Dynamics of Urban Growth: Modeling the Fractal Dimension of the City of Irbid, Jordan

***Imad Al-Hashimi* [1], *Abdul-Sahib El-Azzawi* [2] *and Hassan El-Kasbi* [3]**

[1] Jordan University of Science and Technology, College of Architecture and Design, Dept. of Urban Planning, P.O. Box 3030, Irbid, Jordan. E-Mail: imadhashimi@hotmail.com
[2] Assistant Professor, Jordan University of Science and Technology, Irbid, Jordan
[3] University of Babil, Al-Hilla, Iraq

ABSTRACT

The emergence of fractal geometry engendered a shift from the common view that sees cities as simple, ordered and structured, expressed by smooth lines and shapes, towards a view that cities are complex organisms evolving according to local rules and conditions. The main objectives of this study are first to prove the fractality of the geometry of Irbid, Jordan as a case study and second to provide mathematical procedure and tool (fractal geometry) for urban analysis. The research simulates the growth of Irbid across many scales and times. The main hypothesis asserted here is that Irbid is growing systematically and factually, even though its growth demographically and geometrically hasn't been strictly regular .This supports the argument that cities globally or locally would produce fractal growth at every level of their hierarchy.

KEYWORDS: Geometry, Fractal growth, Self-similar, regression.

INTRODUCTION

Our understanding of cities is still dominated by the search for visual order. This means that our immediate knowledge of the city is visual. Cities display a mixture of organic and pure geometry. Their form is seen as being more organic than purely geometric. To Batty and Longley (1994), form represents the spatial pattern of elements composing the city in terms of its networks, buildings and spaces, defined mainly, but not exclusively, through its geometry, in two rather than in three dimensions. Yet form can never merely be conceived in terms of these local properties, but has a wider significance in the way cities grow and change. Visual form of any system implies and reflects its internal structure (function or behavior) and *vice versa.*

Accepted for Publication on 11/10/2012.

Many people believe that 20th century science will be remembered for three main theories: quantum mechanics, relativity and chaos (Donahue, 1999). The chaos theory is a developing scientific discipline which is focused on the study of nonlinear system (complex and unpredictable systems). It implies order beneath the mess (disorder). Chaos has a sense of order and pattern necessary to explain some common terms. While linearity includes simplicity, nonlinearity refers to complexity which is about the study of nonlinear dynamic systems and deals with irregular and unpredictable behavior rather than trying to reduce complex systems to linear cause and effect relationships (Grace, 1991).

The traditional geometry (smooth geometry) of Euclid which depends on points, lines, planes, volumes with integer dimensions has been shifted and changed towards a new geometry which is fractal geometry.

Fractal geometry enables us to understand order and regularity in what, at first sight, appears irregular and disordered (chaos) (Wahl, 1998). It is an interesting approach to the urban form developed over the past twenty years, which builds on more systematic, mathematical ideas, linking surface and underlying structure and process. Fractal geometry has been utilized in many fields and subjects including, but not restricted to, biology, architecture, construction in addition to urban analysis, and is still a focus of interest at present. Recent work in urban analysis during the past decade focused on the analysis of city form, its sprawl, structure and urban morphology which could be found, for example, in the works of: (Shen, 2002; Benguigui, 2004; Thomas et al., 2007; McAdams, 2009; Terzi and Kaya, 2008; Thomas et al., 2012).

The physical form of the city has come to be regarded as a specialists' concern. It reveals the internal structure and behavior (function) of any city system. The correlative interrelationship between form and function is occurring continuously in time-space context in any urban growth, and the traditional controversial jury about them is still out. Therefore, the physical form as temporal visible product (picture) at a practical point of time reflects a continuous hierarchical and conceivable or detectable micro-level of processes and functions (meanings). Thus, one cannot deal with any of them without dealing with understanding the others at the same time. Fractal dimension D (broken not integral magnitude and orientation within time-space framework) is considered to be a sensitive indicator and parameter to these variables at each level of the hierarchy. The problem is about urban growth; how do cities grow? Can we describe, interpret, simulate or even make some predictions about this growth? How can we calculate the fractal dimension D? Does it truly represent the fingerprint of the city varying according to many scales and sizes without losing its self-similarity? And finally, could a city in a developing world of different socio-economic stands and less strict planning controls also be fractal in its growth?

The main objectives of this research are: first, to prove that the dependent variables (size, spacing and density of cities) are the same (similar) at every level of their hierarchy; therefore the city systems are fractal, and second, to provide mathematical procedure and tool for urban planners and decision makers to deal with in describing, simulating and even forecasting the city systems and urban growth.

The case study for this research is the Jordanian city of Irbid; the main city of the northern part of Jordan which goes back deep in history since the ancient bronze ages. It expands over an area of 40 km² and has a population of about 300000 (2005). The city was established on an artificial hill (which still represents the oldest core of the city). This research will depend on archival records (maps, statistics and historical photographs) and relevant formal and informal institutions to get the necessary needed data and information.

METHODOLOGY

Irregular Urban Boundary as an Index of Urban Form

What seems to be irregular on a large scale (level) might be seen regular on a smaller one (another level) and *vice versa* for all directions of magnification. Cities, like all organisms of natural growth, evolve through commutative addition and deletion of basic units, cells or parcels. Such units may be individuals, households, firms, transportation links… and so on, represented in terms of the immediate space they occupy. Cities thus grow through successive accumulation of these basic scales.

