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Prediction of Flow Duration Curve for Seasonal Rivers in Iraq

Abdulwahab M. Younis[1] and Ihsan F. Hasan [2]

[1] Dams and Water Resources Engineering, College of Engineering, University of Mosul, Iraq.
E-Mail: amyggv56@yahoo.com
[2] Dams and Water Resources Research Center, University of Mosul, Mousl, Iraq.
E-Mail: ihsanfh@yahoo.com

ABSTRACT

This paper presents a new method to estimate the flow duration curves [FDCs] for ungauged river basins whose natural and meterological characteristics are known. This study highlights the modeling of the lower three-quarters of the section of the flow duration curves [FDCs]. Eight sub-catchments were used to develop and evaluate the proposed flow duration model in the north of Iraq. The logarithmic type function has been found appropriate for the lower three-quarters of the section of [FDCs] for all river sub-catchments located in the study area. Parameter values of the logarithmic function model were calculated using topographic, hydrological and climatic characteristics of the basins under study by two regional regression models: first CA-MAP (catchment area-mean annual precipitation) model and second MAF-PE (mean annual flow-potential evapotranspiration) model. Generally, it is found that both models used were predicting a good estimate at the end of the flow duration curve (low-flows). In most cases, the statistics and graphical results showed that the agreement between observed and estimated FDCs is very good by using MAF-PE model as compared to CA-MAP model.

KEYWORDS: Flow durationu curve, Ungauged rivers, Low-flow measures.

INTRODUCTION

Flow-duration curves (FDCs) have been in general use since about 1915. FDC is a key tool for the sustainable management of water resources. Gordon et al. (1992) illustrated the use of FDCs for the assessment of river habitats in the estimation of stream flow requirements. Hughes and Smakhtin (1996) suggested a nonlinear spatial interpolation approach (based on FDCs) for patching and extension of observed daily flow time series, which has later been extended to generation of flow time series at ungauged sites. Lanen et al. (1997) and Smakhtin et al. (1998) used FDCs as a tool for rainfall–runoff model calibration and/or for the comparison of flow-time series simulated for different scenarios of development. Wilby et al. (1994) used FDCs to assess the effects of different climate scenarios on streamflow with particular reference to low-flows. Hughes et al. (1997) developed an operating rule model which is based on FDCs and is designed to convert the original tabulated values of estimated ecological instream flow requirements for each calendar month into a time series of daily reservoir releases.

Prediction of flow duration curves (FDCs) in ungauged basins is an important tool for water resources planning and management. It is clear that FDCs for each catchment are different. For prediction in ungauged basins, it is essential to understand what factors cause FDCs to vary between catchments.

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Understanding the climatic and catchment characteristics controls on the FDCs can enable the extrapolation of empirical FDCs derived from gauged catchments to ungauged catchments within a similar or homogeneous region. The regionalization of flow duration curves appears to be an operative tool when dealing with ungauged catchments or short stream flow records (Castellarin et al., 2004).

Information on flows in rivers, particularly low flows, became a greater priority in the early 1970s in Iraq to protect and quantify water resources and to meet the data requirements of ongoing developmental needs. In this study, low flows are considered as the lowest discharge values observed in a river. The ability to estimate magnitude and frequency of low flows in river streams and ungauged catchments is important for water supply planning, waste-load allocation, reservoir storage design, maintenance of quantity and quality of water for irrigation, recreation and environmental flow requirements for wildlife conservation (Smakhtin, 2001). It is necessary to estimate low flows, not only in observed streams but also in ungauged watersheds.

There are many techniques for estimating low flow regimes at an ungauged site, particularly by classifying catchments into physiographic types and transferring flow data between catchments in the same region. An estimation of low flows by correlation with neighbouring gauged catchment data is described in U.S. Smakhtin (2001) and Demuth and Young (2004) give an extensive list of possible approaches and techniques for low-flows estimation in ungauged catchments, which include regional regression, spatial interpolation, construction of regional curves and time series simulation.

Definition and Construction of Flow Duration Curves

One of the most commonly used techniques in hydrology is the flow duration curve (FDC), which provides a graphical representation of the frequency distribution of the complete flow regime (from low flows to flood events). It is a graph of any given discharge value plotted against percentage of time that this discharge is equalled or exceeded. In other words, the relationship between magnitude and frequency of stream flow discharges is shown (Smakhtin, 2001).

Flow duration curves are widely used by engineers and hydrologists around the world in numerous applications related to water resources management, like hydropower generation and planning, designing of irrigation systems, management of stream pollution, river and reservoir sedimentation and fluvial erosion **(**Castellarin, 2007). Flow duration curve is one of a variety of low flow measures which describe and quantify different properties of flow regimes and has different applications in water resources.

Note that FDCs can be constructed using daily, weekly or monthly stream flow data, depending on the period of records. Even though flow duration curves can be defined and constructed for different time series, our study will focus only on daily stream flows (average daily flow for one-year long period "the average daily flows data for each station averaged over M years of observed daily flow") because FDCs constructed on the basis of daily flow time series provide the most detailed way of examining duration characteristics of a river.

