Reliability-Based Design of Reinforced Concrete Raft Footings Using Finite Element Method

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ABSTRACT

In this study, a FORTRAN-based reliability-based design program was developed for the design of raft footings based on the ultimate and serviceability design requirements of BS8110 (1997). The well-known analysis of plate on elastic foundation using displacement method of analysis was used in conjunction with the design point method. The design point method was adopted for designing to a pre-determined safety level, β_T. Example of the design of a raft footing is included to demonstrate the simplicity of the procedure. It was found among other findings that there is a saving of about 64% of longitudinal reinforcement applied at the column face using the proposed method as compared with the BS8110 design method. Also, the depth of footing required using the proposed procedure was found to be 47% lower than in the deterministic method using BS8110. Also, considering a target safety index of 3.0 was found to be cheaper than considering a target safety index of 4.0 for the same loading, material and geometrical properties of the footing. It is therefore concluded that the proposed procedure is quite suitable for application.

KEYWORDS: Design, Raft footings, BS8110, Reliability, Finite Element Method (FEM).

INTRODUCTION

The aim of a design is the achievement of an acceptable probability that a structure being designed will perform satisfactorily during its intended life (BS8110, 1997). Thus, a design engineer should strive to achieve good design and be creative while at the same time considering the dangers inherent in revolutionary concepts. Ample experiences in the past and in recent times have shown that uncommon designs or unfamiliar constructional methods do increase the risk of failures (Kong and Evans, 1998).

FEM is one of the reliable numerical techniques of analysis of structural continua. The elastic continua of a plate or beam are replaced by a substitute structure of discrete elements connected together at their nodal points in such a manner that the actual continuity of stresses and displacements in the plate or beam is approximately represented by the nodal point displacements.

The basic technique of coupling soil and structure is not new (Cheung and Zienkiewicz, 1965). However, extensive investigations and improvements can lead to the development of powerful computer programs, which enable a wide range of practical raft analysis problems to be solved in a rational and comprehensive manner (Hooper, 1983).

FEM was used for calculating the structural design parameters of moments, shears and deflections of

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reinforced concrete raft foundations (El-Garhy et al., 2000). With the parameters, design of a structural element being analyzed by this method becomes easier and more friendly.

Probabilistic method has a strong logic that can be of a great help in many complex design situations. An example is the design problem involving soil-structure interaction. The dual nature of the soil (its nature can change easily from load to resistance) may make a partial factor analysis quite confusing. However, the problem can be solved in a straight forward manner using a reliability approach, even in conjunction with Finite Element Models (Vrouwenvelder, 2000).

The engineering design decisions are therefore surrounded by uncertainties that result from the random nature of the loading and structural resistance as well as the load and resistance prediction models. The effect of such uncertainties is included in the design through the use of safety factors that are based on the engineering judgement and previous experience with similar structures. Under-estimation of these uncertainties sometimes leads to adverse results like collapse such as those reported by Carino et al. (1983) and Igba (1996). In general, because of uncertainties, the question of safety and performance has arisen.

Hence, it is necessary to devote particular attention to the evaluation of the level of safety implied in the design criteria. The study of structural safety is concerned with the violation of ultimate or serviceability limit states for a structure (Melchers, 1999).

The BS8110 (1997) design criteria for reinforced concrete one-way slabs were shown, using probabilistic concepts, to be fairly consistent (Afolayan and Abubakar, 2003). Also, reliability study of strip footing with pinned column base was reported (Abubakar, 2006), and it was shown that the minimum reinforcement ratio recommended by the code is only safe at higher effective depths while at lower effective depths, reinforcement ratios between 0.3% and 0.4% are safer. The design criteria of fixed column strip footings were also investigated using reliability techniques (Abubakar, 2007), and it was however shown that the minimum reinforcement ratio of 0.2% recommended by the BS8110 for this type of footing, is only safe at higher effective depths. At lower effective depths, reinforcement ratios between 0.35% and 0.5% are safer.

Reliability aspect plays a key role in the development of a prescriptive code to a performance-based code. It is on this basis that Vrouwenvelder (2001) considered these codes from a historical perspective. On one hand, the present day codes will be compared with the allowable stress and load factor methods of the past, while on the other hand, a look into future development like full probabilistic assessment, system approach, risk analysis and the inclusion of durability and maintenance strategies will be considered.