The physical form of cities is the ultimate result of a multitude of social and economic processes, constrained and shaped by the geometry of the natural and man-made world. One can begin to think of cities as systems of organized complexity whose geometry betrays a complexity of form (scale and shape) and function (size and processes with time). The morphology of cities should be understood in terms of their forms and processes, scales and shapes; their statics and dynamics, and this will enable us to map out our approach, which builds our understanding of urban form about the new geometry of the irregular-fractal geometry.

Most towns and cities grew organically as the product of many individual decisions made according to local rules and circumstances. Yet, changes in our conception of space and time in which we see irregularity and discontinuity as reflecting a new underlying order and system, perhaps, provide a fresh perspective as to the impact of physical determinants on social and economic processes. So, the geometry of irregularity is still ordered but only where the order repeats itself across many scales and through time and where such irregularity is clearly constant with observations and measurements for our most interesting systems.

The physical form of the city can be recognized only by conceiving its edge or boundary, which is not just linear but implies area. Thus, it is clearly something more than a one-dimensional line. Urban boundary is the most obvious visual delimiter of the city's size and shape. While urban land-uses describe the explicit zones in a city, their boundaries express the physical form with implicit indices of irregularity, self-similarity and hierarchy, that would be shown by estimating the fractal dimension. The focus here is on the urban boundaries of Irbid because of the lack and scarcity of detailed information, data, maps and archival records about urban land use zones.

Hypotheses

The main hypothesis is that cities' growth implies the same systematic order at each level of their hierarchy according to multi-fractal dimensions. In other words, the hypotheses of this study can be elaborated as:

- The main hypothesis: Cities seem to grow factually (according to multi-fractal-not integer-dimensions).
- The sub-hypothesis: Cities themselves are fractal at every level of their hierarchy (D value is between 1.26 and 1.7). This factuality is the same for each simultaneous process of urban growth.

The estimate of the fractal dimensions of Irbid according to multi-scales will be attempted first, and then follows the estimate of multi-period time depending on the urban boundary of the city.

MODELS

The boundary of the city is a closed line, and without any knowledge of fractal geometry, it seems intuitively obvious that such a line implies an object with a dimension somewhat greater then one. Any layman would probably associate the closed line with an area and argue that the purpose of the line is simply to mark out the area. The hypothesis postulates that there is a fractal, not-integer, indivisible part (motif, seed or archetype). This part (parcel or segment) might be invisible (too tiny to be noticed) or too big, and out of our vision, to be conceived with our own limited sense of vision. That means that there is a sensible range of human's sensory system and there are multi-steps (processes or levels) within this range. Any change, growth or deletion is about adding or subtracting a seed or certain number of these seeds in systematic steps (processes). There are orders of these processes, order within order within order till infinity. The difficulty in solving this conundrum is how to find a complete order with all serial steps and details. The fractal theory argues that there is a repetition of every single order somewhere and somehow within the whole system's processes. In order to complete the missing (in-between) steps or processes of the anti-patterns and unknown order, the theory supposes that each step, process or level (of one or more seeds) is similar to any other step (process or level) after some modification in scale (distance/power of magnification) or size (growth or decline with time). Sometimes, we need to go back deeply in the past, looking for missing data to complete the whole picture, and sometimes we need to use the super-power of magnification to (zoom-into/out of) any system to get this self-similarity. These ideas involve the notion of transient self-similarity and transfer the analysis to models of varying self-similarity with respect to morphology and scale.

The major attribute of the theory is trying to find linear relations (after applying regression operations to exponential equations as will be shown numerically in the next paragraphs) between variables seeming to be related together reasonably in non-linear equations for the first time. The fractal dimension represents the key (the exponent) parameter and index in every relation. It is possible to derive the fractal dimension of a single object by measuring the same object with different scales or by varying the extent or size of the object over which the dimension might be computed.

The research will follow the scientific empirical approach (Koch Curve Model, DLA Model and Richardson Method) (Batty and Longley, 1994) to test and measure the fractal dimension. While the fractal dimension of the Koch Curve Model is calculated theoretically and represents a static model of the way scale is varied, the DLA Model calculates the fractal dimension empirically as a dynamic model where the object is grown by varying its size, and the Richardson Method represents a way (method) of estimating the fractal dimension. The research depends mainly on the proposition of Batty and Longley that "a growth model of a city based on Diffusion Limited Aggregation DLA with a dimension (D = 1.71), represents an idealized model of the way which urban space is filled (approaching the 2-dim.); while a simplified form of the boundary of the space filled was based on the Koch Curve with a dimension (approaching the 1-dim.) of (D= 1.26).

Koch Curve Application

The generation of this fractal is simple. One begins with a straight line of unit length and divides it into three equal-sized parts. The middle section is replaced by an equilateral triangle and its base is removed. After one iteration, the length is increased by four-thirds. Having this process repeated, the length of the figure tends to infinity as the length of the side of each new triangle goes to zero. To calculate the dimension of the Koch curve, we look at the image of the fractal and realize that it has a magnification factor of three and with each iteration, it is divided into four smaller pieces:

$$D = \frac{\ln(4)}{\ln(3)} = \frac{1.3863}{1.0986} = 1.2619 \text{ (Koch curve dimension)}$$

Diffusion Limited Aggregation (DLA)

It is a technique used to link growth to specific geometrical forms. DLA model can be represented in a very tiny lattice, where each occupied cell unit within the lattice grid represents the value (1); while each unoccupied one represents the value (0).

Like all organisms of natural growth, cities evolve through cumulative addition and deletion of basic units, cells or parcels. Such units may be individuals, households, firms, transportation links… and so on, represented in terms of the immediate space they occupy, and cities, thus, grow through successive accumulation of these basic scales. The application of the DLA technique on multi-cities gives a fractal dimension of 1.7 which represents an interesting matter that needs more research and application.