The following steps are followed to construct an FDC:

1. Ranking the observed stream flows in descending order (from the maximum to the minimum value**).**
2. Calculating exceedence probability (P) as follows:

   $$P = 100 * [M/(n+1)] \quad (1)$$

   P: the probability that a given flow will be equalled or exceeded (% of time).

   M: the ranked position on the listing.

   N: the length of the sample.
3. Plotting each ordered observation *versus* its corresponding duration or exceedence probability.

Measures and Indices of Low Flows

Low flow could mean different things to the hydrologist. It may be defined as the actual flow in a river occurring during the dry season of the year, others may consider it as the length of time and the conditions occurring between flood events. The World Meterological Organization (WMO) defines low flow as the flow of water in a stream during prolonged dry weather.

To define specific values derived from any low-flow measure, we used the term "low flow indices". There are three standard statistically defined low flow indices of rivers:

1. Annual minimum N-day moving average flows with N= 3, 5, 7, 10, 15, 30 and 90 days. The AM (N-day) moving average can easily be calculated by applying a moving-average filter of N days on a daily discharge series and selecting the minimum of the filtered series. The annual minimum of 7-day moving-average flows (AM7) is one of the most widely used indices.
2. Annual minimum N-day sustained low flows with N = 1, 5, 7, 10, 15, 30 and 90 days. The AM (N-day) is the lowest flow of N consecutive days within one year. It is a way to study low flow characteristics as a time series. The case N= 1 corresponds to the annual minimum mean daily flow series; it can be derived from a daily flow series by selecting the lowest flow every year. Kobold and Brilly (1994) analyzed the relationship between different low flows durations at regional scale by using the mean annual 10-day minimum as a key variable. In the United States, the most widely used low flow index is the 10-year annual minimum 7-day flows AM (7-day) 10 (Hisdal et al., 2004), it is mean AM (7-day) with a return period of 10 years. At the same time, there is no big difference between 1-day and 7-day low flows (Smakhtin, 2001).
3. Low flow percentiles from the flow duration curve (FDC) are often used as low flow indices, such as Q75% and Q95% percentiles from the (FDC) which describe the low flow part of the river flow regime. Q95% is most commonly used to characterize the low flow and is defined as the flow equalled or exceeded 95% of the time. Other percentiles can similarly be derived from the flow duration curve. According to (WMO-No. 50), the mean annual minimum 7-day is numerically similar to Q95% for most flow records.

Choice of Percentiles as Low Flow Indices

The ‘low flow section’ of the FDC is the most important section to predict the complete range of river discharges from low-flows to flood events, which is determined as part of the FDC with flows below mean flow (discharge equalled or exceeded 50% of the time, Q_{50}).

Various other low-flow indices may be estimated from ‘low flow section’ of the FDC. The most widely used as design low flows range of a flow duration curve is within the range of 70%-99% time exceedence. Some conventional indices include the percentage of time that 25% average flow is exceeded. The Q95 and Q90 flows are most often used as low flow indices in the academic sources (WSC Report No. 04-2004).

Low flow percentiles from the FDC are often used as key indices of low flow; for example, the 75 percentile flow Q75 or 95 percentile flow Q95, the flow that is exceeded for 75 percent or 95 percent of the period of record. This discharge is a useful general index of low flow. In semi-arid areas, the river has zero flows for the time useful percentiles will be higher; for example Q25 and Q50. Because of its relevance for multiple topics of water resources management, we used in this study the sections (Q_{25}, Q_{50}, Q_{75} and Q_{95}) of the FDC; i.e., the discharge equalled or exceeded on 25% of all days of the measurement period, as indicators for low flow regime. These particular exceedence values were chosen because these percentages are important in the sizing of hydropower plants and designing of irrigation systems.

FDC and Low Flows Applications

FDCs are widely used in hydrological practice. Vogel and Fennessey (1994) refer to several early studies related to the theory and applications of FDCs. Searcy (1959) was the first researcher who summarized a number of FDC applications including the analysis of low flow, hydropower and stream water quality studies. The FDC is the conventional method for describing water availability in a river for hydropower design and water supply. Warnick (1984) illustrated the application of FDCs to hydropower feasibility studies for run-of-river operations. The FDC is also used to estimate the dilution of domestic or industrial discharge destined for a river (Vogel and Fennessey, 1994). The FDC is commonly used for the preliminary design of simple abstraction schemes (Mhango and Joy, 1998). Alaouze (1991) developed the procedures based on FDC for estimation of optimal release schedule from reservoirs for agriculture. The FDC application for agricultural use is to supply water for irrigation. FDC can be used for the assessment of river habitats in estimation of stream flow requirements for ecosystem protection and fish farming requirements (WMO, 2009).

Low flow applications are approximately similar to FDC applications. Low flow information is required for a wide range of applications such as sustainable management requirements of both surface water and groundwater resources and long-term river basin plans. One of the most common uses of low flow information is the design and operation of public water supply schemes and irrigation water demand. Design of hydropower schemes is dependent on the complete range of flows, low flows can be critical in determining how much water most bypasses a run of river hydroplant to maintain downstream river ecology and how much is available for power generation in the dry season. A common application of low flow information is that of estimating the dilution of domestic or industrial discharge released in a river. Low flow is important for navigation; it is interrupted during low flow periods. Ecosystem protection is very vulnerable during low flow periods (WMO, 2009).

