is presented to find the exact relation between the total damage and the damage of concrete in a reinforced concrete beam. It is shown that the reinforcement ratio has a clear but small effect on the total damage variable of the reinforced concrete beam. As the reinforcement ratio increases, the total damage in the beam decreases. Although this effect is small, it becomes more pronounced at higher levels of damage in the beam.

KEYWORDS: Reinforcement Ratio, Damage, Reinforced Concrete Beams, Damage Mechanics Approach.

1. INTRODUCTION

Kachanov (1958) pioneered the subject of damage mechanics by introducing the concept of effective stress. This concept is based on considering a fictitious undamaged configuration of a body and comparing it with the actual damaged configuration. The damage variable was defined in terms of both the damaged and effective cross-sectional areas of the body. Kachanov (1958) originally formulated his theory by using simple uniaxial tension. Following Kachanov's work, researchers in different fields applied damage mechanics to their areas in fields like brittle materials.

(Krajcinovic and Foneska, 1981; Krajcinovic, 1988) and ductile materials (Lemaitre, 1984, 1985, 1986; Kachanov, 1986; Murakami, 1988). In the 1990's, applications of damage mechanics to plasticity and composite materials have appeared (Voyiadjis and Kattan, 1990, 1993, 1999; Kattan and Voyiadjis, 1990, 1993a, 1993b, 1996; Voyiadjis and Park, 1997a, 1997b; Voyiadjis and Thiagarajan, 1996; Voyiadjis et al., 1995.

The effect of the reinforcement ratio on various aspects of the behavior of reinforced concrete has been studied in the past. Ashour (2000) and Ashour et al. (2000) studied the effect of tensile reinforcement ratio and compressive strength on the flexural behavior of high-strength concrete beams. Theriault and Benmokrane (1998a, 1998b) examined the effects of FRP reinforcement ratio and concrete strength on the flexural behavior of concrete beams. Dancygier (1997) studied the effect of reinforcement ratio on the resistance of reinforced concrete to hard projectile impact. Al-Shaikh and Al-Zaid (1993) studied the effect of reinforcement ratio on the effective moment of inertia of reinforced concrete beams. Also, Yamada et al. (1992) examined the influence of reinforcement type and ratio on the punching shear resistance of flat slabs. Finally, Grzybowski and Meyer (1993) investigated the damage accumulation in concrete with and without fiber reinforcement. However, the effect of reinforcement ratio on the total damage in reinforced

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concrete beams has not appeared in the before literature.

A consistent and simple mathematical derivation is presented to find the exact relation between the total damage and the damage of concrete in a reinforced concrete beam. The derivation is elementary and based on the principles of damage mechanics. Although damage mechanics was applied in the past to reinforced concrete, the study of the effect of reinforcement ratio on total damage has not been attempted before. It is well known that the maximum width of cracks in reinforced concrete beams can be estimated by using the Gergely-Lutz equation (Gergely and Lutz, 1968; McCormac, 1998). However, the reinforcement ratio does not appear in this famous equation. Instead of that, the number of reinforcing bars plays an important role in this equation. The relation of the Gergely-Lutz equation to this work is also examined.

2. BRIEF REVIEW OF DAMAGE MECHANICS

The principles of damage mechanics are first reviewed for the case of uniaxial tension. In this case, isotropic damage is assumed throughout. Consider a cylindrical bar subjected to a uniaxial tensile force T as shown in Figure 1a. The cross-sectional area of the bar is A and it is assumed that both voids and cracks appear as damage in the bar. The uniaxial stress σ in the bar is found easily from the formula $T = \sigma A$. In order to use the principles of damage mechanics, we consider a fictitious undamaged configuration of the bar as shown in Figure 1b. In this configuration, all types of damage, including both voids and cracks, are removed from the bar. The effective cross-sectional area of the bar in this configuration is denoted by $\overline{A}$ and the effective uniaxial stress is $\overline{\sigma}$. The bars in both the damaged configuration and the effective undamaged configuration are subjected to the same tensile force T. Therefore, considering the effective undamaged configuration, we have the formula $T = \overline{\sigma}\overline{A}$. Equating the two expressions of T obtained from both configurations, one obtains the following expression for the effective uniaxial stress $\overline{\sigma}$:

$$\overline{\sigma} = \frac{A}{\overline{A}}\sigma \tag{1}$$

Next, one uses the definition of the damage variable ϕ as originally proposed by Kachanov (1958):

$$\phi = \frac{A - \overline{A}}{A} \tag{2}$$

Thus, the damage variable is defined as the ratio of the total area of voids and cracks to the total area. Its value ranges from zero (for the case of an undamaged specimen) to one (for the case of complete rupture). Substituting for $A/\overline{A}$ from equation (2) into equation (1), one obtains the following expression for the effective uniaxial stress:

$$\overline{\sigma} = \frac{\sigma}{1-\phi} \tag{3}$$

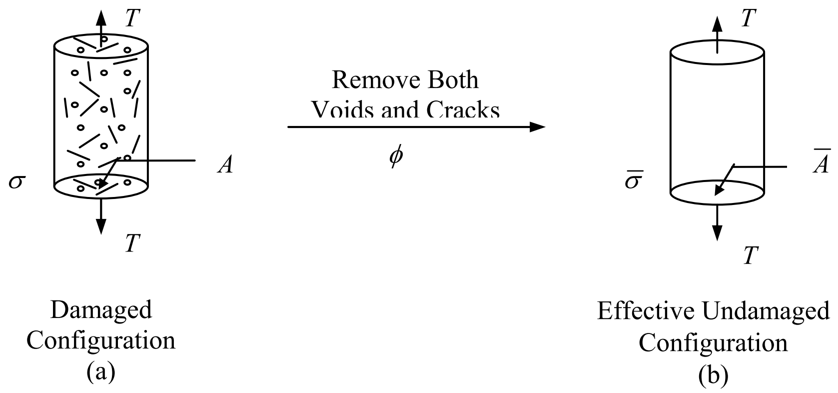

Fig. 1: A Cylindrical Bar Subjected to Uniaxial Tension.

Equation (3) above was originally derived by Kachanov in 1958. It is clear from equation (3) that the case of complete rupture ($\phi = 1$) is unattainable, because the damage variable ϕ is not allowed to take the value 1 in the denominator.

For the uniaxial tension case shown in Figure 1, the constitutive relation is the Hooke's law of linear elasticity given by:

$$\sigma = E\varepsilon \tag{4}$$

where ε is the strain and E is the modulus of elasticity (Young's modulus). The same form of the linear elastic constitutive relation applies to the effective (undamaged) state, i.e:

$$\overline{\sigma} = \overline{E}\overline{\varepsilon} \tag{5}$$

Where $\overline{\varepsilon}$ and $\overline{E}$ are the effective counterparts of ε and E, respectively. Next, we will derive the necessary transformation equations between the damaged and the hypothetical undamaged states of the material. In the derivation, the following assumptions are incorporated: (1) The elastic deformations are small (infinitesimal) compared with the plastic deformations (finite), and (2) there exists an elastic strain energy scalar function U. This function is assumed based on the linear relation between the Cauchy stress σ and the engineering strain ε given by equation (4). The elastic strain energy function U is defined by:

$$U = \frac{1}{2}\sigma\varepsilon \tag{6}$$

It is clear from equations (4) and (6) that $\sigma = dU/d\varepsilon$ and $\varepsilon = dU/d\sigma$. Sidoroff (1981) proposed the hypothesis of elastic energy equivalence. The latter hypothesis assumes that "the elastic energy for a damaged material is equivalent in form to that of the undamaged (effective) material except that the stress is replaced by the effective stress in the energy formulation". Thus, according to this hypothesis, the elastic strain energy $U = \frac{1}{2}\sigma\varepsilon$ is equated to the effective elastic strain energy $\overline{U} = \frac{1}{2}\overline{\sigma}\overline{\varepsilon}$ as follows:

$$\frac{1}{2}\sigma\varepsilon = \frac{1}{2}\overline{\sigma}\overline{\varepsilon} \tag{7}$$