The DLA model generates self-similar form which provides a baseline for comparison with real growth. The DLA technique is an empirical dynamic model, where the object is grown by varying size, with a dimension (D=1.7), which approaches two-dimensional objects.

The research hypothesis proposed that the estimation of dynamic systems' dimensions (cities' growth) would not be more than (1.7). Thus, it could be argued that an estimation of fractal dimensions in any urban growth system is between 1.26 and 1.7.

Richardson Method of Varied Measured Lengths

This method shows that the length of coastline depends upon the scale with which its length was measured. The method is a typical approximation for the length of boundary. It is about walking on the edge line (boundary) with some divisions (r0) and calculating the number of divisions (segments/N0), then repeating this step by taking another proportional division to the first one (r1) and calculating the number of divisions (N1). By continuing these processes, a visually appropriate matching between the segments and the edge line could be seen. Thus one can approximately calculate the length by multiplying the number of segments by the length of each one at that level, see Figure 1. It is possible to calculate the fractal dimension as:

$$D = \frac{\log \frac{N1}{N0}}{\log \frac{r0}{r1}} \quad \text{.......................... (1)}$$

where: D/ fractal dimension;

N1/ accumulative number of segments at level (1);

N0/ accumulative number of segments at the original level;

r0/ length of each segment at level (1);

r1/ length of each segment at level (2).

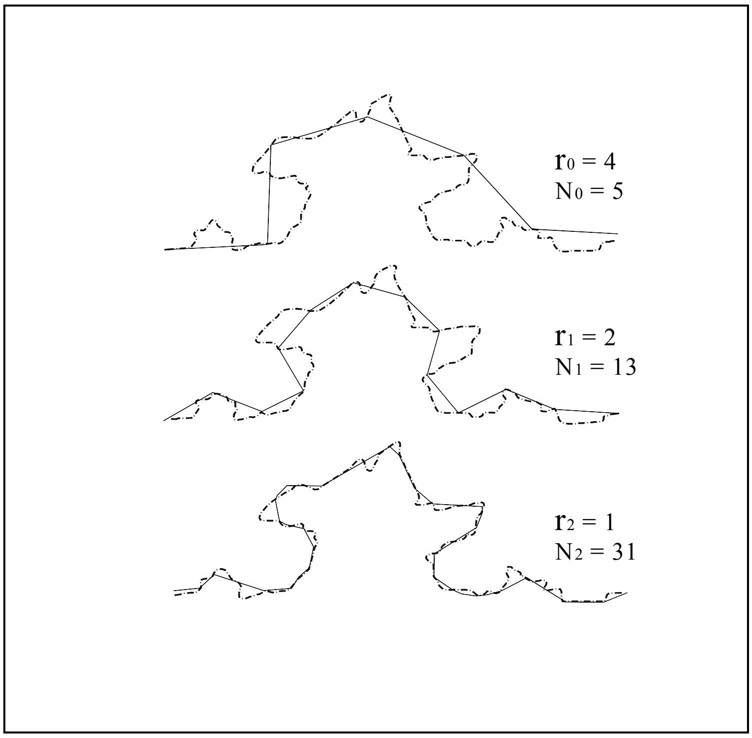

Figure 1: Approximating an irregular line and measuring perimeter length at three scales (Batty and Longley, 1994)

It is intuitively obvious that if halving the scale gives exactly twice the number of chords, then equation 1 implies that (D = 1) and that the line would be straight. If halving the scale gives four times the number of chords, the line would enclose the space and the fractal dimension would be (D = 2). If the line is fractal, then it is clear that halving the interval always gives more than twice the number of steps, since more and more of the self-similar detail is picked up. Thus, this equation is suitable for measuring the fractal dimension of static systems (no growth) by varying their scales (multi-levels of magnification). Therefore, it is possible to translate the sub-hypothesis (cities themselves are fractal at every level of their hierarchy, and this fractality is the same) into equations.

Cities do not have strictly self-similar fractals. When a set of objects (maps in various interval periods of time) of the same type is available which are all measured at the same scale, the fractal dimension of the set can be computed by examining changes in form at a fixed scale of measurement, which is controllable here. Each single object (city's map) will be defined by variables N(r): the number of parts or chords composing the object and L(r): the total lengths of these parts or chords at a given scale (r) to measure the geometric properties of that object.

It is clearly reasonable that the relation between the number of parts (N) and the scaling ratio (r), as the sub-hypothesis proposes, can be expressed as:

$$N \propto \frac{1}{r^{D}}$$

$$N = \alpha r^{-D}$$

$$\therefore\ N(r) = \alpha r^{-D} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

where:

N(r): accumulative number of parts till (r) level;

r : scaling ratio (the length of each part or chords or segments);

D : fractal dimension;

α : constant.

But, it is obvious that:

$$L(r) = N * r \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

where:

L(r): accumaltive length of the urban boundary till (r) level.

$$\therefore\ N(r) = \frac{L(r)}{r} = L(r)r^{-1}$$

By substituting this value of (N) in equation 2, the result will be:

$$L(r) = \alpha r^{1-D} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

It is possible to work out the relationship between the variables N(r) and L(r) for Irbid at each period of five serial time-period intervals (for the years 1953, 1961, 1978, 1984 and 1992) by halving ten different scales (levels of magnification) of each year's map, thus approximating its boundaries using AutoCAD programme.

The fractal dimension D can be calculated by logarithmic application (linear regression analysis) as detailed in the next section. Another set of equations is used for measuring the fractal dimensions of dynamic systems (with growth) by varying their size (multi-period-interval time, 1953, 1961, 1978, 1984 and 1992).