STUDY AREA AND DATA

The study area is bounded between 34º 10' and 38º 06' N latitudes, 43º 25' and 46º 05' E longitudes, covering approximately 51984 km^2 located on the North-Eastern part of Iraq, as shown in Figure 1. Most of the study basins are mountainous catchments for which streamflow generation is mainly controlled by precipitation. Precipitation presents seasonal variations over the study area, being the highest in winter, high in spring, low in autumn and the lowest in summer; i.e. low flows occur during summer. For this reason, the analysis of low flows regime is both complex and interesting.

Average daily stream flow data series for 7 catchments (which lie on Tigris river tributaries) were obtained from the gauge stations (Hydrological survey of Iraq), the catchments' area ranges from 1020 to 17330 km^2, elevations range from 86 m to 570 m a.s.l. and historical streamflow data length varies from a minimum of 14 years to a maximum of 20 years.

In the map given in Figure 1, gauged stations are extracted according to their coordinates. Those coordinates were extracted for each catchment from digital elevation model (DEM) databases (with an accuracy of 30 meters), and by using these DEM data in WMS v7.1, we can identify the morphological characteristics such as basin areas, stream network and delineate sub-basins within a watershed, creating the basin outlet point… etc.

Table 1 gives some physiographic and climatic characteristics of the selected basins for the proposed model studies, such as mean annual precipitation (MAP), mean annual flow (MAF), catchment area (CA), the specific discharges Q_{25},Q_{50},Q_{75},Q_{95} and the time series length.

The length of streamflow data series is related to the data availability, (Castellarin et al., 2004; Ganora et al., 2009) showing that five years of observed streamflows are generally sufficient to obtain good estimates of the long-term flow duration curve.