Substituting equation (3) into equation (7) and simplifying, we obtain the following relation between the strain ε and the effective strain $\overline{\varepsilon}$:

$$\overline{\varepsilon} = (1-\phi)\varepsilon \tag{8}$$

Continuing further, we substitute equations (3) and (8) into equation (5), simplify the result and compare it with equation (4) to obtain:

$$E = \overline{E}(1-\phi)^2 \tag{9}$$

Equation (9) represents the transformation law for the modulus of elasticity. It is now clear that Young's modulus for the damaged material depends on the value of the damage variable ϕ. Solving equation (9) for ϕ, one obtains:

$$\phi = 1 - \sqrt{\frac{E}{\overline{E}}} \tag{10}$$

Once the values of E are experimentally measured for the damaged material, one can use equation (10) to obtain values of the damage variable ϕ. It should be noted that the value of $\overline{E}$ is constant for the effective (undamaged) material.

3. DAMAGE IN REINFORCED CONCRETE BEAMS

Consider a reinforced concrete beam with a rectangular cross-section. The beam is singly-reinforced with tension reinforcement only. The dimensions of the cross-section are the width *b* and depth *h* with *d* being the effective depth. The area of reinforcement is A_s The reinforcement ratio ρ is defined as:

$$\rho = \frac{A_s}{bd} \tag{11a}$$

We will also define the alternate reinforcement ratio ρ_h as:

$$\rho_h = \frac{A_s}{bh} \tag{11b}$$

In the following mathematical derivation we will directly use the ratio ρ_h and relate the result to the ratio ρ at the end of the derivation.

We introduce three damage variables; namely, the total damage variable ϕ_t the damage variable of concrete ϕ_c and the damage variable of steel ϕ_s.The three variables are defined in terms of the respective cross-sectional areas following equation (2):

$$\phi_t = \frac{A_t - \overline{A}_t}{A_t} \tag{12a}$$

$$\phi_c = \frac{A_c - \overline{A}_c}{A_c} \tag{12b}$$

$$\phi_s = \frac{A_s - \overline{A}_s}{A_s} \tag{12c}$$

where A_t is the total area of the cross-section $(A_t = bh)$, A_c is the area of concrete $(A_c = A_t - A_s)$ and A_s is the area of steel reinforcement. We also have $\overline{A}_t$, $\overline{A}_c$ and $\overline{A}_s$ as the effective areas corresponding to A_t, A_c and A_s, respectively.

Equations (12a), (12b) and (12c) are now rewritten in the following form, where the ratios of the two areas are emphasized:

$$\frac{\overline{A}_t}{A_t} = 1 - \phi_t \tag{13a}$$

$$\frac{\overline{A}_c}{A_c} = 1 - \phi_c \tag{13b}$$

$$\frac{\overline{A}_s}{A_s} = 1 - \phi_s \tag{13c}$$

We will now start the derivation with the following two obvious relations:

$$A_t = A_c + A_s \tag{14a}$$

$$\overline{A}_t = \overline{A}_c + \overline{A}_s \tag{14b}$$

Dividing equation (14b) by (14a), we obtain:

$$\frac{\overline{A}_t}{A_t} = \frac{\overline{A}_c}{A_c + A_s} + \frac{\overline{A}_s}{A_c + A_s} \tag{15}$$

We will now find appropriate expressions for the two terms on the right-hand-side of equation (15).