The main hypothesis (cities seem to grow fractally/ according to multi-fractal, not-integer dimension) can be expressed by equations. An object is constructed or measured at the same scale with changing its size in some regular way (growth or decline). Its mass or the number of its parts (N) increases or decreases as the object grows or diminishes.

It is assumed that the size of the object is proportional to (R: Radius), which is here proposed to be a linear measure appropriate to its measurement between the oldest core in the urban area and the farthest point on the boundary.

The research tries to explore what happens to its geometry, as its size, which is proportional to R changes. It is possible to assume that N(R) represents the number of parts till the accumulative radius R. Thus, if the scale is fixed and the size of the object is simply increased by R, then the number of parts increases in direct proportion.

It is obvious that the relation between the number of parts N and the radius R, as assumed in the main hypothesis, can be expressed as:

$$N = \delta R^{D}$$

$$\therefore\ N(R) = \delta R^{D} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

where:

N(R): accumulative number of parts till total radius R;

R : urban cluster radius (it is important here to know

the historic and oldest center of the cluster to measure the radius approximately);
D : fractal dimension;
δ : constant.

Also, it is clear that the relation between the urban boundary L and the radius R, as proposed before in the main hypothesis, can be presented as:

$$L(R) = \beta R^{D} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

where:
L (R): accumulative length of the urban boundary till R;
β : constant.

The concentration here is upon the urban boundaries (2-dimesional) as an implication of other variants such as areas and urban land-uses. Some future researches could be capable of focusing upon other variants such as transportation systems, urban land-uses, and (3-dimensional) geometric features, of the urban pattern. Equations 5 and 6 will be used for estimation when the city grows within time and has (at least two) maps of the same scale at different time periods for that city.

Ultimately, the variables which will be used in estimating the fractal dimension of Irbid city are:

- Dependent variables: Urban size, spacing and density, which can be represented by the fractal dimension D.
- Independent variables: Urban boundaries, parameters, scaling ratio and radius.

Estimation

Two basic properties of urban form will be dealt with which involve, on the one hand, boundaries to urban development, and on the other hand the growth of the city, its size and shape as we might perceive in terms of fractal geometry. The fractal dimension of a single object can be derived by measuring the same object (static) at different scales or by varying the extent or size of the object (dynamic/growth) over which the dimension might be computed. These two methods for estimation will be adopted .

Stabilizing the Size and Changing the Scale (Static/No Growth)

As previously mentioned, the variables N, L and (r) would be used to define the number of parts composing an object and the total length of these parts at a given scale (r). As the scale becomes finer, the number of parts of the object and the total length increase without bound, but the area and the density, which are proportional to one another, decrease to zero. This situation, of course, is based on the assumption that the dimension D is between (1) and (2) for the two-dimensional representative (maps, plans, pictures …) of any system.

The fractal dimension D for equations 2 and 4 would be computed by examining the city at ten different scales, taking the ratio of the number of its repetition to its scaling factor.

Figure 2 shows the application of the Richardson Method on the AutoCAD drawing of the city of Irbid (1953-1992) with 10 observations of changing scale. If a series of observations of N(r) and L(r) at different scales (r) is obtained, then the fractal dimensions can be derived by taking (log) transforms of each of the equations 2 and 4 and applying a regression of these relationships. One might wonder: what would be the relations between (N and r) if their values are directly represented as functions on the (X-Y) coordinates? It is clear that the results are non-linear and unpredictable relationships. Thus, it is necessary to take the (log) transformation in order to get rid of the exponents and to alter the values of the variables into more linear and predictable forms. Then, respectively, equations 2 and 4 would become:

$$\log N(r) = \log \alpha - D \log r \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

$$\log L(r) = \log \alpha + (1-D) \log r \quad \ldots\ldots\ldots\ldots\ldots \quad (8)$$

Figures 3 and 4 illustrate graphically the logarithmic relationship between (log N and log r) and (log L and log r), respectively, having applied the regression analysis on equations 7 and 8.

Figure 2: Richardson method approximation of Irbid boundaries for the years 1953, 1961, 1978, 1984 and 1992

Thus, the urban pattern of Irbid (1992) shows the same self-similarity under multi-levels of magnification (varying the scale) of (D = 1.256), which asserts the sub-hypothesis that this fractality is the same for each simultaneous process of urban growth.

Stabilizing the Scale and Changing the Size (with Growth)

Here, the same object is considered as fixed scale (r). Consider that the number of parts into which the object is divided based on equation 5 and without loss of generality is N. If the scale is fixed and the size is simply increased (the growth for the years 1953, 1961, 1978, 1984 and 1992 indicated by its radius R), then the number of parts increases in direct proportion. The same is said about the total length of the boundary L, mentioned in equation 6, with the same proportion.

The application of Richarson Method on the AutoCAD drawing through the years (1953, 1961, 1978, 1984 and 1992) can be made. Again, the fractal dimensions can be computed, by taking transforms of each of the equations 5 and 6 and applying the regression analysis at different variants of size R.