The Joint Committee on Structural Safety (JCSS, 2001) developed the first complete model code for the probabilistic design. This code offers a general probabilistic design philosophy and a set of operational models or loads (self, wind, snow, live load,.. etc), materials (steel, concrete,...etc) and model uncertainties (for beam models, columns, plates,...etc). It is assumed that this code will be improved and extended in the years to come.

Based on the foregoing, this paper therefore proposes the reliability-based design of raft footings using FEM analysis. This approach allows the analysis of raft footings using the well-known analysis of plate on elastic foundation using displacement method of analysis of FEM.

The finite element procedure is based on a purely analytical treatment of a reinforced concrete plate on an elastic layer of soil. Design variables such as bending moments, shear forces, displacements, as well as soil pressures at each node of the footing under a static loading were determined for the design. The design variables were determined by discretizing the structure into various elements using as inputs the thickness of the footing, dimensions of the footing, soil pressure, Poisson ratio of the layer of soil and its elastic modulus, and magnitudes of concentrated loading and bending moments on each of the columns on the footing. The reliability analysis adopted in the proposed procedure is by the use of the FORM.

METHODOLOGY

Finite Element Analysis of Raft Footings

Many finite element foundation problems of considerable practical importance can be treated to the solutions of plates on elastic foundation. To simplify the inherently complex problem, the supporting medium was assumed to be isotropic, homogeneous and linearly elastic. Such a type of sub-base is called a Winkler type foundation (Vlasov, 1964). The foundation's reaction q(x, y) can be described by the following equation:

$$q(x,y) = kw \quad (1)$$

where k represents the modulus of sub-grade reaction of the foundation material (Hetenyi, 1961) and w is the deflection of the plate.

The hypothesis of linear elastic, isotropic foundation material of soils is only an approximation of the real condition; thus higher accuracy can be obtained by considering the actual elasto-plastic deformation of the soils (Selvadurai, 1979). The Winkler model has the advantages in obtaining fast solutions, sometimes analytical to more complicated soil-structure interaction problems (Yin and Huang, 2000).

Equation (1) can be solved only for relatively few combinations of the load and boundary conditions by any of the classical numerical methods (Szilard, 1974). The deflection of the plate as given by Timoshenko and Woinowsky-Krieger (1959) is given by equation (2).

In equation (2), A_m and B_m are constants of integration, γ_m and β_m are tangents of angles of inclination of the finite element to the horizontal and vertical axes, respectively, w_o is the initial deflection of the footing, and m = 1,3,5,7,………………n. D is the flexural rigidity of the footing.

The plate shown in Figure 1 rests on elastic foundation and transmits four column point loads as indicated. The plate consists of sixty equal elements, and has thirty six nodes as shown. Each node consists of two rotational vectors to the right and upward, and a downward translational vector. Each element therefore has six global degrees of freedom (DOF) at both the initial and terminal points.

The total DOF of the footing (plane grid) is 108. The total DOF becomes 104 when the nodes carrying the column loadings were considered fixed at column points, thereby limiting rotations at nodes 15, 16, 21 and 22.

$$w = \sum_{m=1,3,5,\ldots}^{\infty} \sin\frac{m\pi x}{a}\left[\frac{4kw_o}{D\pi}\frac{1}{m\left(\frac{m^4\pi^4}{a^4}+\frac{k}{D}\right)} + A_m\cosh\beta_m y\cos\gamma_m y + B_m\sinh\beta_m y\sin\gamma_m y\right] \quad (2)$$

Solution Procedure

The procedure adopted for this work was conducted by the use of the matrix displacement method. The combined beam and torsion elements have a very high degree of indeterminacy, and the compatibility conditions usually require the determination of many deformation quantities due to the applied loads and the redundant actions.

A typical member in a plane grid with nodes 1 and 2, as shown in Figure 2a, has a total of six DOF as described earlier. The internal bending moments at nodes 1 and 2 (shown in Figure 2b) are given as F_1 and F_2, while the torsional moment is given as F_3. The vertical shear forces at the two nodes are each equal to $(F_1+F_2)/L$.

