Influence of Soil Plasticity on the Seismic Performance of Pile Foundations – a 3D Numerical Analysis

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ABSTRACT

This paper presents a detailed analysis of the influence of plasticity on the seismic response of pile foundations. Since soils exhibit nonlinear and irreversible behaviour, it is of major interest to consider plasticity in the design of pile foundations for structures which may be subjected to severe earthquake loading. The study is carried out using a full three-dimensional finite difference modeling using a real earthquake record. The influence of soil plasticity is investigated on the seismic response of soil-pile-structure system for two idealized soil deposits: cohesive and frictional soils. Analyses provide valuable information on the influence of plasticity on the seismic response of soil-pile-structure systems. They show that the soil state in the vicinity of piles head dominates the piles response and may lead to higher deflection and bending stresses.

KEYWORDS: Foundations, Piles, Superstructure, Seismic, Plasticity, Three-dimensional, Modelling.

INTRODUCTION

Analysis of seismic response of soil-pile-structure systems constitutes a complex problem in earthquake engineering. In addition to post-earthquake investigations, analytical and numerical analyses show that the damage of piles in seismic areas is mainly attributed to the kinematic interaction between piles and soils and/or to the inertial interaction between the superstructure and the pile foundation which may cause foundation damages, in particular at the pile-cap connection (Gazetas and Mylonakis, 1998; Nikolaou and Mylonakis, 2001; Sadek and Shahrour, 2006). Seismic damage also depends on the governing frequencies such as natural frequency of the soil and dominant frequency of loading (Shahrour et al., 2001; Alsaleh and Shahrour, 2009).

Methods based on the Winkler model are widely used

for the analysis of seismic response of pile foundations (Gazetas, 1991; Makris and Gazetas, 1992). These methods are based on simplified hypotheses of the soil media, which permit to conduct analyses with little computation cost. More rigorous methods were also used to analyze the seismic response of pile foundations, mainly the finite element method (Sen et al., 1985; Fan et al., 1991; Ousta and Shahrour, 2001).

Nonlinear full 3D analyses considering the soil, piles and the superstructure are still limited. Such studies were conducted in the linear domain by Sadek and Shahrour (2004, 2006) to analyze the influence of micropiles inclination and boundary conditions on the seismic behaviour of the soil-micropile structure system. Gerolymos et al. (2009) used a full 3D finite element analysis to study the seismic performance of inclined piles assuming a linear behaviour of the soil and the structure. On the other hand, it is well known

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b) 3D numerical mesh with adsorbing boundaries (138 beam elements and 6978 nodes) Figure 1: Problem under consideration



Figure 2: Kocaeli earthquake record (1999) a) Displacement, b) Velocity, c) Acceleration, d) Fourier spectra of velocity component

Material	Diameter (m)	Mass Density ρ (kg/m ³)	Young Modulus E (Mpa)	Poisson ratio v	Damping ratio ξ (%)	Height (m)
Pile	0.80	2500	20000	0.3	2	10
Soil		1700	8	0.45	5	15

Table 1. Elastic property of the soil and piles materials

that the soil material exhibits a non-linear and irreversible behaviour, even at low levels of deformation. Observations on recent devastating earthquakes show that soil nonlinearity should be taken into consideration in the design of pile foundations. Analyses of record events (Loma Prieta, 1989; Northridge, 1994; Hyogoken-Nambu, 1995) revealed significant non-linear soil response in both deamplification and degradation of wave velocities in the ground motion with peak ground acceleration above 0.1–0.3g (Chin and Aki, 1991; Satoh et al., 1995; Trifunac and Todorovska, 1996; Field et al., 1997). Gerolymos et al. (2008) showed that the response of piles subjected to cyclic lateral loading is governed strongly by the nonlinear stress–strain soil behaviour that occurs even at low levels of loading.

