

## A Notion in Modeling Concrete Members

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### ABSTRACT

In this paper, the influence of aggregate size on width of fracture process zone  $w_c$  is considered. Some researchers observed that the greater the grains of aggregate, the wider the fracture process zone (FPZ). The average value of the FPZ width taken from tests performed by Woliński was 26.6 mm and it did not depend on maximum aggregate size  $D_{max}$ . There are no consistent conclusions as to whether the width of FPZ depends on aggregate size, and there are no standard methods of FPZ width measurement. The problem arises how to choose the width of FPZ in numerical modeling of concrete structures. For example, Bažant and Oh proposed to take  $w_c = 3D_{max}$  in numerical calculations. To discuss this problem, the authors' own numerical simulations concerning bent concrete members with different widths of FPZ: 5, 10, 20, 26.5, 50 and 100 mm were performed. On the basis of the comparison of obtained results, significant differences dependent on  $w_c$  have been observed. Taking into account the minimum potential energy in a member, it can be said that the most rational thing to do is to take the smallest elongation within the localized microcracking. This condition takes place in the analyzed beam when  $w_c = 50$  mm. The assumption  $w_c = 3D_{max}$  does not fit this criterion. Also, the width from the experiment performed by Woliński is not in good relation to obtained numerical results. The main conclusion from this paper is that the width of FPZ does have an influence on obtained numerical results performed by crack band model. The problem of estimating the width of FPZ in numerical simulations exists and requires further research.

**KEYWORDS:** Fracture process zone, Concrete, FEM, Nonlinear fracture mechanics, Crack.

### INTRODUCTION

Recent advances in nonlinear fracture mechanics give a possibility to analyze crack propagation in concrete structures. There are two ways of modeling cracking using finite element analysis (FEM). In the first concept, crack is considered as densely distributed throughout the finite area of element. The alternative approach assumes an isolated sharp interelement crack. The first concept of smeared crack has practical advantages and is mostly used in numerical computations. In fracture model proposed by

Hillerborg et al. (1976), concrete fracture properties are characterized by three main parameters: axial tensile strength ( $f_{ct}$ ), fracture energy ( $G_F$ ) and shape of the stress deformation diagram given by two curves: stress-strain curve ( $\sigma$ - $\epsilon$ ) and stress-crack opening curve ( $\sigma$ - $w$ ). The decrease in stress under increasing deformation is called strain softening and it takes place in the narrow zone where the progressive microcracking appears, see Fig. 1.

The width of the microcrack band, which is called the width of fracture process zone ( $w_c$ ), is the additional parameter taken into account when fracture in concrete is modeled as a smeared crack band. There are different opinions about this parameter. Sometimes,

it is treated as concrete property, but sometimes it is assumed to be dependent on size, geometry and static scheme of the structural member. In practice, tensile concrete strength is determined by splitting tensile test and fracture energy in three-point bend test, but there is no standard method of FPZ measurement. The values of  $w_c$  obtained during experiments vary significantly due to the use of different types of tests and different

shapes and volumes of tested specimens (Tang et al., 1999). Some experiments on standardization of FPZ width test have been performed, for example (Cedolin et al., 1983; Shah, 1990), yet they have not led to finalization. The question is how to take FPZ width in numerical computations of concrete members using the crack band model. This problem is discussed in this paper.

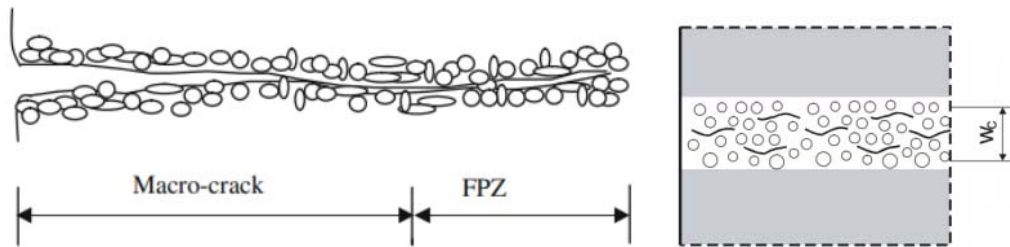


Figure (1): Fracture process zone in concrete

### STUDY OF FPZ WIDTH ESTIMATION

As there are no standard methods to determine the width of FPZ experimentally, it may be approximately calculated from the equation:

$$w_c = \frac{2G_F}{f_{ct}^2} \left( \frac{1}{E_c} - \frac{1}{E_t} \right)^{-1}$$

where  $E_c$  is the elastic modulus of concrete and  $E_t$  is the tangent softening modulus of a declining segment of the stress-strain diagram. This is just an effective width corresponding to linear stress-strain diagram and to assumed uniform strain distribution within the fracture process zone. Some authors point out that  $w_c$  is an independent material parameter, which can differ from concrete to concrete, and it depends on  $D_{max}$ . The ratio of  $w_c$  to  $D_{max}$ , presented in professional technical literature (Bažant and Oh, 1983), ranges from 1.0 to 5.0 for various kinds of concrete. Bažant and Oh (1983) came to a conclusion that the boundary of the localized cracking region should not be limited only as the boundary of visible microcracks, but as the boundary

of the whole strain softening region. In their opinion, it is generally possible to assume, in practical cases, that the optimum width of FPZ is about three-times the maximum aggregate size. The possible reason for the influence of aggregate graining on the value of fracture energy and the width of FPZ, given by (Hu and Duan, 2004), is the nonuniform distribution of local fracture energy. The presence of large size aggregates prevents the crack from opening and results in wider FPZ. Interesting experiments were performed by Otsuka and Date (2000). When comparing fracture process zone traced from X-ray films, they observed a significant influence of aggregate size on the width of the microcrack zone. The results obtained by acoustic emission technique showed the relationship between the width and the length of FPZ. With the increase of maximum aggregate size, the width of FPZ increased whereas the length of FPZ decreased. From Mihashi et al. (1991) tests, it was found that the length of FPZ was independent of heterogeneity, but the width was obviously influenced by the aggregate size. Quite different experimental results were obtained in tests performed by Woliński (Zhang and Wu, 1999). He did not find marked relationships between fracture concrete

parameters and maximum aggregate size. The obtained mean value of the width of FPZ was 26.6 mm and it did not depend on  $D_{max}$ . Some researchers, for example (Jankowski and Styś, 1990; Bažant and Planas, 1998), observed that the dimensions of FPZ were greatly influenced by the specimen size. The size effect on fracture properties of concrete was broadly described by Bažant and Planas (1998). There are no consistent conclusions as to whether the width of FPZ depends on the maximum aggregate size. The task of standardizing the testing procedure and the method of estimating the

width of fracture process zone have not been undertaken yet. Therefore, there are difficulties with performing numerical simulations of concrete structures based on crack band model of nonlinear fracture mechanics in which it is necessary to model the width of FPZ. The question arises as to how the choice of the width of fracture process zone influences the results of numerical calculations. To analyze this problem, the own numerical simulation was performed in case of concrete beams.