Using equations (13b) and (13c), one can easily derive the following relations:

$$\frac{A_c + A_s}{\overline{A}_c} = \frac{1}{1-\phi_c} + \frac{A_s}{\overline{A}_c} \tag{16a}$$

$$\frac{A_c + A_s}{\overline{A}_s} = \frac{A_c}{\overline{A}_s} + \frac{1}{1-\phi_s} \tag{16b}$$

Inverting both equations (16a) and (16b) substituting the result in equation (15) and comparing with equation (13a), we finally obtain:

$$1-\phi_t = \frac{1}{\dfrac{1}{1-\phi_c} + \dfrac{A_s}{\overline{A}_c}} + \frac{1}{\dfrac{A_c}{\overline{A}_s} + \dfrac{1}{1-\phi_s}} \tag{17}$$

It is clear from equation (17) that the total damage variable ϕ_t is a function of the damage variable in concrete ϕ_c , the damage variable in steel ϕ_s and the interactions between concrete and steel given by the area ratios $A_s/\overline{A}_c$ and $A_c/\overline{A}_s$. It is also clear that the above relation is complex and cannot be directly used in practical applications. Therefore, we will investigate equation (17) for a special but an important case.

We will consider the case when damage occurs in the concrete only, i.e. the steel reinforcing bars remain undamaged. In this case, it is obvious that $\phi_s = 0$ and $A_s = \overline{A}_s$. Therefore, the two area ratios $A_s/\overline{A}_c$ and

$A_c / \overline{A}_s$ appearing in equation (17) reduce to the following:

$$\frac{A_s}{A_c} = \frac{\overline{A}_s}{\overline{A}_c} = \frac{\overline{\rho}_h}{1-\overline{\rho}_h} \tag{18a}$$

$$\frac{A_c}{A_s} = \frac{A_c}{A_s} = \frac{1-\rho_h}{\rho_h} \tag{18b}$$

where we have used the relations $A_c = bh - A_s$ and $\overline{A}_c = \overline{b}\,\overline{h} - \overline{A}_s$. In equation (18a), $\overline{\rho}_h$ is the effective reinforcement ratio as defined in equation (11b). At this point, we will make the assumption that the reinforcement ratio remains constant during the process of damage, i.e. $\overline{\rho}_h = \rho_h$. Therefore, equation (18a) becomes:

$$\frac{A_s}{\overline{A}_c} = \frac{\rho_h}{1-\rho_h} \tag{19}$$

The final step in the derivation involves the substitution of equations (18b) and (19) along with $\phi_s = 0$ into equation (17) to obtain:

$$1-\phi_t = \frac{(1-\rho_h)(1-\phi_c)}{1-\rho_h-\rho_h(1-\phi_c)} + \rho_h \tag{20}$$

Simplifying equation (20) by performing some algebraic manipulations, we obtain the following explicit relation:

$$\phi_t = \frac{(1-\rho_h)^2\phi_c}{1-\rho_h\phi_c} \tag{21}$$

Equation (21) gives an explicit relation between the total damage variable ϕ_t, the concrete damage variable ϕ_c and the reinforcement ratio ρ_h. It is clear that for a homogeneous concrete beam with no steel reinforcement (i.e. $\rho_h = 0$), equation (21) reduces to the obvious relation $\phi_t = \phi_c$. However, in the presence of steel reinforcement we obtain the result of equation (21) which is not at all obvious. Equation (21) implies that $\phi_t \neq \phi_c$ when $\phi_s = 0$ for a reinforced concrete beam. The reinforcement ratio ρ_h is an important factor in the determination of the total damage in the concrete beam. The total damage in a reinforced concrete beam is not a function of the concrete damage only, but also a function of the reinforcement ratio ρ_h. Please note that when using valid numerical values in equation (21), we can safely substitute ρ for ρ_h in that equation to obtain:

$$\phi_t = \frac{(1-\rho)^2\phi_c}{1-\rho\phi_c} \tag{22}$$

Equation (22) is clearly a nonlinear relation between the two damage variables and the reinforcement ratio.

4. DISCUSSION OF RESULTS

Equation (22) is now investigated by performing some numerical computations. Using practical numerical values for ρ ranging from 0.005 to 0.035 and the value $\phi_c = 0.05$, we obtain the results shown in Figure 2. It is clear from the figure that in the range of valid numerical values, the nonlinear equation (22) behaves linearly. The total amount of damage decreases due to the reinforcement ratio. It is also clear that the effect of ρ at this low level of damage is very small.