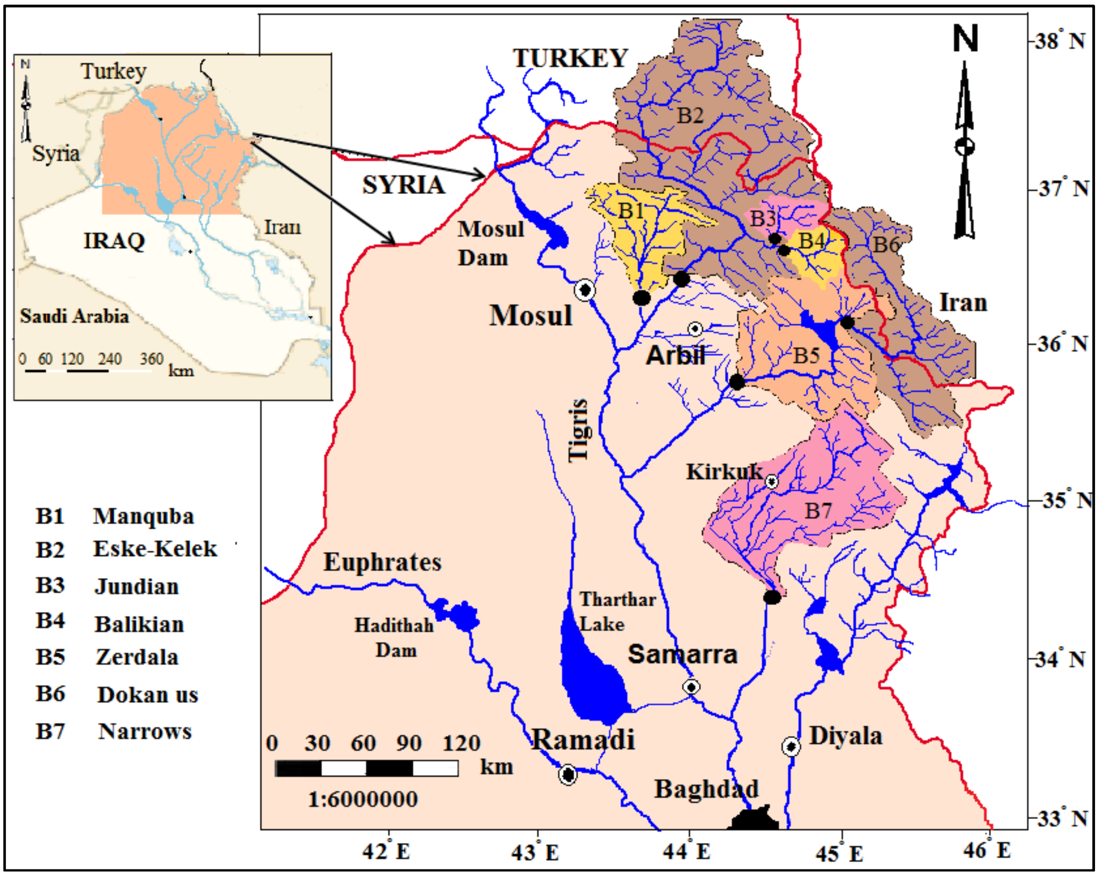

Figure (1): Locations of study catchment areas

Table 1. Data used in the study for parameter estimation of the proposed model

Index / Gauging stations	CA km^2	MAP mm	MAF m^3/s	Elev. m	L year	Q_{25} (m^3/s)	Q_{50} (m^3/s)	Q_{75} (m^3/s)	Q_{95} (m^3/s)
Manquba	3187	400	29.06	255	20	61	38	21	7.5
Eske-Kelek	17330	375	418.0	299	20	535	340	167	87
Balikian	1020	900	26.5	535	16	58	35	17	2.5
Jundian	1197	900	20.2	570	14	26.3	12.2	5.8	4
Dokan	7000	700	527	418	20	910	482	257	110
Zerdala	13800	370	227.4	256	20	265	166	89	39
Narrows	8450	320	33.3	86	18	129	50	22	5

CA: Catchment area (km^2).
MAP: Mean annual precipitation (mm).
MAF: Mean annual flow (m^3/s).
Elev.: Elevations of gauging stations (m).
L: Time series length of the available data (years).

Modelling the Flow Duration Curve

Step I: Obtaining parameters for each gauging station: In order to obtain Parameters **α** and **β** in Equation (2), an empirical FDC was developed using the average daily flows data for each station (Average daily flows for one-year long period are put into an order from maximum to minimum). The lower three-quarter section (25% to the end) of developed FDC of each gauging station was fitted. Based on the shape of this FDC curve section, a 2-parameter logarithmic function appears to be the most appropriate one to choose (among: power, exponential, linear rational, hyperbolic and rational functions) in order to represent the lower three-quarter section of empirical FDC of each gauging station under study. The chosen function is shown in Eq. (2).

$$Q_p=\beta+\alpha*Ln\,(P) \qquad (2)$$

Q: the flow (m^3/sec).
α and β: FDC parameters.
P: percentile exceedence of flow.

Figures 2 and 3 show that the fitted logarithmic and measured sections (25% to the end) of FDC curve are in good agreement for stations Eske-KeleK and Dokan.

The Model Parameters Relationship

In general, these parameters are related to each other; i.e., these parameters are inter-dependent. The correlation between model parameters *α* and β is given by the approximated relationship function shown in Eq. (3).

$$\beta = 4.6916 * \alpha + 14.555 \qquad (3)$$

This correlation could help exclude one and depend on one of the 2 parameters in the approximation of the analytical lower three-quarter part of the FDC, by using single parameter 'α' as follows:

$$Q_P = \alpha*(4.446-\ln(p)) + 14.55 \qquad (4)$$

Estimating parameters *α* and *β* is obtained first for each of the stations, and then the regionalization is made in step II.