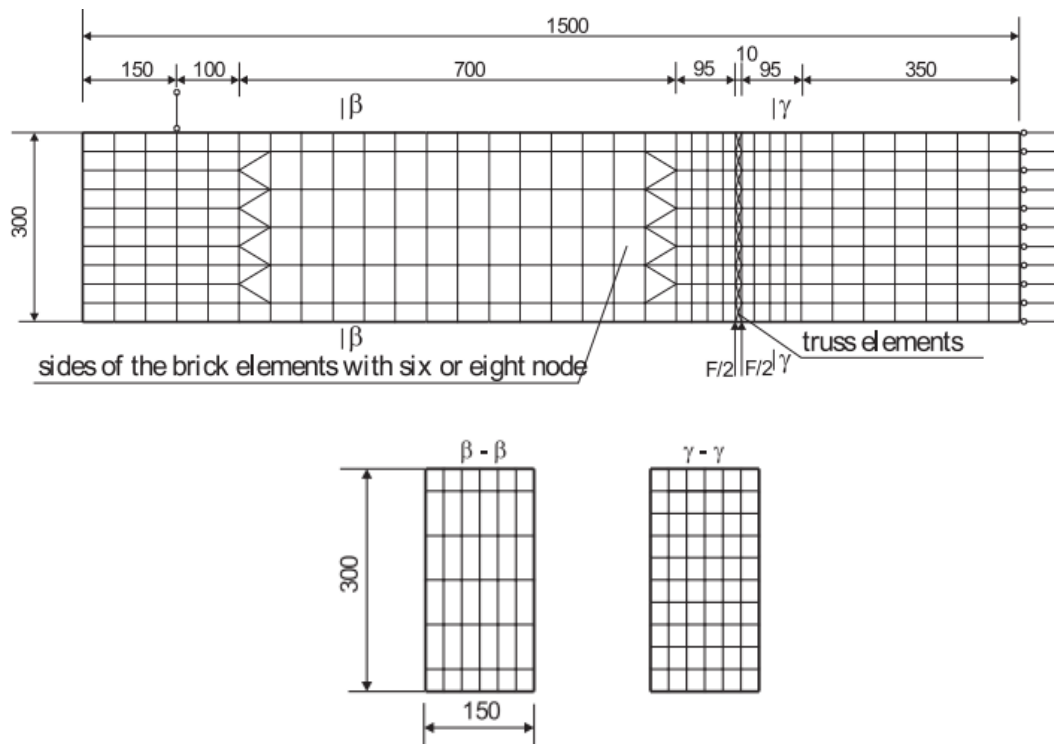


Figure (2): The FEM-mesh for a beam with  $w_c = 10$  mm

### NUMERICAL SIMULATIONS

The numerical calculations were performed using the FEM program OOFEM. A concrete beam was computed with rectangular cross-section and the following dimensions: width  $b = 0.15$  m, height  $h = 0.30$  m, total length  $L = 3.00$  m, span  $l = 2.70$  m. The specimen was unnotched. The four-point bend test was

chosen for the simulations. The beam was loaded symmetrically by two concentrated forces, which were applied from bottom towards top. The FEM analysis was performed on one half of the concrete beam since the four-point test is symmetrical. FEM beam was made by three-dimensional brick elements and truss elements. Brick elements were used in the bulk material behind the fracture process zone and truss

elements were used only in FPZ. The dimensions of brick elements in the support zone and in the region of the crack were twice as small as in the rest of the beam. The assumed FEM mesh allowed to obtain nonlinear stress distribution in FPZ. The FPZ was modeled in the region of the biggest bending moment. The biggest values of bending moment were obtained in the sections applying forces due to the fact of the reversed load scheme and the influence of weight of the beam. To analyze the influence of the FPZ width on the numerical calculation results, different widths were taken for modeling this zone:  $w_c = 5; 10; 20; 26.5; 50$  and  $100$  mm. Two of the chosen values of  $w_c$  are characteristic,  $w_c = 26.5$  mm – the value of the crack

band width obtained experimentally by Woliński (Zhang and Wu, 1999) and  $w_c = 100$  mm – the value equaling three times the maximum aggregate size as it was proposed by Bažant and Oh (1983). The finite element mesh for the analyzed beam in case of  $w_c = 10$  mm is shown in Fig. 2. While performing FEM calculations, the following material properties were used: the tensile strength  $f_{ct} = 1.5$  MPa; – the compressive strength  $f_c = 20.5$  MPa; – the modulus of elasticity  $E_{cm} = 22$  GPa; – the fracture energy  $G_F = 83$  Nm/m<sup>2</sup>; – the maximum size of aggregate  $D_{max} = 32$  mm. In the region of the fracture zone, the concrete was modeled as a nonlinear material whereas outside this zone it was modeled as a linear elastic one.

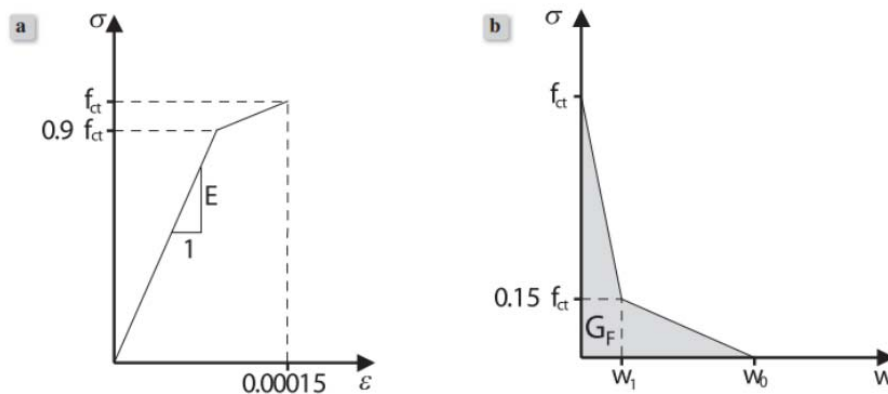


Figure (3): (a) Stress versus strain, (b) Stress versus crack opening

To describe concrete in the fracture region, the tensile concrete model given in Fig. 3. It has been shown that this model is suitable for FEM analysis of cracking in flexural members.