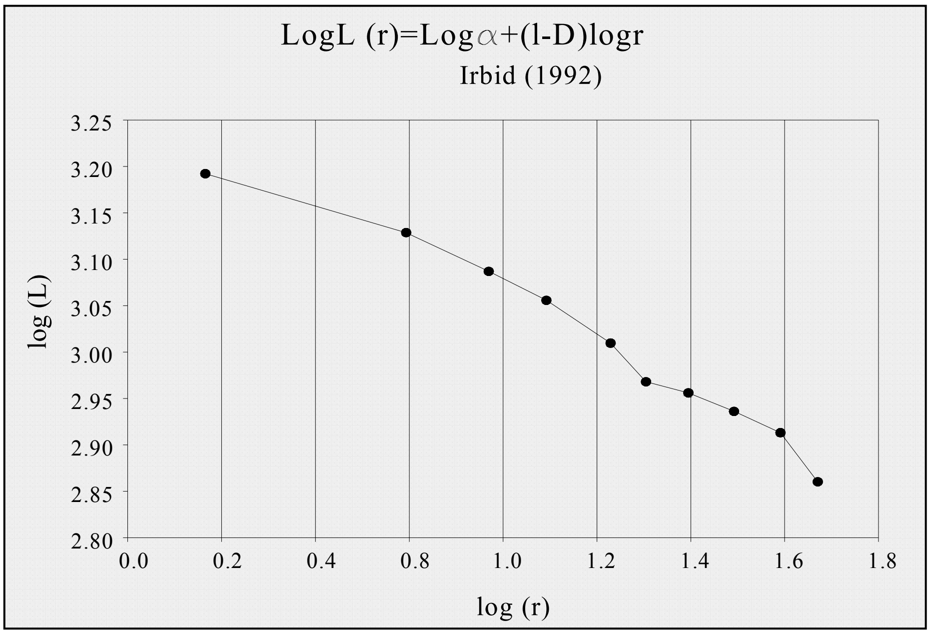

Figure 3: Logarithmic graph between log (L) and log (r), Irbid 1992

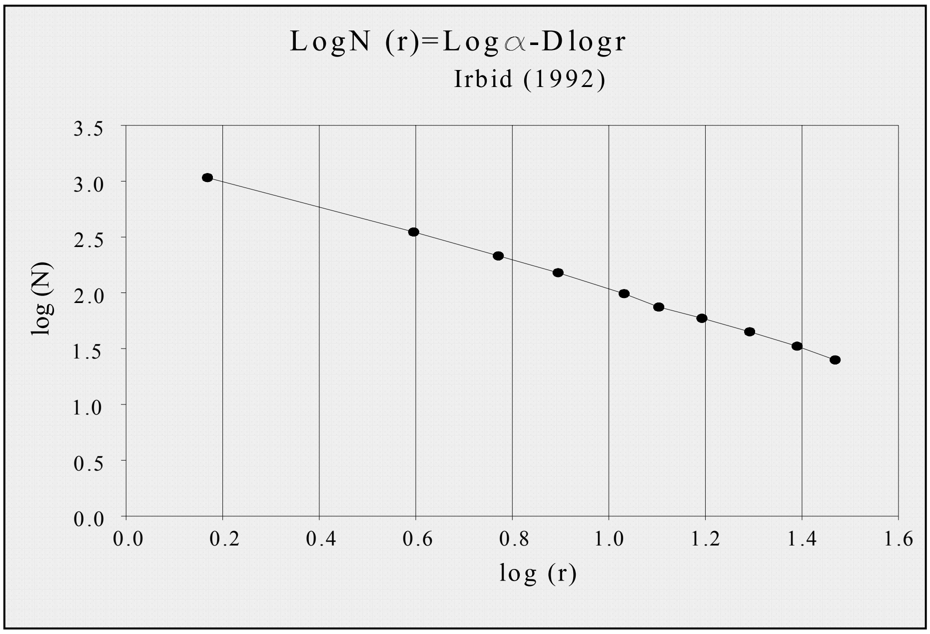

Figure 4: Logarithmic graph between log (N) and log (r), Irbid 1992

Table 1. Results of the whole estimation

growth / no growth				Stabilizing the scale and changing the size (growth)					D	
				Year (Size / Radius R)					log N= logδ+D log R	log L =logβ+D log R
				1953	1961	1978	1984	1992		
no growth				54.9355	60.3232	75.7056	100.0792	172.8535		
Stabilizing the size and changing the scale (no growth)	Scaling Ratio (r)	30	N	5	6	10	17	24	1.4	1.35
			L	155.6876	191.8477	304.1389	521.2534	721.5516		
		25	N	5	8	12	22	32	1.5	1.5
			L	134.7644	214.1003	308.5005	566.338	814.2143		
		20	N	8	10	17	29	42	1.4	1.4
			L	168.377	212.8474	353.119	589.1807	859.3204		
		16	N	9	122	23	39	56	1.5	1.5
			L	152.4652	206.7046	377.0255	632.4898	898.6837		
		13	N	12	18	31	54	71	1.5	1.46
			L	162.7237	244.3375	411.2763	708.1062	923.6563		
		11	N	15	19	39	67	92	1.5	1.5
			L	168.692	218.5459	430.285	737.009	1018.0842		
		8	N	21	31	58	106	141	1.6	1.6
			L	172.3064	253.2048	468.6175	855.8075	1130.0394		
		6	N	29	46	85	166	202	1.6	1.6
			L	168.692	278.2776	513.7572	997.9123	1216.9681		
		4	N	47	69	147	287	334	1.6	1.6
			L	188.8577	278.7407	589.1995	1149.938	1336.1466		
		1.5	N	141	258	479	981	1031	1.6	1.6
			L	212.2448	387.7509	719.5191	1471.995	1547.1296		
D	log N = logα - D logr			1.12	1.217	1.3	1.365	1.256		
	log N=logα+(1-D) logr			1.12	1.217	1.3	1.365	1.256		

Equations 5 and 6 can be re-arranged after taking the (log) transforms as follows:

$$\log N(R) = \log \delta + D \log R \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

$$\log L(R) = \log \beta + D \log R \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

Each of the above equations has the same slope D of the regression lines, which facilitates the direct derivation of the dimension. By the regression analysis, the values of the fractal dimensions are computed for the (10) scaling ratios (as will be shown later). The results of D would be (1.4, 1.5, 1.4, 1.5, 1.5, 1.5, 1.6, 1.6, 1.6 and 1.6) for the scaling ratios (r) of (30, 25, 20, 16, 13, 11, 8, 6, 4 and 1.6), respectively.