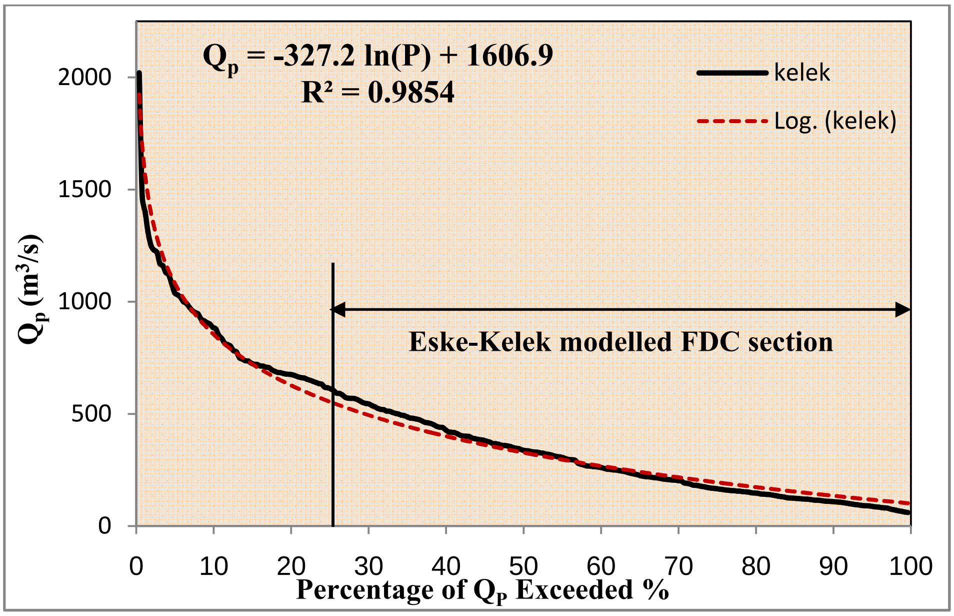

Figure (2): Fitted log-model to Eske-Kelek empirical FDC

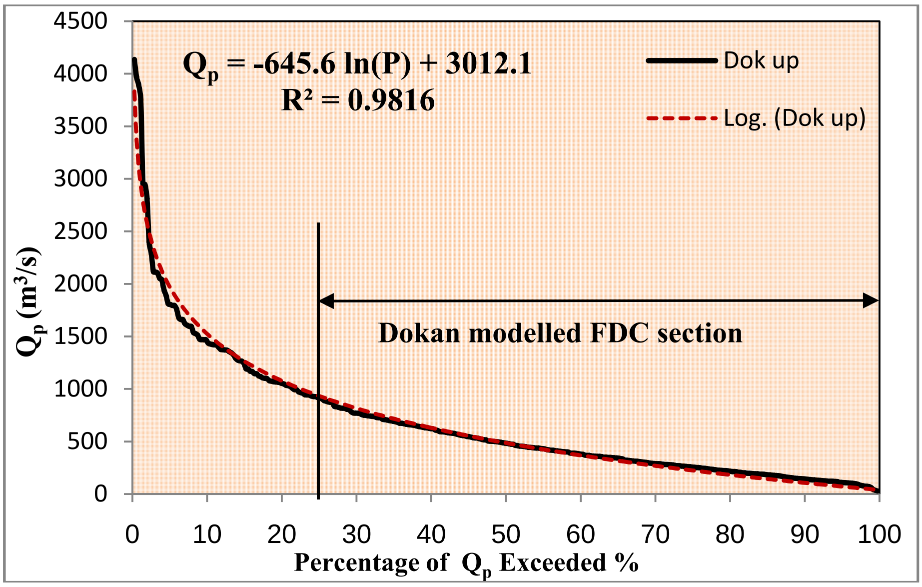

Figure (3): Fitted log-model to Dokan empirical FDC

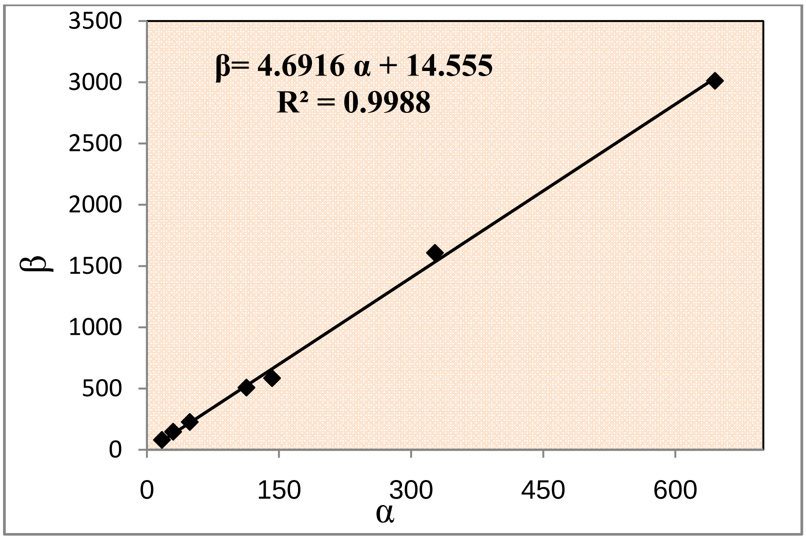

Figure (4): Correlation between model parameters 'α' & 'β'

Step II. Regionalization

The prediction of flow characteristics in ungauged catchments is usually based on transferring or extrapolating information from gauged to ungauged sites. This process is called regionalization; i.e. use of estimated parameter values of hydrological predictive models for gauged catchments in ungauged catchments without needing observed data. To be able to derive the flow duration curves for ungauged catchments, we need to derive the values of the model parameter **α** to substitute it in equation (4), and then test its performance by employing it in gauged catchments. The shape of the FDC is governed by interplaying both of (catchment physiographic and climatic parameters). In the study area catchments, the values of **α** depend on the following regional parameters:

CA: Catchment Area (km^2).
DD: Drainage Density (km/km^2).
BFI: Baseflow Index.
MAP: Mean Annual Precipitation (mm).
PE: Mean Annual Potential Evapotranspiration (mm).

The values of 'α' and 'β' also depend on the region on which the catchment is located; its latitude, longitude and elevation above sea level. To predict the value of model parameter *α* by using the above regional parameters, we suggest the following two regional regression equations.

First Model

(CA-MAP) Model: It's expected that the value of α is dependent on the mean annual precipitation (MAP) within the catchment area. David Post (2005) shows that *α* has the same relationship with mean annual precipitation and catchment area. Figure 5 shows the plot of ln(CA) x ln(MAP) *versus* α. The exponential equation type of Catchment Area and Mean Annual Precipitation model possibly leads to a good correlation with parameter 'α', the value of coefficient of determination (R^2) is about (0.87) for our following suggested regional regression model:

$$\alpha = c_1 * e^{C_2 * (\ln CA) * (\ln MAP)} \quad (5)$$

c_1, c_2: coefficients of regression, the optimized values of which are 0.0002 and 0.253, respectively.