However, Figure 3 shows a more pronounced effect when the value $\phi = 0.3$ is used. It is concluded that while the effect of reinforcement ratio is small, this effect increases at higher levels of damage. The decrease in the total damage is almost negligible at the low damage value of 0.05, but it becomes appreciable at higher levels of the damage variable especially when $\phi_c > 0.5$.

Figure 4 shows the effect of the reinforcement ratio ρ at different damage levels ranging from $\phi_c = 0.05$ to $\phi_c = 0.3$ in increments of 0.05. It is clearly seen that the slopes of the lines are higher at the high damage levels and become extremely small (approaching zero) at the lower damage levels. The slope of the straight line at $\phi_c = 0.3$ is -0.5040601573 compared to a slope of -0.09569196314 at $\phi_c = 0.05$. This is almost a five times increase in the slope of the line.

Finally, we discuss the relevance of this work to the famous Gergely-Lutz equation. Using the Gergely-Lutz equation, we can estimate the maximum width of cracks in the concrete in the tension faces of flexural members. The maximum width of cracks is given by (Gergely and Lutz, 1968; McCormac, 1998):

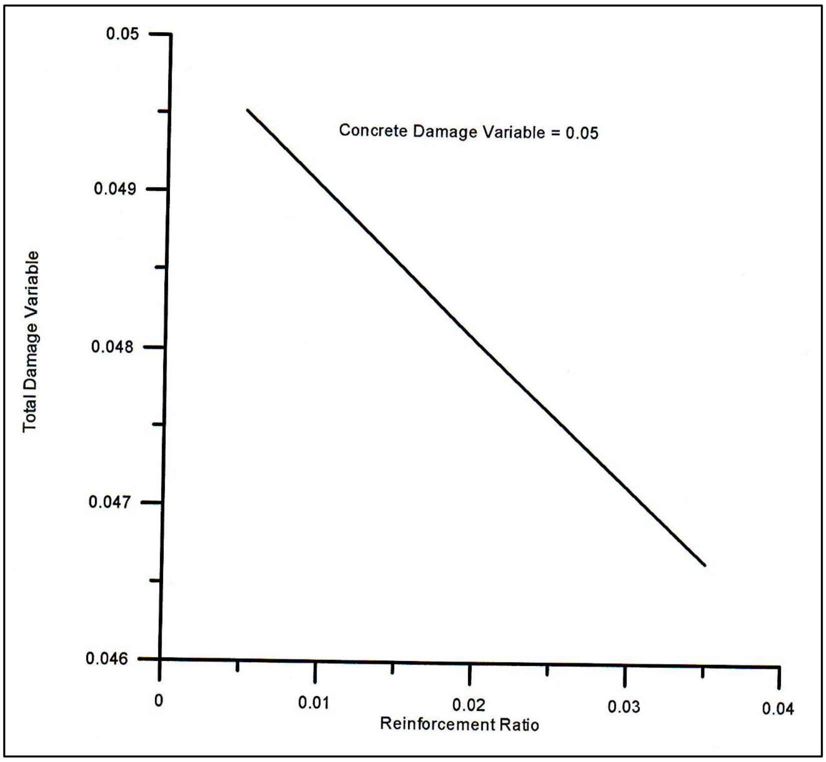

Fig. 2: Relation of Total Damage Variable and Reinforcement Ratio for ϕ_c = 0.05.

$$w = 0.076\beta_h f_s \sqrt[3]{d_c A} \tag{23}$$

where

w = the estimated cracking width in thousands of inches.

p_h = ratio of the distance to the neutral axis from the extreme tension concrete fiber to the distance from the neutral axis to the centroid of the tensile steel.

f_s = steel stress, in kips per square inch at service loads.

d_c = the cover of the outermost bar measured from the center of the bar.

A = the effective tension area of concrete around the main reinforcement (having the same centroid as the reinforcement) divided by the number of bars.