From Figure 2, the equilibrium equations can be resolved as (Wang, 1970):

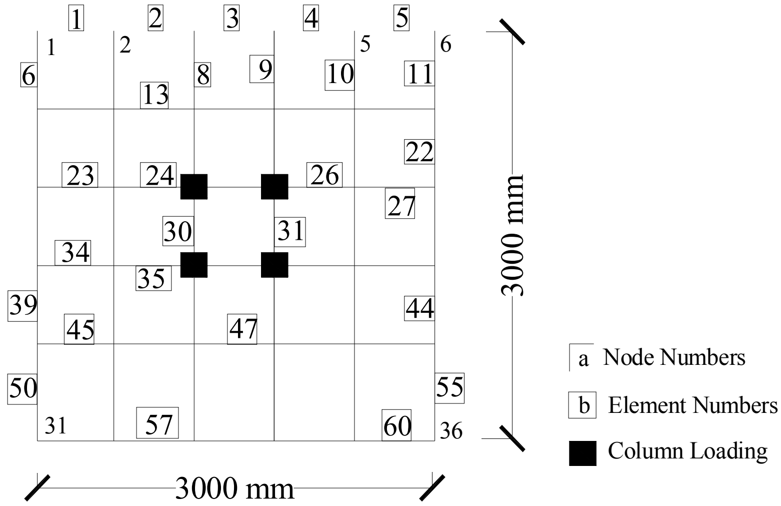
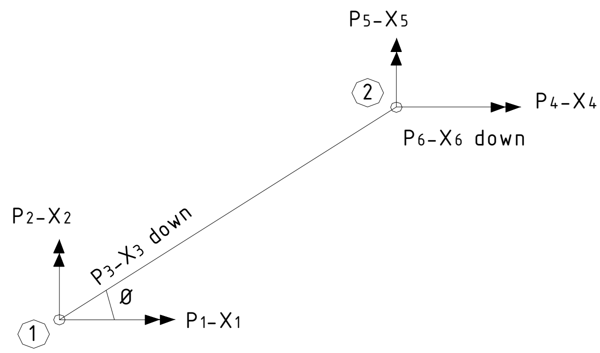
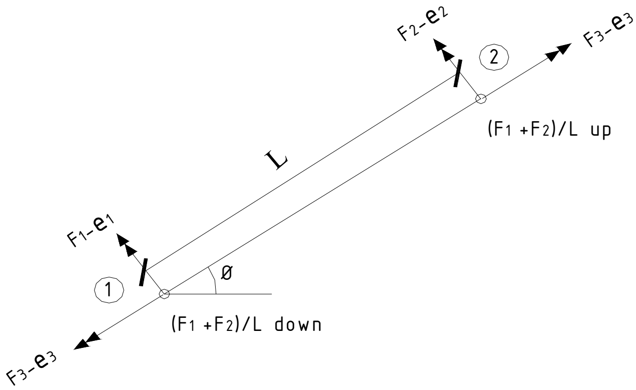


Figure (1): Plate on Elastic Foundation (Bowles, 1997)



a) External Forces (P) – Displacements (X) Diagram



b) Internal Moments (F) – Deformations (e) Diagram
Figure (2):Typical Member in Plane Grid (Wang, 1970)

$$P_1 = -F_1 \sin\phi - F_3 \cos\phi$$
$$P_2 = -F_1 \cos\phi - F_3 \sin\phi$$
$$P_3 = (F_1 + F_2)/L$$
$$P_4 = -F_2 \sin\phi + F_3 \cos\phi$$
$$P_5 = -F_2 \cos\phi + F_3 \sin\phi$$
$$P_6 = -(F_1 + F_2)/L \quad (3)$$

Also, the compatibility equations are:

$$e_1 = -X_1 \sin\phi + X_2 \cos\phi + X_3/L - X_6/L$$
$$e_2 = X_3/L - X_4 \sin\phi + X_5 \cos\phi - X_6/L$$
$$e_3 = -X_1 \cos\phi - X_2 \sin\phi + X_4 \cos\phi + X_5 \sin\phi \quad (4)$$

The stiffness matrix of the member is the matrix in which all the internal end moments are expressed in terms of all internal end rotations, it is obtained from equations (3) and (4) and given as:

$$[S] = \begin{bmatrix} s_1 & s_2 & 0 & 0 & 0 \\ s_2 & s_1 & 0 & 0 & 0 \\ 0 & 0 & s_3 & 0 & 0 \\ 0 & 0 & 0 & s_4 & 0 \\ 0 & 0 & 0 & 0 & s_4 \end{bmatrix} \quad (5)$$

where in equation (5):

$$s_1 = +\frac{4EI}{L}$$
$$s_2 = +\frac{2EI}{L}$$
$$s_3 = +\frac{GJ}{L}$$
$$s_4 = +\frac{kLB}{4} \quad (6)$$

In equation (6), E is the elastic modulus of the foundation material, I and G are its sectional and shear moduli, L and B are the length and width of the finite element, J is the torsion rigidity, and k is the modulus of sub-grade reaction.