According to Finn (2005), the influence of the soil non-linearity has been firstly introduced in the

simplified pseudo-static approaches via non-linear springs (p-y curves). Results of full-scale tests on piles embedded in cohesionless and clayey soils (Gazioglu and O'Neill, 1984; Murchison and O'Neill, 1984) showed that the use of p-y curves gave poor predictions with large errors. Centrifuge test data, obtained by Wilson (1998), illustrated the uncertainty associated with such pseudo-static analysis.



c) C=100 KPa Figure 3: Distribution of plasticity for different cohesive soils

A nonlinear simplified 3D continuum method using strain-dependent moduli and damping was presented for the dynamic nonlinear effective stress analysis of pile foundations under earthquake excitation by Wu and Finn (1997a, b). It emphasized the importance of inertial interaction between the pile and the superstructure. Maheshwari (2004) conducted a 3D nonlinear analysis of the seismic soil-pile-structure interaction using a subsystem model. Material nonlinearity was considered using an advanced plasticity-based soil model (HiSS). Analyses showed that the soil nonlinearity increased the pile head and structural responses at low frequencies.

This paper includes a full 3D coupled modelling of

the soil-pile-superstructure interaction under seismic loading considering the elastoplastic behaviour of the soil material. The study provides valuable information about the domain of validity of the linear theory. The influence of plasticity is investigated for two idealized soil deposits: cohesive and frictional soils. The soil behaviour is described using the non-associated Mohr– Coulomb criterion.

SOIL-PILE STRUCTURE SYSTEM AND NUMERICAL MODEL

The problem under consideration consists of a bridge structure supported by a group of 6 vertical piles

embedded in a homogeneous soil layer underlined by rigid bedrock (Figure 1a, b). The thickness of the soil layer is $H_s=15$ m; its natural frequency (f₁) is equal to 0.67 Hz (f₁= $V_s/4H_s$, where V_s is the shear wave velocity in the soil layer). An elastoplastic constitutive

relation based on the non-associated Mohr-Coulomb criterion is used for the soil material. Table 1 summarizes the mechanical properties of the soil and structure materials.



Figure 4: Influence of the plasticity of cohesive soil on the lateral displacement of piles and free field (z denotes for the depth from the pile head as shown in Figure 1.a)



Figure 5: Influence of the plasticity of cohesive soil on the amplification of lateral movement at the superstructure head for different cohesions



Figure 6: Influence of the plasticity of cohesive soil on seismic induced internal forces in the corner piles



Figure 7: Distribution of plasticity in frictional soil (ψ =0° and 7°)

The behaviour of the piles and superstructure components is assumed to be elastic with Rayleigh material damping. The reinforced concrete piles length is equal to L_p = 10 m, its section is assumed to be circular with a diameter D_p =80 cm. The piles spacing ratio is taken to be S/D_p= 3.75 (S: center-to-center piles

spacing). They are rigidly connected to a massive cap, 1 m thick, which is supposed free of contact with the soil. The superstructure consists of a massive rectangular bridge pier of 4 m height supporting a bridge deck of 350 tons. It is modeled as a singledegree-of-freedom system composed of a column and a concentrated mass. Pier inertia has been fixed in order to obtain a flexible fundamental frequency of the superstructure $f_{st, flex}$ = 1.1 Hz (including SSI). The static load supported by each pile is equal to 80 tons.



Figure 8: Influence of the plasticity of frictional soil on the seismic induced internal forces in the corner piles

Numerical analyses are conducted using the finite difference FLAC3D program, which is based on a continuum finite difference discretization using the Lagrangian approach (Flac3D, 2005). For elastic materials, a Rayleigh damping of 5% is used in the analyses to compensate the energy dissipation through the medium (Lokmer et al., 2002; Paolucci, 2002). The finite difference mesh used in the numerical simulation is illustrated in Figure 1b. It includes 6978 8-node elements for the soil and the superstructure and 138 beam elements for the pile foundation. The mesh has been refined at the vicinity of the piles and the superstructure where inertial forces induce significant stress concentration. Calculations are performed with the following boundary conditions:

The base of the soil mass is assumed to be rigid. The seismic loading is applied at the base of the soil mass as velocity excitation.