### ANALYSIS OF NUMERICAL RESULTS

As a result of numerical simulations, the displacements of nodes and stress components along three axes of the global coordinate system were obtained. On the basis of a comparison of numerical results for six computed concrete beams with different

widths of FPZ used in calculations, the influence of the FPZ width on obtained numerical results is discussed below. Firstly, the elongation on the base 250 mm long was calculated in all beams with different widths of FPZ. The base was situated in the tensile zone where the crack was modeled. The results of calculations in succeeding load stages were compared and they are presented in Fig. 4. Furthermore, diagrams of normal stress distribution along the height of the cross-section in FPZ for analyzed beams were made. The obtained normal stress diagrams are juxtaposed in Fig. 5 for concrete beams with different  $w_c$ . When analyzing the

diagrams presented in Fig. 4 and Fig. 5, the differences in calculation results when compared with the FPZ width used in FEM calculations are noted. In Fig. 4, we can observe that greater concrete elongations were obtained in cases of the modeled beams, where  $w_c$  was more than 20 mm. Comparing the stress distributions presented in Fig. 5, we can notice that the greater the width of fracture zone used in FEM calculation, the less intensive strain softening of tension concrete and the slower the crack formation. The confrontation of obtained results presented in Fig. 4 and Fig. 5 points out that a choice of the width of FPZ is an important parameter in numerical modeling of concrete cracking. In order to analyze the influence of the fracture zone width on numerical results more precisely, concrete strains within the modeled FPZ were calculated (see Fig. 6). On the basis of concrete strains in the fracture zone presented in Fig. 6, we can see a significant influence of  $w_c$  on the obtained results. At the beginning load levels, when  $F = 3.0$  and  $4.5$  kN, strain

values were similar. At higher load levels, concrete strains were much greater in beams with  $w_c = 5$  and  $10$  mm compared with other beams. Although differences of concrete strains for beams with  $w_c = 26.5$ ,  $50$  and  $100$  mm were not significant, it may be noticed that the smallest strain was reached in the beam with  $w_c = 50$  mm. Taking into account the minimum potential energy in a member, it may be said that the most rational thing to do is to take the smallest elongation within the localized microcracking where the crack appears. In the analyzed beams, this condition takes place when  $w_c$  is  $50$  mm. If we take  $w_c = 3D_{\max}$  as proposed in literature (Bažant and Oh, 1983) (in the analyzed beams it would be  $w_c = 100$  mm, because  $D_{\max} = 32$  mm) such an assumption does not fit this criterion. Also, the width from the experiment performed by Woliński (Zhang and Wu, 1999),  $w_c = 26.6$  mm, is not in good relation to obtained FEM calculation results.