Also, the statics matrix which is solely based on the equilibrium conditions of statics, is obtained considering equation (3) and given by:

$$[A] = \begin{bmatrix} -\sin\phi & 0 & -\cos\phi & 0 & 0 \\ \cos\phi & 0 & -\sin\phi & 0 & 0 \\ 1/L & 1/L & 0 & -1.0 & 0 \\ 0 & -\sin\phi & \cos\phi & 0 & 0 \\ 0 & \cos\phi & \sin\phi & 0 & 0 \\ 1/L & -1/L & 0 & 0 & -1.0 \end{bmatrix} \quad (7)$$

where ϕ is the angle of inclination of the finite element to the global axis. Both [S] and [A] matrices are generated for each finite element and used in the computation of the fixed end moments and displacements of the element.

The overall stiffness matrix of the structure, $[K_{ij}]$ is obtained from the stiffness matrices of the structure's elements (k_{ij}) by simple algebraic summation of the element stiffness matrices. This is given as:

$$[K_{ij}] = \Sigma k_{ij} \quad (8)$$

First Order Reliability Procedure

Probabilistic design entails the realization of acceptable probability for the designed structure to fulfil its intended purpose. In this work, the reliability method employed is briefly reviewed.

The carrying capacity, Q and the structural response, R, are both functions of design variables. A design is said to be safe when the magnitude of Q is greater than that of R. The variables are related using a performance function expressed as:

$$g(x_i) = Q - R, \quad (9)$$

The performance function can also be expressed as:

$$g(x_i) = g(x_1, x_2, ..., x_n) = 0 \quad (10)$$

where the values for **X** represent the basic design variables.

The performance function, $g(x_i)=0$ corresponds to the failure surface while $g(x_i) > 0$ corresponds to the safe region and $g(x_i) < 0$ represents the failure region.

Introducing the set of standardized variates.

$$x'_i = \frac{(X_i - \mu_{x_i})}{\sigma_{x_i}}, i = 1,2,....n \quad (11)$$

Substituting equation (11) into equation (10), we have:

$$g\,(\sigma_{xi}X'_1 + \mu_{xi}, ..., \sigma_{xn}X'_n + \mu_{xn}) = 0 \quad (12)$$

where μ and σ are the means and standard deviations of the design decision variables.

The reliability index β considering equation (12) can be obtained either using the invariant solution by (Hasofer and Lind, 1974) or using the second moment method described by (Afolayan and Nwaiwu, 2005). The reliability index β considering equation (13) can be obtained either using the invariant solution by (Hasofer and Lind, 1974) or using the second moment method described by (Afolayan and Nwaiwu, 2005). The reliability index based on the FORM model is given by:

$$\beta = \min_{x\varepsilon F} \sqrt{\left(\left(X_1'\right)^2 + \left(X_2'\right)^2 + ... + \left(X_n'\right)^2\right)} \quad (13)$$

where $X'_1, X'_2, ………., X'_n$ are the random variables in the limit state function given by G(**X**)=0.

The reliability index is obtained by minimizing equation (13) through an optimization procedure over the failure domain F corresponding to G(**X**)=0 using FORM5 (Gollwitzer et al., 1988). FORM5 is a program written in FORTRAN that can give a solution to the minimization problem by transforming correlated and non-normal variables (Gollwitzer et al., 1988), and then calculating the probability of failure, P_f using the equation:

$$P_f = \Phi(-\beta) \quad (14)$$

The reliability index can therefore be obtained from (Thoft-Christensen and Baker, 1982):

$$\beta = -\Phi\,(P_f) \quad (15)$$

where $\Phi(.)$ is the standard normal integral and β is the reliability index.