Free-field boundaries are used with the aim to absorb outward waves originating from the structure. The procedure involves the execution of free-field calculations in parallel with the main-grid analysis. The lateral boundaries of the main grid are coupled to the free-field grid by viscous dashpots to simulate a quiet boundary.

Numerical simulations are conducted for two idealized soils: frictional and purely cohesive soils. The system is subjected to an earthquake loading representative of the 1999 Kocaeli earthquake in Turkey (Mw=7.4, Chen and Scawthorm, 2003; Parish

et al., 2009). The estimated peak velocity is equal to 40 cm/sec (peak acceleration 0.247g); the loading duration is equal to 30 sec. However, the analysis will be focused on the first 10 sec where the input loading is more significant.

The record for the base acceleration, velocity and displacement waves is shown in Figure 2a-b-c. Fourier

analysis of the record of the earthquake's velocity results in a power spectrum as depicted in Figure 2d. The velocity spectrum reveals a dominant frequency at f = 0.9 Hz (lower peaks are observed at 0.6 and 1.3 Hz) to be compared with the natural frequencies of the soil (0.67 Hz) and the superstructure (1.1 Hz).



a) $S = 3 D_{p}$

b) $S=3.75 D_p$



Figure 9: Influence of piles spacing on the plasticity distribution of cohesive soil (C=50 KPa, ϕ =0°)

INFLUENCE OF PLASTICITY ON THE SEISMIC REPONSE OF THE SOIL-PILES-STRUCTURE SYSTEM

Cohesive Soil

Numerical simulations were conducted for three values of soil cohesion: C=20, 50 and 100 kPa. Figure

3 shows the distribution of plasticity in the soil at the peak of the seismic excitation. It can be observed that for the high cohesion C=100 kPa, the soil mass seems to remain in elastic domain. For lower cohesion (C = 20 kPa), plasticity is induced at the base and spreads to the surface. For a purely cohesive soil, the plasticity criterion is firstly reached at the base. The extension of

plasticity from the base induces a dissipation of the energy and reduces the wave transmission to the surface. This result is confirmed in Figure 4, which compares the amplification of the piles lateral motion to the free field. Results show a significant decrease in the lateral amplification with the decrease in the cohesion. The amplification at the pile head for C=100 kPa (U/U_g = 4.85) is 85% higher than that obtained with C=20 kPa. On the other hand, the comparison of elastic and elastoplastic response (Figure 5) reveals a comparable trend at the beginning of shaking followed by a discrepancy for higher load level sufficient to produce the soil plasticity and consequently an additional hysteretic damping in the soil with low cohesion. The Fourier spectrum illustrates large components of lateral amplification at natural frequency of the soil and dominant frequency of the loading record. However, observed peak values are significantly reduced for low cohesion C=20 kPa where plasticity is extended to the whole soil mass.

The influence of plasticity on the seismic induced internal forces in the piles is summarized in Figure 6 and Table 2. The profile of the shearing force shows a regular decrease in the maximum force with the decrease in the cohesion. The shearing force at the piles head is related to the inertial force at the superstructure that decreases with the increase in the plasticity of the soil mass.

The profile of the bending moment does not show a regular trend. Indeed, the bending moment is not governed only by the superstructure acceleration but also by the soil state around the pile and especially at the vicinity of the pile head. For example, when the cohesion decreases from 100 kPa to 20 kPa, a significant decrease in the acceleration of the superstructure head (about 60%) is observed, while the decrease in the maximum bending moment is negligible (less than 10%). The normalized bending moment confirms this trend (Table 2). The soil weakening at the vicinity of the piles head leads to higher flexural strain in this zone. In some cases, seismic observations of damaged piles show a gap formation in the soil around the pile head. Cyclic experiment tests conducted by Rabin et al. (2008) showed a gap formation for laterally loaded pile in cohesive soils.