Thus, the city of Irbid is said to grow systematically (fractally, according to multi-fractal, not-integer dimensions). The main hypothesis of the research is asserted here, and the values of the fractal dimensions are ($1.26 < D < 1.7$).

RESULTS AND DISCUSSION

The graphical representations after taking the logarithmic values of the variables for each level of magnification (in case of neutralizing the size) and each process of urban growth (in case of stabilizing the scale) show that the followed procedures are incomplete and unable to draw and specify a linear relationship between the variables in order to measure and predict the behavior of the system with a sense of control. Thus, applying linear regression analysis is needed here. Table 1 illustrates the results of performing the regression analysis by varying the scale and size, respectively, of the whole estimation. All results of the tests can be briefly elaborated as: $t > 2$, $R^2 > 0.5$, $F > 10$, $1 < D.W < 2$. Therefore, these results are reliable and all regression tests are significant.

The results also show that the values of the fractal dimension give some hints and indications that assert the main and the sub-hypothesis of the research (cities seem to grow fractally, according to multi-fractal not-integer dimension) and (cities themselves are fractal at every level of their hierarchy; D values are between 1.26 and 1.7), respectively. However, these values do not reflect the actual relationships between the scaling ratio (r) and D on one hand, and between the growth parameter (Radius/R) and D on the other hand. It is important to notice and test these relationships directly. At first, from equation 8, stabilize the size (R) and change the scale (r), let: $1-D = \xi$

$$\therefore D = 1 - \xi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

Assume that the direct relationship between (ξ : as index of D) and (r) would be:

$$\xi = \lambda + \Phi r \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

where (λ, Φ) are constants. By substituting equation 12 into equation 11, the fractal dimension can be calculated as:

$$D = 1 - \lambda - \Phi r \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

Substituting equation 13 into equation 8 leads to:

$$\log L = \log \alpha + \lambda \log r + \Phi r \log r \quad \ldots\ldots\ldots\ldots \quad (14)$$

Table 2 shows the results of applying the linear regression on equation 14, and then the values of D can be computed easily from equation 13. Figure 5 demonstrates the graphical representation of the direct resultant relationship between D and (r). As for equation 10; stabilize the scale (r) and change the size R, assuming that the direct relationship between D and R would be:

$$D = \chi - \kappa R \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (15)$$

where (χ, κ) are some constants. Substituting equation 15 into equation 10 leads to:

$$\log L = \log \varsigma + \chi \log R - \kappa R \log R \quad \ldots\ldots\ldots\ldots \quad (16)$$

Table 2. Linear regression results of (logL = logα +λ logr + Φr logr)
by stabilizing the size and changing the scale

Dependent Variable	Independent Variable (s)	Tests				Coefficients			r	D=1-λ -ør	Seed
		t	R^2	F	D-W	log α (a)	λ	ø			
log L_{1953}	log r , rlog r	73.34	0.828	14.45	–	2.338	-0.104	-0.00097	30	1.1331	**0.00097**
									25	1.12825	
									20	1.1234	
									16	1.11952	
									13	1.11661	
									11	1.11476	
									8	1.11176	
									6	1.10982	
									4	1.10788	
									1.5	1.105455	
log L_{1961}		0.00265	-0.288	2.635	–	47.4	0.94	88.6	30	1.2085	**0.00265**
									25	1.22175	
									20	1.235	
									16	1.2456	
									13	1.25335	
									11	1.25885	
									8	1.2668	
									6	1.2721	
									4	1.2774	
									1.5	1.284025	
log L_{1978}		328	0.997	982.2	–	2.9	-0.224	-0.00267	30	1.3041	**0.00267**
									25	1.29075	
									20	1.2774	
									16	1.26672	
									13	1.25871	
									11	1.25337	
									8	1.24536	
									6	1.24002	
									4	1.23468	
									1.5	1.228005	
log L_{1984}		143.6	0.987	226.6	–	3.243	-0.333	-0.00119	30	1.3687	**0.00119**
									25	1.36275	
									20	1.3568	
									16	1.35204	
									13	1.34847	
									11	1.34609	
									8	1.34252	
									6	1.34014	
									4	1.33776	
									1.5	1.334785	
log L_{1992}		176	0.981	153.4	–	3.237	-0.2	-0.00163	30	1.2489	**0.00163**
									25	1.24075	
									20	1.2326	
									16	1.22608	
									13	1.22119	
									11	1.21793	
									8	1.21304	
									6	1.20978	
									4	1.20652	
									1.5	1.202445	