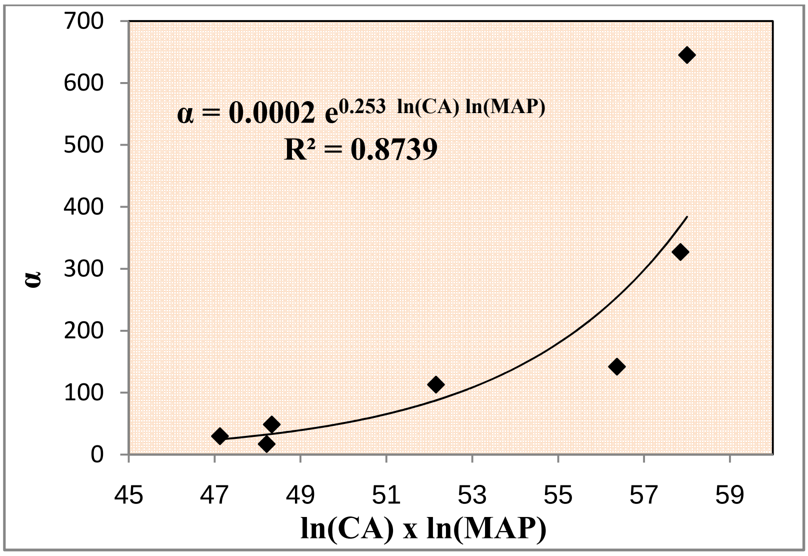

Figure (5): Relationship between parameter 'α' and 'ln (CA) x ln(MAP)'

Second Model

(MAF-PE) Model: The linear regression equation type of Mean Annual Flow **(MAF)** in m^3/s and mean annual Potential Evapotranspiration **(PE)** in mm model has good correlation with parameter 'α'. From Figure (6), the relationship between α and Mean Annual Flow which would possibly lead to a better fit is linear. The value of coefficient of determination (R^2) is about (0.88) for our following suggested regional regression model:

$$\alpha = a_1 * MAF + a_2 \quad (6)$$

a_1, a_2: coefficients of regression, the optimized values of which are 1.0019 and 5.4542, respectively.

MAF in m^3/s at any ungauged site can be calculated from catchment area in km^2 and the known mean annual potential evapotranspiration in mm and mean annual precipitation in mm by using Eq.7, then substituting the value of **MAF** in Equation 6 to calculate the value of α.

MAF = CA*(MAP- PE)/ (365*24*60*60)*10^3 (7)

The performance test of the above equation was carried out by employing it in gauged catchments under study, which gave a good agreement between the observed and predicted values of **MAF** where the correlation coefficient of the observed and predicted values of **MAF** was **(93.8%)**. After calculating the value of **α** in ungauged catchments by using Eq. (5) and Eq. (6), we substitute it in Equation (4) to predict Q_P for an ungauged catchment.

Performance Measures

In this section, for measuring the performance of the models, a combination of numerical measures and graphical plots is presented and evaluated. Dawson et al. (2007) assembled 20 performance measures used in assessing the performance of the hydrological models; several of these performance measures are mainly based on the absolute or squared error; e.g. the mean absolute error MAE and the root mean squared error RMSE. Most of these measures are designed to capture the degree of exact agreement between modelled and observed values. In this study, three statistic measures are computed, including the correlation coefficient R, MAE and RMSE as follows:

$$R = \frac{\sum(O_i-\bar{O})(P_i-\bar{P})}{\sqrt{\sum(O_i-\bar{O})^2}\ \sqrt{\sum(P_i-\bar{P})^2}} \quad (8)$$

$$MAE = \frac{\sum|O_i - P_i|}{N} \quad (9)$$

$$RMSE = \sqrt{\frac{\sum(O_i - P_i)^2}{N}} \quad (10)$$

with O observed and P predicted values of flow (perfect agreement for R = 1). The results are summarized in Table 2.

The errors and correlation observed for the Q_{95} flow was better than for the Q_{75}, Q_{50} and Q_{25} flow in both parameter estimation models. For all catchments, the performance of **(MAF-PE)** model was better than that of **(CA- MAP)** model. In all four cases, the agreement between the observed and the predicted FDCs for the **(MAF-PE)** model was greater than that of **(CA- MAP)** model.

To support the numerical measures, graphical measures are plotted; the graphical methods show how a model prediction fits the available observations. The predicted and the observed lower three-quarter sections of FDC (25%ile to the end of the curve) are plotted. These plots show better agreement between the observed and predicted values by using **(MAF-PE)** Model compared to **(CA-MAP)** Model, as shown in Figures 7 and 8.

The above criteria thus measure the extent to which the models are able to provide an accurate representation of the overall ungauged river catchments in the neighboring regions. In spite of their incompleteness, the above criteria provide a reasonable summary of the overall model performance.

Measuring the agreement between the predicted and observed flow duration curves for all gauging stations and by using both proposed models shows that predicting at the low-flow (the last part of the FDC) was accurate, particularly at Q_{95}.

Model Performance Evaluation

To identify the best performance and examine which of the two (CA-MAP) and (MAF-PE) models is more suitable for predicting FDCs in ungauged catchments at the individual gauging stations, we selected an evaluation which was not used in the calibration of the above two models. Evaluation consisted of comparing the reconstructed FDCs predicted by (CA-MAP) and (MAF-PE) methods; with the observed FDC from **(Zakho)** gauging station. This is located in khabur river basin 37º 08' N - 42º 41' E, with an elevation of 440 m and a catchment area of 3530 km^2 as shown in Fig. 9.