Several parameters are used in this equation including the effect of the number of reinforcing bars. As the number of reinforcing bars increases, the maximum crack width decreases thus leading to a smaller amount of concrete damage. This corroborates the result reached in this work. However, no direct relation can be made between equation (22) and the Gergely-Lutz equation, because the two equations measure different quantities. Equation (22) uses the reinforcement ratio and calculates the total damage ϕ_t in the beam. However, the Gergely-Lutz equation uses the number of reinforcing bars and calculates the damage in concrete ϕ_c. This is because this equation was established based on statistical considerations. Therefore, we need to use both equations consecutively.

First, we use the Gergely-Lutz equation to estimate the maximum width of cracks in the concrete and thus obtain an approximate value for ϕ_c. We then substitute this value into equation (22) to obtain an estimate of the total damage ϕ_t in the beam. Therefore, equation (22) is not a replacement for the Gergely-Lutz equation, but it represents an additional step to be taken in the calculation of total damage in reinforced concrete beams.

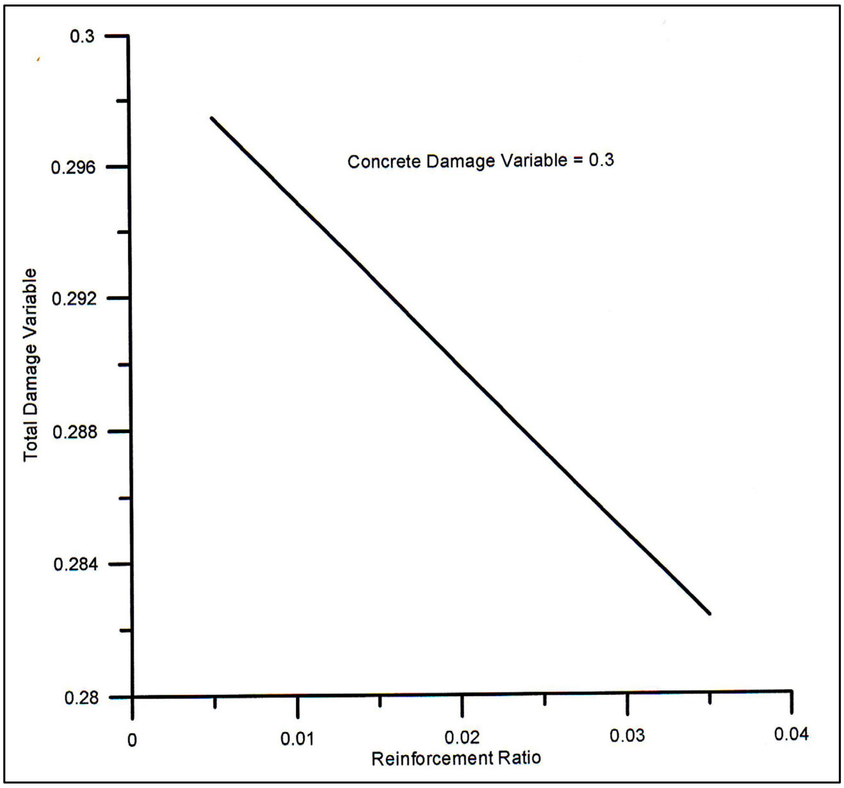

Fig. 3: Relation of Total Damage Variable and Reinforcement Ratio for ϕ_c = 0.3.

5. ESTIMATION OF CONCRETE AND STEEL DAMAGE

In this section, we present new equations for the estimation of the concrete damage variable ϕ_c and the steel damage variable ϕ_s based on the stiffness degradation of both materials. We start by applying equation (9) to both concrete and steel as follows:

$$E_c = \bar{E}_c(1-\phi_c)^2 \tag{24a}$$

$$E_s = \bar{E}_s(1-\phi_s)^2 \tag{24b}$$

Also, we will apply equation (3) to the compressive strength of concrete f_c' as follows:

$$\bar{f}_c' = \frac{f_c'}{1-\phi_c} \tag{25}$$

where $\bar{f}_c'$ is the effective compressive strength of concrete (for the undamaged material).