Reliability-based Design

The main objective of a reliability-based design is to ensure that the safety index of a component does not exceed the threshold level. Various methods of determining target safety index exist (Ellingwood et al., 1980; Whitman, 1984; Mortensen, 1993). A realistic interpretation of the design objective would include the implicit requirement that the safety index does not depart significantly from the threshold (Phoon, 2005).

For a design to be satisfactory, it was proposed in the current study that (JCSS, 2001):

$$\beta \approx \beta_T \quad (16)$$

In equation (16), β is the reliability index calculated using FORM5 (Gollwitzer at al., 1988) considering the values of the input design variables and β_T is the target safety index (JCSS, 2001).

PROPOSED NUMERICAL PROCEDURE

The numerical procedure adopted for solving the finite element of raft footings is by the use of the matrix displacement method as given above. The matrix displacement method adopted for the analysis of the footing which is a two-dimensional plate on elastic foundation problem, is in accordance with Bowles (1997). The raft footing was thereafter designed in accordance with BS8110 (1997). FORM5 (Gollwitzer et al., 1988) was adopted for the reliability analysis of the designed section of the raft, which was built into the design program. Design is said to be satisfactory when equation (16) is satisfied. The flowchart of the program is as shown in Figure 3. The detail of the procedure followed is therefore given as:

a) Considering a finite element of dimension L, the internal joint moments [F] due to applied loads and moments is calculated from the equation:

$$[F] = [S].[e] \quad (17)$$

In equation (17), [e] is the matrix of the end rotations of the finite elements and [S] was calculated using equation (5).

b) Again, considering the finite element, the statics matrix [A] was determined using equation (7).
c) Other transition matrices were computed. Reliability– based finite element design of the entire footing consisting of N elements has the following procedure:
   i) Sum of the elemental stiffness matrices to external stiffness matrix of the total elements in the structure using equation (8).
   ii) Compilation of the external joint moments and shears due to any applied loading to form the [P] matrix as:
   $$[P] = [ASA^T].[X] \quad (18)$$
   iii) Computation of the external displacement matrix, [X]as:
   $$[X]=[ASA^T]^{-1}.[P] \quad (19)$$
   iv) Finally, the mid-span moments acting on each segment were obtained using equation (17), by applying the laws of statics.
   v) Using the optimum results obtained in (iv) above, the footing was designed in accordance with the design requirements of BS8110.
   vi) The implied safety of the designed section was obtained using FORM (Gollwitzer et al., 1988) built into the design program.
   vii)Final check using equation (16) was carried out. A design was considered satisfactory if equation (16) was satisfied; else the procedure was repeated for varying values of the design variables until equation (16) was satisfied.

Performance Functions

Bending Moment Failure Modes

The calculation of the performance function is performed for discrete combination of basic variables into the bending moment failure mode for a strip width of the raft footing in accordance with BS8110 (1997), as given by:

$$G(X) = 0.9 f_y \rho L d^2 [1 - 1.0556 f_y \frac{\rho}{f_{cu}}] - 1.6L\,[0.875\alpha + 1] FN + M_A \quad (20)$$

Shear Failure Modes

The performance function considering the shear failure mode for a strip width of the raft footing in accordance with BS8110 (1997) is given by:

$$G(X) = \frac{0.79}{\gamma}\left[\left(\frac{100 A_S}{bd}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4}\right] - q_u\left(LS_{coeff} - d - h/2\right) \quad (21)$$

In equations (20) and (21), f_y is the characteristic strength of the reinforcing tension steel, ρ is the reinforcement ratio of the designed section, L is the effective span, b is the strip width of the footing, d is the effective depth of the section, h is the depth of column section, A_s is the area of longitudinal tension steel provided, q_u is the bearing pressure at Ultimate Limit State (ULS), S_{coeff} is the shear stress coefficient obtainable from BS8110 (1997), f_{cu} is the characteristic strength of concrete, α (Alpha) is the ratio of dead-to-live loads, M_A is the magnitude of the factored applied column moment and FN is a moment coefficient. In the case at hand, values of FN for both span and column face reinforcements were selected from the BS8110.

Design Data

It is required to analyze and design a raft footing shown in Figure 4. Assuming that f_{cu} and f_y are 30 MPa and 460 MPa, respectively, and concrete cover is 50mm.