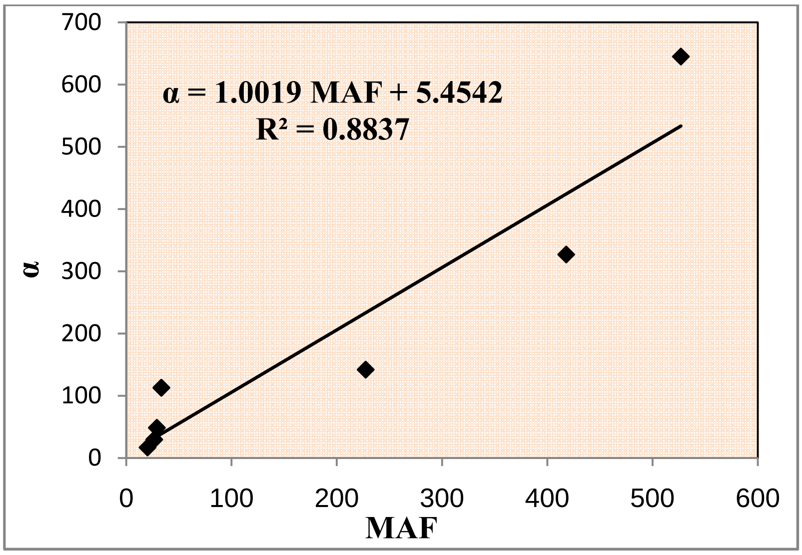

Figure (6): Relationship between parameter 'α' and 'MAF'

Table 2. Statistical comparison of model performance

(CA-MAP) Model	R	MAE	RMSE	(MAF-PE) Model	R	MAE	RMSE
Q_{25}	0.92	95.09	125.7	Q_{25}	0.972	57.10	71.40
Q_{50}	0.95	45.05	57.0	Q_{50}	0.992	17.66	24.04
Q_{75}	0.94	28.83	34.2	Q_{75}	0.989	14.93	18.68
Q_{95}	0.96	18.08	19.2	Q_{95}	0.997	14.08	14.47