Table 2. Influence of the plasticity of cohesive soil on the seismic induced response of soil- pile-superstructure system $(T^* = \frac{T}{T_{cap}}; M^* = \frac{M}{m_{st}a_{st}H_{st}}; T_{cap}$ and a_{st} denote the inertial force induced at the cap and the acceleration

	2	a _{Cap} (m/s²)		Interna	Normalized forces			
C (kPa)	a_{st} (m/s ²)		Central piles		Corner piles		Corner piles	
			T _{max} (kN)	M _{max} (kN.m)	T _{max} (kN)	M _{max} (kN.m)	T* _{max}	M* _{max}
elastic	11.28	8.385	675.8	954.4	1016.1	1099	0.196	0.05
20	4.694	3.422	259.2	632	342.6	986.4	0.159	0.109
50	8.793	6.367	502.8	793.7	737	898	0.183	0.053
100	11.06	7.902	642.1	949.2	984.7	1050	0.195	0.049

of the superstructure mass.	H _{st} :	Superstructure	height)
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Ψ (°)	a _{st} (m/s²)	a _{Cap} (m/s ²)		Interna	Normalized forces			
			Central piles		Corner piles		Corner piles	
			T _{max} (kN)	M _{max} (kN.m)	T _{max} (kN)	M _{max} (kN.m)	T* _{max}	M* _{max}
elastic	11.28	8.385	675.8	954.4	1016.1	1099	0.196	0.05
0	9.669	6.321	531.8	999.2	931.6	1118	0.2149	0.06
7	9.567	6.592	511.7	897.9	917.3	1140	0.211	0.062

Table 3. Influence of the plasticity of frictional soil on the seismic induced response of soil-pile-superstructure system

Table 4. Influence of piles spacing on the seismic induced response of soil-pile-superstructure system – cohesive soil (C=50 kPa, φ=0°)

S		a _{Cap} (m/s²)	Internal forces					
	a_{st}		Centra	al piles	Corner piles			
	(11/8)		T _{max} (kN)	M _{max} (kN.m)	T _{max} (kN)	M _{max} (kN.m)		
3D _p	9.002	6.12	483.1	1309	651.6	1363		
3.75D _p	8.793	6.367	502.8	793.7	737	898		
4D _p	8.312	6.272	482.8	888.4	693.7	998.2		
5D _p	6.582	5.493	406.4	1211	529.1	1310		

Frictional Soil

Analyses are conducted with a friction angle $\varphi=30^{\circ}$ and a soil cohesion C=2 kPa with two values of the dilatancy angle $\psi=0^{\circ}$ and 8°. Elastic properties of the soil are unchanged (see Table 1). Figure 7 depicts the plasticity distribution in the frictional soil for two values of the dilatancy angle. Oppositely to cohesive soil, it can be observed that plasticity is generated at the top of the soil due to the low soil confinement in this zone. The decrease in the lateral acceleration at the superstructure head (15 %) is not significant when compared to the elastic solution (Table 3). When the soil dilatancy is taken into account, the plasticity extension is reduced. The seismic induced internal forces in the piles are summarized in Figure 8 and Table 3. The variation in the maximum shearing force at the pile's head is not important since it is directly related to the superstructure acceleration. However, it can be noticed that plasticity induces an increase in the ratio of maximum shearing force between the corner pile and center pile (about 1.75 frictional soil compared to 1.5 for elastic soil). The profile of bending moment shows that maximum values obtained at the head are not affected while a discrepancy is observed for lower depth, in particular in the case of zero dilatancy: at the vicinity of the pile center (z = 6 m), the bending moment for frictional soil is 200% greater than that obtained for elastic soil. The soil state around the pile is of major importance.

GROUP EFFECT

Analysis shows that the seismic induced bending stress in the piles is directly related to the soil state around the pile's head. For this reason, it is of main interest to investigate the group effect in the case of closely spaced piles where kinematic interaction may play a key role in the overall system behaviour.