The dimensions of the raft are 3m x 3m, where the modulus of elasticity of concrete is 22 400 MPa, the allowable soil pressure is 190 kN/m^2, the shear modulus is 9740 MPa, the Poisson's ratio is 0.15, and the maximum allowable settlement of the raft is 50mm.

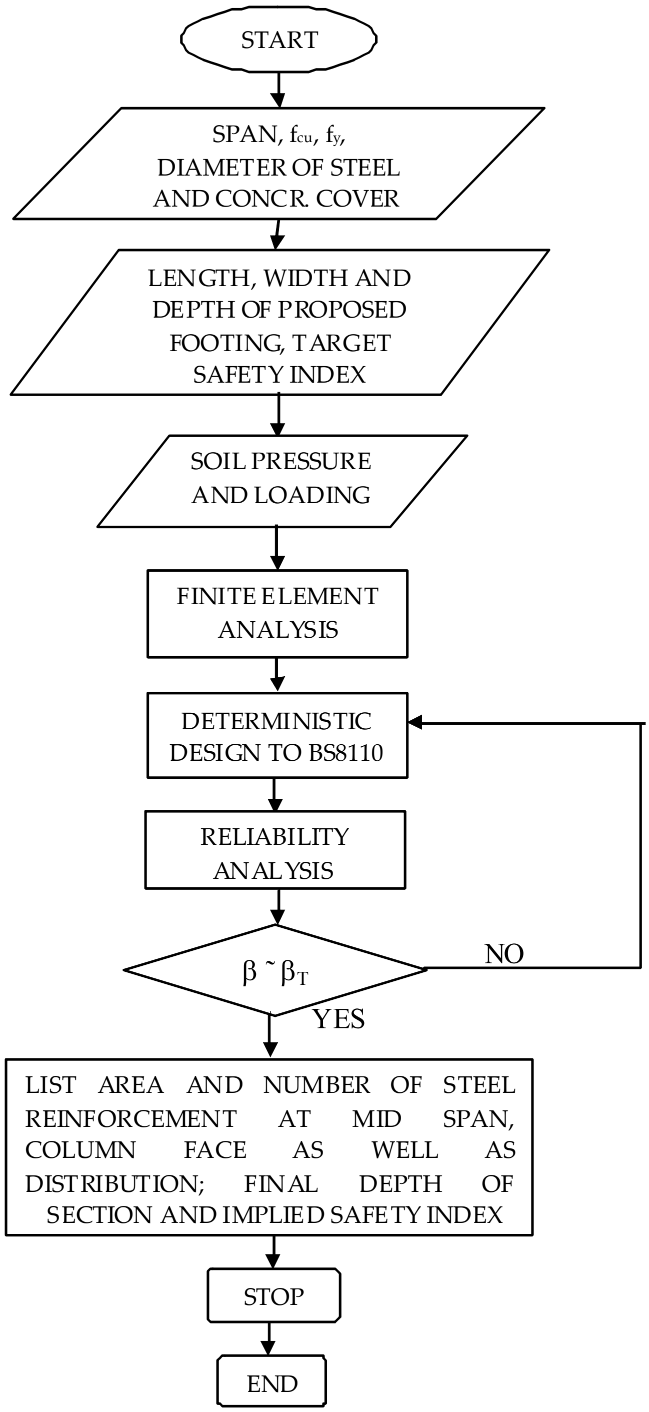


Figure (3): Program Flowchart

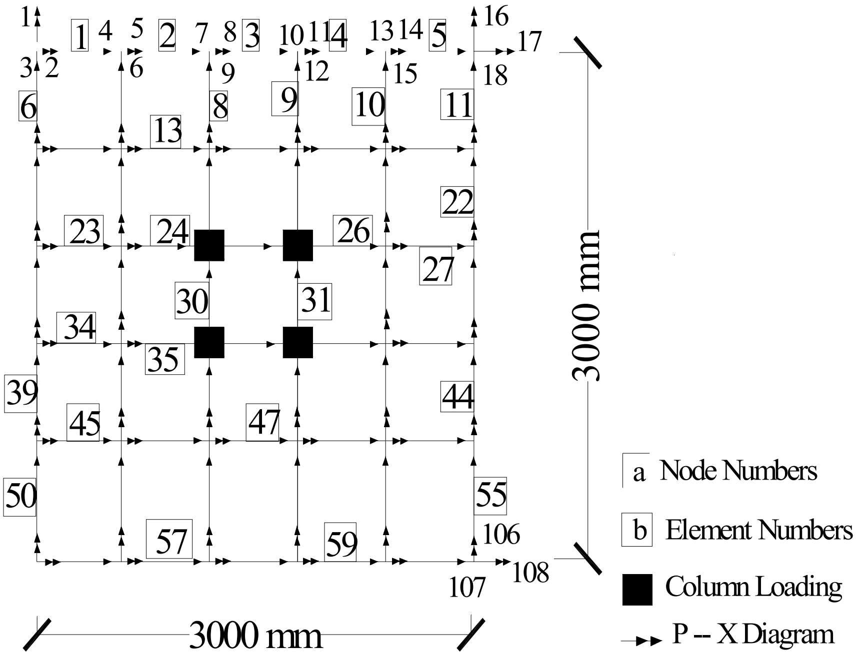


Figure (4): Raft Footing

For analysis purpose, the footing was divided with equal grids of 0.6m and with P – X diagram as shown in Figure 4. The total applied axial column load is 550 kN located at the middle of the raft.

Stochastic Model

The stochastic model considering the limit state functions given in the previous section was prepared in accordance with FORM5 (Gollwitzer et al., 1988). This is as presented in Table 1. Coefficients of variation were obtained from (Phoon, 2005).

RESULTS

Results of Proposed Procedure

The developed program was used to carry out the analysis with the optimum design variables automatically selected by the program for the design. The program displays design results in less than 5 seconds on a Pentium III personal computer.

The BS8110 (1997) design procedure was adopted for the design of the footing. In addition, the safety procedure of FORM was adopted. The execution of the program considering the given loading gave the design solutions as shown in Table 2. The symbol 'T' in the Table signifies the type of deformed high yield reinforcement produced in accordance with BS4449 (1985) as set out in BS8110 (1997).

Results of Deterministic Design