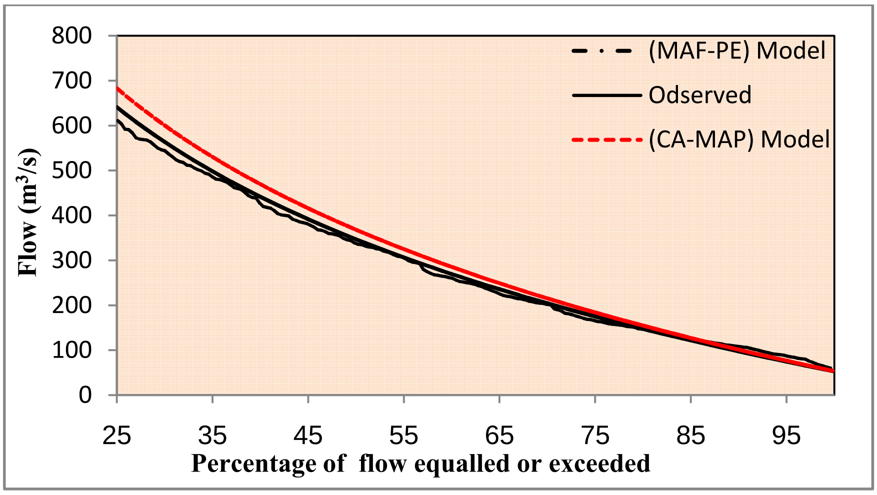

Figure (7): Observed and predicted FDC for Eske-Kelek gauging station

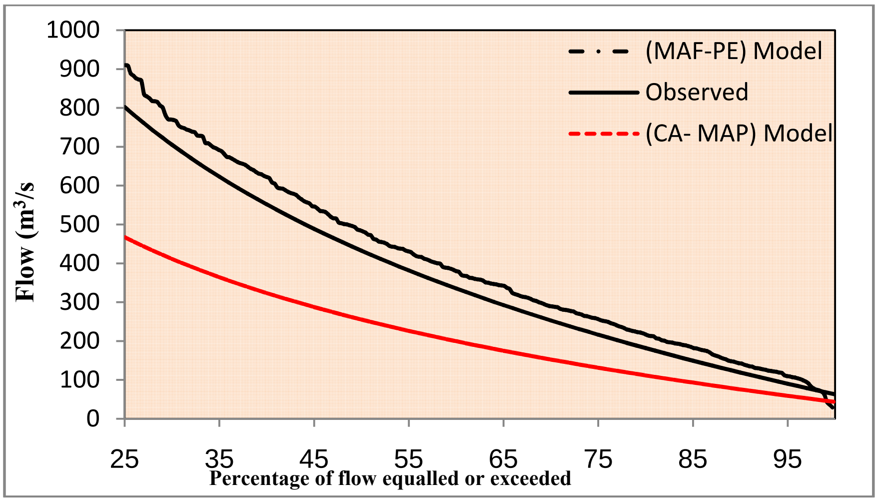

Figure (8): Observed and predicted FDC for Dokan gauging station

Table 3. Value of NE, RMSE and MAE

Method	NE	RMSE	MAE
CA-MAP	-3.1	32.2	28.2
MAF-PE	-1.6	25.5	22.4

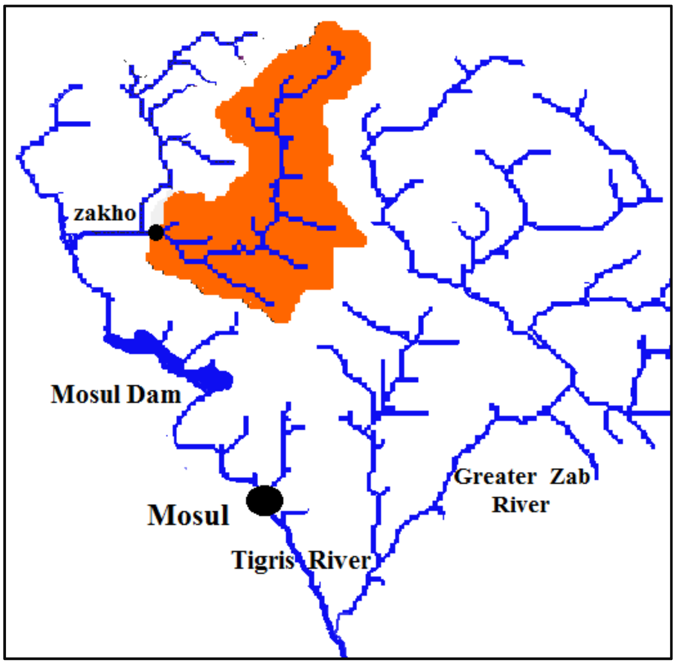

Figure (9): Location of Khabur river basin

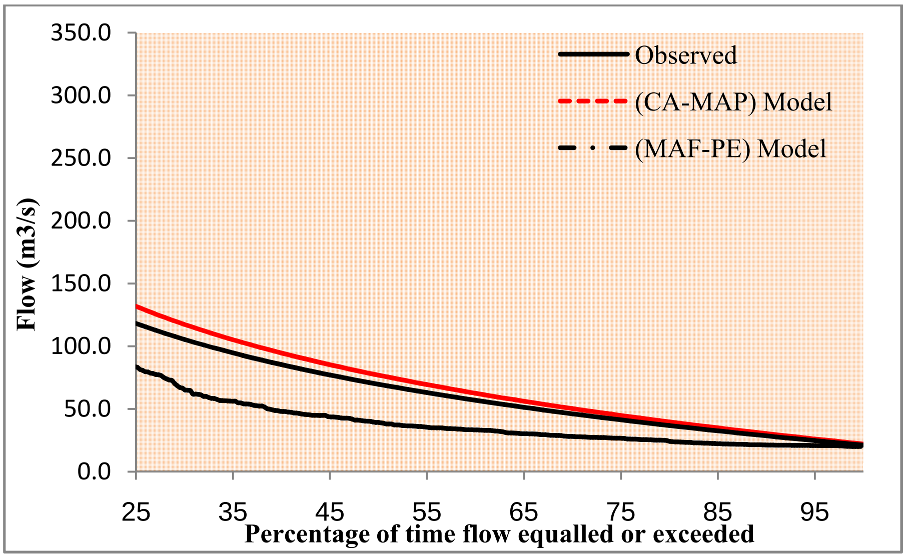

Figure (10): Observed FDC and predicted FDC for Zakho gauge station

Performance evaluation criteria, or goodness-of-fit criteria, were the Nash-Sutcliffe efficiency (NE) (Nash and Sutcliffe, 1970). The range of E lies between 1.0 (perfect fit) and $-\infty$ (Krause, 2005). The root mean square error (RMSE) and (MAE) are as reported in Table 3.

As can be seen from Table 3 and Figure 10, the predictive performance for the (MAF-PE) method was better with a closer agreement with the observed FDC compared to the (CA-MAP) method.

CONCLUSIONS

The models described in this study are simple approaches to estimate the FDCs of ungauged catchments for which monthly stream flow data are available. In the analytical approach, two performance indicators (average MAE and RMSE) were used. Table 2 presents the estimated MAE and RMSE values for the proposed FDCs from both models. In general, the agreement between the observed and predicted FDCs is reasonably good; where R is greater than 0.90 for all percentages (Q_p) of FDC. The MAF-PE (mean annual flow-potential evapotranspiration) model provides a better performance than CA-MAP (catchment area-mean annual precipitation) model in estimating FDCs. Figures 7 and 8 show good agreement between the observed and estimated FDCs. In most cases, this agreement is very good by using MAF-PE model as compared to using CA-MAP model. However, both models predict a slightly better estimation at the low-flow end of the curves, particularly at Q_{95}. To test the validity of the models they were applied on the Khabur River at Zakho gauging station. The results show that MAF-PE model is more valid than CA-MAP model.

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