Now, we use the relation between the compressive strength of concrete f_c' and its modulus of elasticity E_c as follows expressed for the effective undamaged material:

$$\bar{E}_c = 4700\sqrt{\bar{f}_c'} \tag{26}$$

where both $\bar{f}_{c'}$ and $\bar{E}_c$ are in MPa. Substituting for $\bar{E}_c$ from equation (24a) and for $\bar{f}_c'$ from equation (25), and simplifying, we obtain:

$$E_c = 4700\sqrt{f_c'(1-\phi_c)^3} \tag{27}$$

The above equation gives the modulus of elasticity for the damaged concrete in terms of the damage variable ϕ_c

and the concrete compressive strength f_c'. Solving equation (27) for ϕ_c , we obtain:

$$\phi_c = 1 - \sqrt[3]{\frac{E_c^2}{2.209x10^7 f_c'}} \tag{28}$$

Equation (28) can be used to estimate the concrete damage variable ϕ_c once the modulus of elasticity E_c and the compressive strength f_c' are experimentally measured.

Finally, solving equation (24b) for ϕ_s , we obtain:

$$\phi_s = 1 - \sqrt{\frac{E_s}{\overline{E}_s}} \tag{29}$$

Equation (29) can be used to estimate the steel damage variable ϕ_s once the modulus of elasticity E_s is experimentally measured. It is noted that the effective modulus $\overline{E}_s$ is constant for undamaged steel.

6. CONCLUSION

The effect of the reinforcement ratio on the total damage of reinforced concrete beams is investigated using a damage mechanics approach. A new formula is derived showing the exact relation between the total damage variable and the concrete damage variable. It is concluded that the total amount of damage is reduced with the increase in reinforcement ratio. This reduction is very small at lower levels of damage. However, the effect becomes more pronounced at higher levels of damage. Finally, the new formula can be used to support the famous Gergely-Lutz equation in order to slightly reduce the amount of damage predicted by that equation.

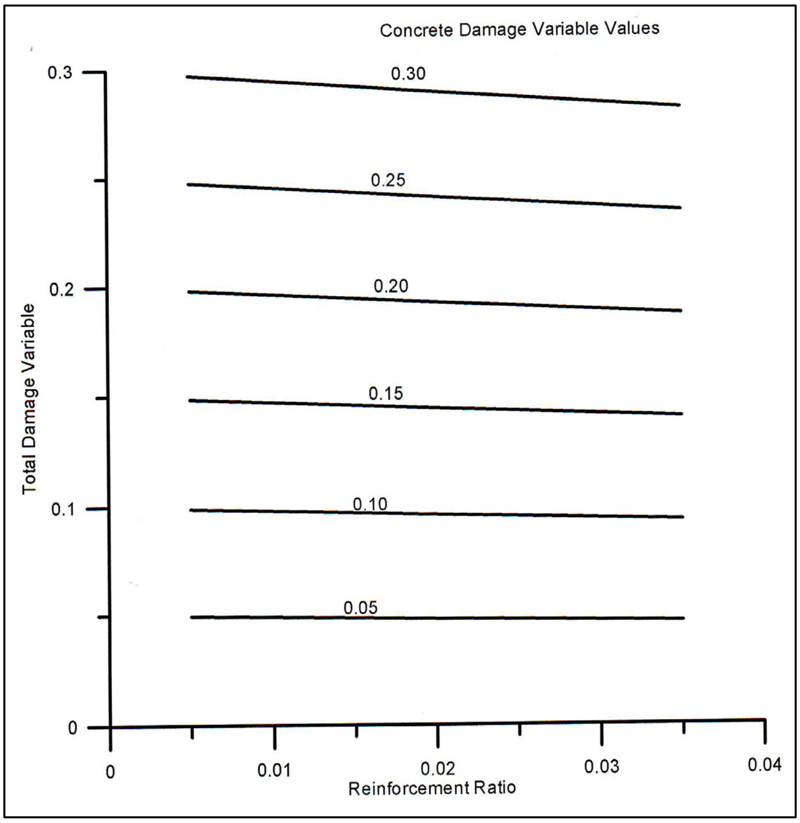

Fig. 4: Relation of Total Damage Variable and Reinforcement Ratio for Different Values of the Concrete Damage Variable.

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