The BS8110 requires that raft footings are designed for the serviceability limit state of deflection and cracking due to shrinkage, and the ULS of bending. In addition, raft slabs with concentrated loads need to be designed for ULS of shear. Following the BS8110 (1997) procedure for the design of raft footing, the results obtained are as shown in Table 2. All design checks were adequate.

Table 1. Stochastic Model Considering Failure Criteria of the Raft Footing

S.No.	Basic Variable	Distribution Type	Mean	Coefficient of Variation	Standard Deviation
1	Steel Strength, f_y	Normal	460 N/mm^2	0.015	6.9 N/mm^2
2	Rho(ρ)	Lognormal	0.0018	0.01	1.8 x 10^5
3	Span of Footing, L	Normal	3000	0.01	300 mm
4	Footing Effective depth, d	Normal	750 mm	0.01	7.5 mm
	Width of Footing, b	Normal	3000 mm	0.01	30 mm
5	Effective Depth, d	Normal	690 mm	0.01	6.9 mm
6	Concrete Strength, f_{cu}	Lognormal	30 N/mm^2	0.015	0.45 N/mm^2
7	Imposed Load	Lognormal	2200kN	0.065	143kN
8	Area of Longitudinal Reinforcement Provided, A_S	Lognormal	2830 mm^2	0.015	42.45 mm^2
9	Unit Weight of Soil, γ	Lognormal	18.9kN/m^3	0.065	1.23 kN/m^3
10	Bearing Pressure at ULS, q_u	Lognormal	233.54kN/m^2	0.065	15.18kN/m^2
11	Depth of Column, h	Normal	300 mm	0.01	3 mm

Table 2. Results of Design of Raft Footing

Design Details	Deterministic Design		Probabilistic Design
Longitudinal Reinforcement	Column Face	9T20 (2830 mm^2)	10T12 (1020 mm^2)
	Span	9T20 (2510 mm^2)	9T20 (2510 mm^2)
Transverse Reinforcement	T20 @ 300 mm c/c (1050 mm^2/m)		T12 @ 275mm c/c (1020 mm^2/m)
Torsion Reinforcement	No Provision		T12 @ 200mm c/c (792 mm^2/m)
Final Depth of Section (mm)	750		395
Prob. of Failure	1 x 10^{-6}(1.86 x 10^{-2})		2.64 x 10^{-3}
Safety Index	6.13		2.790