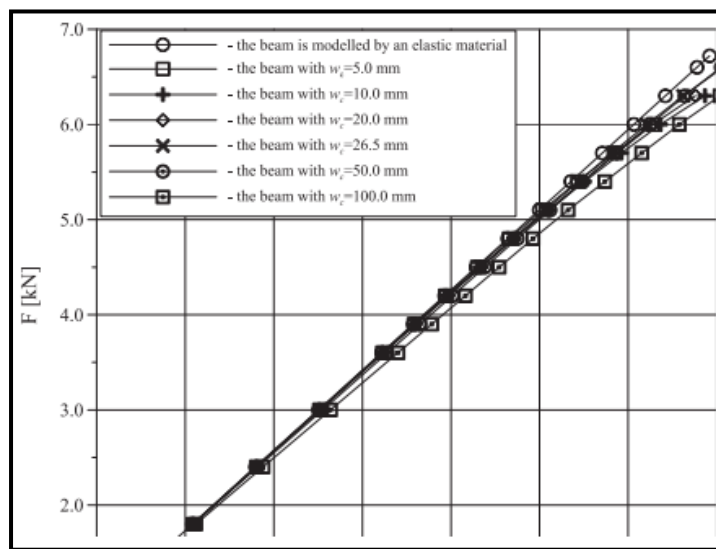


Figure (4): Comparison of the elongation for beams with different  $w_c$  values

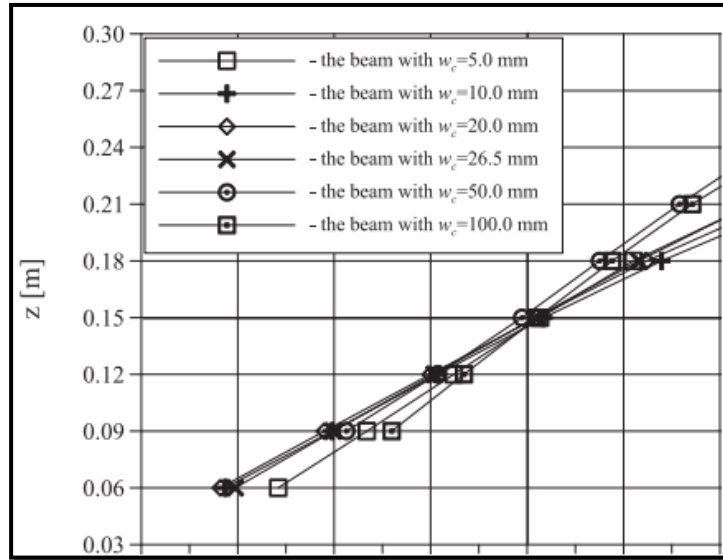


Figure (5): Comparison of normal stress distribution along the fracture zone in beams with different width  $w_c$  values at the same load stage  $F = 6$  kN

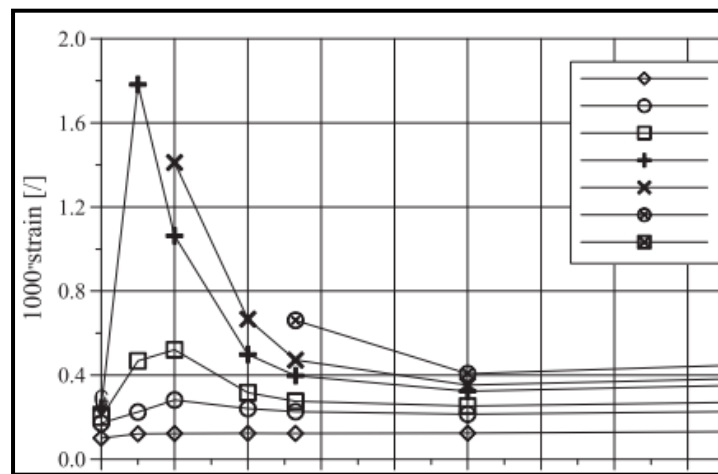


Figure (6): Concrete strain within the fracture zone at different  $w_c$  values

### CONCLUSIONS

As the width of FPZ is the additional parameter described in crack band model, it should be properly applied in numerical simulations of concrete structures. The adequate choice of this parameter during the numerical calculation is a condition of obtaining

correct results performed by finite element method (FEM). The numerical analysis presented in this paper shows that the width of FPZ is an important fracture parameter of concrete which has an influence on the FEM results. Significant differences in obtained results which were observed especially in concrete strain within the microcracked zone prove that this parameter

has an influence on FEM results. There are no definite conclusions as far as the influence of aggregate size on fracture process zone dimensions is concerned, and there are no rules how to determine a width of microcracked zone. The performed numerical simulations were not wide enough to conclude about the relation between maximum aggregate size and width of FPZ. In case of the calculated beam, the obtained numerical results did not confirm the Bažant and Oh proposition to use  $w_c = 3D_{\max}$  and they were

not in good relation to Woliński experiment in which  $w_c = 26.6$  mm. The main conclusion from the performed analysis is that the width of FPZ does have an influence on obtained numerical results performed by crack band model. The problem of estimating the width of FPZ in numerical simulations exists and requires further research. The authors plan to deal with this problem in future, in particular to elaborate the FEM results dependence on the scale effect and the type of FEM mesh used in numerical simulations.

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