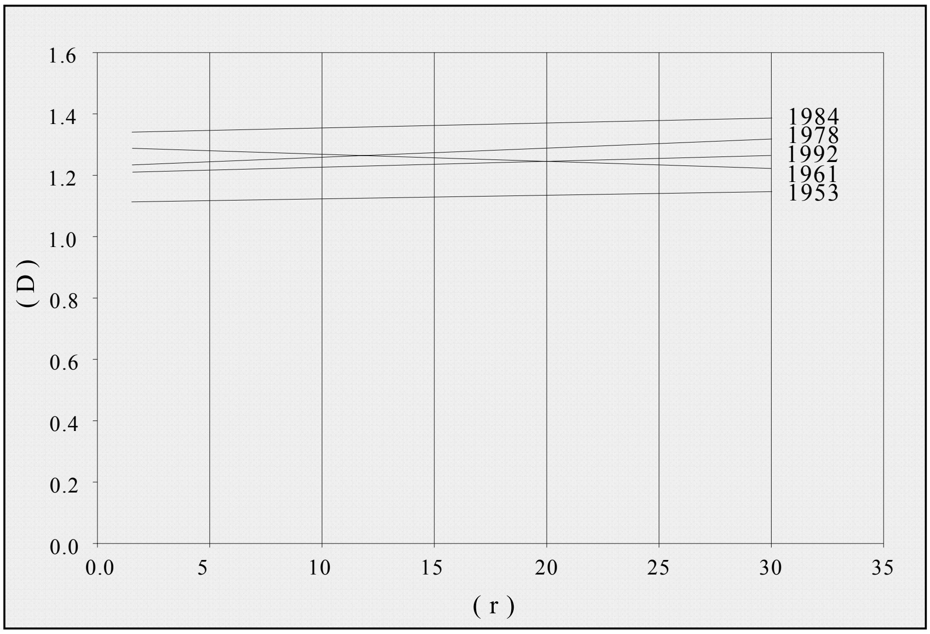

Figure 5: The direct graphical relationship between (D) and (r)

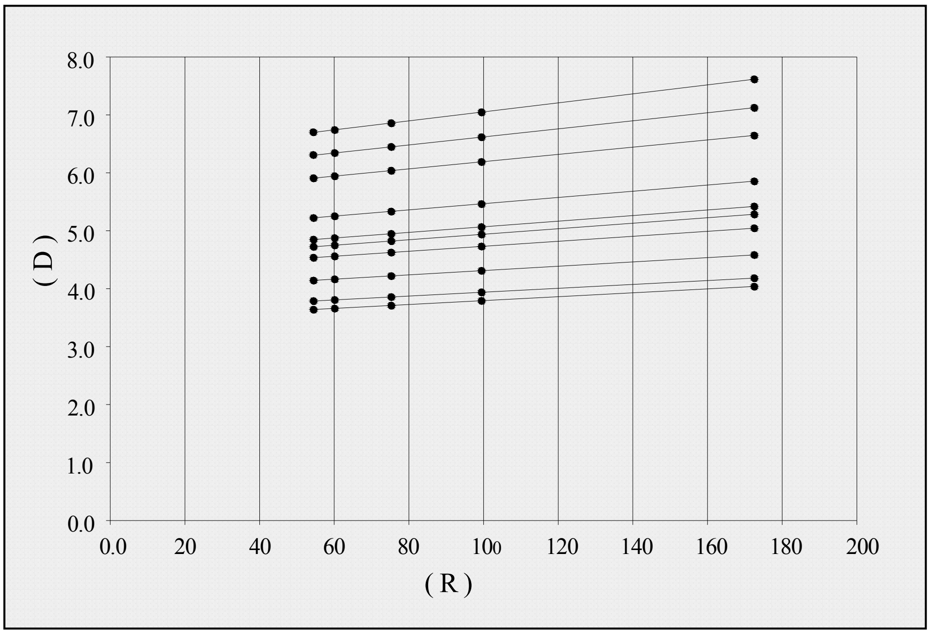

Figure 6: The direct graphical relationship between (D) and (R)

Table 3. Linear regression results of ($\log L = \log \varsigma + \chi \log R - {}_{K}R \log R$) by stabilizing the scale and changing the size

Dependent Variable	Independent Variable (s)	Tests				Coefficients			Year	Radius (R)	$D = \chi - {}_{K}R$	Seed
		t	R^2	F	D-W	log ς (a)	χ	K				
$\log L_1$	log R , R log R	11.643	0.999	929.3	2.819	-3.407	3.408	-0.0035	1953	54.9355	3.6	0.003526551
									1961	60.3232	3.619	
									1978	75.7056	3.6729	
									1984	100.0792	3.757	
									1992	172.8535	4.01198	
$\log L_2$		3.695	0.987	76.8	3.141	-4.17	3.868	-0.004	1953	54.9355	4.087	0.004120496
									1961	60.3232	4.1092	
									1978	75.7056	4.17	
									1984	100.0792	4.268	
									1992	172.8535	4.5594	
$\log L_3$		89.54	1.00	58254	3.406	-3.575	3.532	-0.0036	1953	54.9355	3.7297	0.003582233
									1961	60.3232	3.749	
									1978	75.7056	3.8	
									1984	100.0796	3.892	
									1992	172.8535	4.154	
$\log L_4$		15.5	0.999	1272.8	2.391	-4.725	4.23	-0.0046	1953	54.9355	4.4827	0.004510273
									1961	60.3232	4.507	
									1978	75.7056	4.578	
									1984	100.0796	4.69	
									1992	172.8535	5.025	
$\log L_5$		7.4	0.996	228.3	2.6	-4.942	4.402	-0.005	1953	54.9355	4.674	0.005122779
									1961	60.3232	4.7016	
									1978	75.7056	4.778	
									1984	100.0796	4.9	
									1992	172.8535	5.264	
$\log L_6$		16	0.999	1220	3.5	-5.17	4.528	-0.005	1953	54.9355	4.8	0.005493995
									1961	60.3232	4.8296	
									1978	75.7056	4.9	
									1984	100.0796	5.028	
									1992	172.8535	5.392	
$\log L_7$		13.3	0.999	726.8	2.5	-5.698	4.877	0.0056	1953	54.9355	5.1846	0.00560536
									1961	60.3232	5.2148	
									1978	75.7056	5.3	
									1984	100.0796	5.437	
									1992	172.8535	5.845	
$\log L_8$		7.7	0.995	196	2.77	-6.7	5.512	-0.0056	1953	54.9355	5.8734	0.006589082
									1961	60.3232	5.9089	
									1978	75.7056	6.01	
									1984	100.0796	6.17	
									1992	172.8535	6.649	
$\log L_9$		36.5	1.00	3941	2.3	-7.24	5.866	-0.00714	1953	54.9355	6.2582	0.007145906
									1961	60.3232	6.2967	
									1978	75.7056	6.4	
									1984	100.0796	6.58	
									1992	172.8535	7.1	
$\log L_{10}$		6.4	0.99	100.4	2.86	-7.734	6.238	-0.00785	1953	54.9355	6.6692	0.00775841
									1961	60.3232	6.711	
									1978	75.7056	6.832	
									1984	100.0796	7.023	
									1992	172.8535	7.594	