Figure 10: Influence of piles spacing on the lateral displacement of the piles – cohesive soil (C=50 kPa, φ=0°)



a) Maximum bending moment Figure 11: Influence of piles spacing on the on the seismic induced internal forces in the corner piles - cohesive soil (C=50 kPa, φ=0°)

s	a _{st} (m/s ²)	a _{Cap} (m/s ²)	Internal forces					
			Centra	al piles	Corner piles			
			T _{max} (kN)	M _{max} (kN.m)	T _{max} (kN)	M _{max} (kN.m)		
3D	10.05	6.54	540.5	1383	869.2	1595		
3.75D	9.567	6.592	511.7	897.9	917.3	1140		
4D	9.042	6.844	502.5	1002	879.9	1285		
5D	7.787	6.204	462.1	1425	696	1530		

Table 5. Influence of piles spacing on the seismic induced response of soil-pile-superstructure system – frictional soil (C=2 kPa, φ=30°, ψ=7°)



c) $S = 4 D_{p}$



Figure 12: Influence of piles spacing on the plasticity distribution of frictional soil (C=2 kPa, φ=30°, ψ=7°)

Cohesive Soil

Numerical simulations are carried out for the piles spacing (S= $3D_p$, $3.75D_p$, $4D_p$ and $5D_p$) with a soil C=50 kPa.

The seismic response of the system is summarized in Table 4 and Figures 9 to 11 which show a significant influence of the piles spacing on the overall response of the system. The amplification in the lateral displacement at the pile's head decreases with the increase in piles spacing, due to an increase in the lateral rigidity of the system.

The maximum shear force and bending moment don't show a regular trend with the piles spacing. The decrease of S from $5D_p$ to $3.75D_p$ leads to a reduction of about 35% in the bending moment. This result agrees with those obtained in elastic medium in earlier research works. Conversely, a lower spacing S=3D_p induces a drastic change in the profile of the bending moment that approaches that of isolated piles. This result disagrees with those obtained for elastic soils (see for example Gazetas et al., 1991; Shahrour et al., 2001). Figure 8 illustrates the plasticity extension in the soil for different piles spacing. Note that for $S=3D_p$,

plasticity is extended to the zone located at the vicinity of the pile's head. The piles less protected by the surrounding soil are subjected to higher deflection and bending stresses.



Figure 13: Influence of piles spacing on the lateral displacement of the piles - frictional soil $(C=2 \text{ kPa}, \phi=30^{\circ}, \psi=7^{\circ})$

Frictional Soil

The influence of piles spacing on the group effect is analyzed for the frictional soil (C=2 kPa; φ =30°; ψ =7°). Results are presented in Figures 12-14 and Table 5. They confirm the findings obtained for cohesive soil. For low spacing, the positive group effect is not maintained. For S=3D_p, plasticity is extended to a larger zone around the piles due to the high kinematic interaction. This effect may also result in a gap development behind the closely spaced piles which induces an increase in the group deflection as shown in Figure 9 for S=3D_p; the bending moment profile is close to that of isolated piles.

CONCLUSIONS

This paper has presented a full three- dimensional modeling on the influence of plasticity on the seismic response of soil-pile-superstructure system. Analysis was conducted for two idealized soil deposits: cohesive soil and frictional soil. Results show that viscoelastic analysis is not always satisfactory for a seismic input with frequency content close to resonant frequency of the soil. In this case, the plasticity leads to an attenuation of the superstructure lateral movement due to higher soil damping and consequently to a reduction in the shear force at the piles head. However, the profile of bending moment does not show constant trends since it mainly depends on soil state around the pile especially at the top of the pile. The nonlinear phenomena dominate the piles response and the soil weakening around the piles leads to higher deflection and to higher bending stresses. This conclusion is confirmed for closely spaced piles where kinematic interaction may result in a soil plasticity between the piles resulting in a higher bending moment for low spacing (S=3Dp). These findings disagree with the results of earlier elastic analysis.



Figure 14: Influence of piles spacing on the seismic induced internal forces in the corner piles-frictional soil (C=2 kPa, φ=30°, ψ=7°)

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