Table 3. Probabilistic Design of a Raft Footing at Varying Reliability Levels

Design Details	$\beta_T = 3.0$		$\beta_T = 4.0$
Longitudinal Reinforcement	Column Face	10 T12 (1020 mm^2)	10T12 (1020 mm^2)
	Span	9T20 (2510 mm^2)	10T20 (2789 mm^2)
Transverse Reinforcement	T12 @ 175 mm c/c (1020 mm^2/m)		T12 @ 175mm c/c (1020 mm^2/m)
Torsion Reinforcement	T12 @ 200 mm c/c (792 mm^2/m)		T12 @ 275mm c/c (679 mm^2/m)
Final Depth of Section (mm)	395		425
Prob. of Failure	2.64 x 10^{-3}		6.58 x 10^{-5}
Safety Index	2.790		3.824

DISCUSSION OF RESULTS

Based on JCSS (2001), a target safety index of 3.0 was assumed and the following observations were made:

1) The probabilistic method of design as proposed in this work gives a final depth of concrete section of 395 mm; while the deterministic method of design gives a 750 mm thick section. There is, therefore, a difference of 355 mm of the overall depth of concrete. This gives about (47%) savings.
2) Again, there is about (64%) discount in the longitudinal reinforcement applied at the column face.
3) Span longitudinal reinforcements obtained from the design methods are the same.
4) Transverse reinforcement in the proposed method is about (3%) cheaper than in the deterministic method. There is therefore no significant difference between the design methods considering the magnitudes of the transverse reinforcements.
5) Torsional reinforcement of $792 mm^2/m$ is obtained in the probabilistic design method. On the other hand, there is no provision of torsional reinforcement in the deterministic method.
6) Also, safety index in the proposed method falls within the values recommended by JCSS (2001). The safety level associated with the BS8110 design on the other hand falls within the range. Therefore, BS8110 (1997) seems uneconomical with respect to design of raft footings. (Values of probabilities of failure in brackets indicate the design probabilities of failure using the deterministic code, which is about 10000 times bigger than the assumed deterministic-based value of 1×10^{-6}).
7) Based on item (6) above, the implied safety indices of the deterministic design methods gave design solutions that are not economical considering the values recommended by JCSS (2001).
8) Generally, the proposed probabilistic method is cheaper than the two deterministic design methods. Also, the designer is assured that, with the use of the proposed method, the design has undergone and satisfied the design requirements of BS8110 (1997), as well as safety index format, using FORM.

Effect of Variation of Target Safety Index

It is required to compare the design of the raft footing considered in the example given above at target safety levels of 3.0 and 4.0. The results obtained from the program considering the two safety indices are as shown in Table 3. It is shown that:

1) Longitudinal reinforcements at the column face are the same for the two safety levels;
2) Longitudinal span reinforcements differ by 9% with the safety level of 4.0 having the higher value;
3) Transverse reinforcements are the same for the two safety levels considered;
4) However, torsional reinforcement is higher when $\beta_T = 3.0$. The reinforcements differ by 113 mm^2;
5) Overall depth of section is lower at a target safety level of 3.0. There is a difference of 30 mm concrete.
6) The results obtained considering a target safety index of 3.0 are cheaper than those of target safety index of 4.0. This justifies the assertion of Vrounwenvelder (2001) in which, in a rational reliability analysis, the target safety index which is considered as a control parameter, assigns a particular investment to the material placed in the structure. The more material invested in the right places, the less is the expected loss.

CONCLUSION

Reliability-based design of raft footings using displacement method of finite element analysis was presented with the aid of a computer program in FORTRAN. The finite element analysis was used with the BS8110 (1997) design criteria of raft footings founded on soils of known bearing capacity. In addition, the FORM was strictly followed in the program. It was found among other findings that there was a saving of about 64% of longitudinal reinforcement applied at the column face using the proposed method as compared with the BS8110 (1997) design method. Also, the depth of footing required using the proposed procedure was found to be 47% lower than in the deterministic method using BS8110. Finally, the results obtained considering a target safety index of 3.0 are cheaper than those of target safety index of 4.0 for the same loading and geometrical arrangements of the raft footing.

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