Once again, the linear regression analysis for equation 16 leads to estimate the values of (χ, κ) and then allows computing the value of D from equation 15; Table 3. The graphical representation of the direst resultant relationship between D and R is shown in Figure 6. One might deduce and statistically verify that the direct relationship of the estimated fractal dimension with the scaling ratio (r) and with the radius (R) would be linear, as shown in Figures 5 and 6. The values of seeds (motifs or archetypes) are the same for each period of time (in case of stabilizing the size and changing the scale) as shown in Table 2 and for each scaling ratio (in case of stabilizing the scale and changing the size) as shown in Table 3. That would assert the research hypotheses.

As Figure 5 shows, the smallest scale dimension declines over time except for years 1953 and 1961 because the region had witnessed very profound, complicated and accumulated events of the Israeli-Arab 1948 war and the fuzzy grown refuge accompanied. The refugees preferred to settle in the inhabited areas within the urban fabric, causing the processes of filling the space to increase. The deviated behavior in 1961 results mainly from the imprecision and distortion of the obtained aerial photograph of the city from that time.

Figure 6 provides a reference graph to make some estimation for future predictions. The horizontal extension between the years 1978, 1984 and 1992 is very clear when the greatest changes in transport technology occurred.

The mutual inter-correlated and chronological processes of extending and filling the gaps or holes (form and function) are permanent. Each process reflects the other. This is just like blowing up a balloon. The size and the surface area of the balloon increases noticeably by gradual blowing till it reaches a range (threshold) in which the tiny changes in shape, size and area (because of blowing) become invisible but detectable. The city in this case is said to be compact.

Most of the work in this research is based on conceiving the city and sets of lines and areas, and thus the related geometry is based upon one dimension and two dimensions, not zero or three. Yet, there are arguments which suggest that cities might be treated as points or volumes, thus composing fruitful extensions to the new geometry. Finally, the results also point out the fact that the growth of a "local" city of irregular nature and of different developmental backgrounds as the study area, is also fractal. This supports the argument that cities globally or locally would produce fractal growth at every level of their hierarchy.

Acknowledgment

Our sincere gratitude to Professor Michael Batty (Center for Advanced Spatial Analysis, University College London) and Professor Nikos A. Salingar (the University of Texas, San Antonio, USA) for their precious advice and assistance.

REFERENCES

Batty, M. and Longley, J. 1994. Fractal Cities: A Geometry of Form and Function, Academic Press, Limited, San Diego.

Batty, M. and Xie, Y. 1996. Preliminary Evidence for Theory of the Fractal City, *Environment and Planning (A)*, 28 (October): 1745-1762.

Benguigui, L. 2004. Simulation Analysis of the Fractality of Cities, *Geographic Analysis,* 36 (1): 69-84.

Donahue, J. M. 1999. Chaos Theory and Fractal Phenomena, USA.

McAdams, M. 2009. The Application of Fractal Analysis and Spatial Technologies for Urban Analysis. *Journal of Applied Functional Analysis*, 4 (4): 569-579.

Mikiten, T., Salingaros, N. and Sing-Yu, H. 1997. Pavements as Embodiments of Meaning for Fractal Mind, San Antonio, Texas, USA.

Montgomery, J. 1998. Making a City: Urbanity, Vitality and Urban Design, *Journal of Urban Design*, 3 (1).

Rice, T.J. 1989. Ulysses, Chaos and Complexity, University of South Carolina, USA.

Salingaros, N.A. and Tejada, D. 1998. Notes on the Geometry of Cities, University of Texas, San Antonio, USA.

Shen, G. 2002. Fractal Dimension and Fractal Growth of Urbanized Areas, *International Journal of Geographical Information Sciences*, 16 (5): 419-437.

Terzi, F. and Kaya, H. 2008. Analyzing Urban Sprawl Patterns through Fractal Geometry: The Case of Istanbul Metropolitan Area, UCL Working Papers Series, Paper 144.

Thomas, I., Frankhauser, B. and Badariotti, D. 2012. Comparing the Fractality of European Urban Neighbourhoods: Do National Contexts Matter? *Journal of Geographical Systems*, 14 (2): 189-208.

Thomas, I., Frankhouser, P. and De Keersmaeker, M. 1997. Fractal Dimension *versus* Density of Build-up Surfaces in the Periphery of Brussels, *Papers in Regional Science*, 86 (2): 287-308.

Wahl, B. 1998. Exploring Fractal on the Macintosh, Addison-Wesley Publishing Company, USA.