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Determinants of Students' Logical Reasoning and Mathematics Achievement

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Abstract
This study examined the determinants of students' logical reasoning and mathematics achievement. Three factors; age, sex and class level were viewed as determinants of students' logical reasoning. Ex-post-facto research design was used for the study and the sample size was 420 senior secondary school students in Olamaboro Local Government Area of Kogi State-Nigeria. Three research questions were raised and three hypotheses also stated and tested at 0.05 level of significance. The instrument used for data collection was "Mathematical Reasoning Test (MRT)" developed by the researchers and validated by two mathematics Educators and one measurement and evaluation lecturer. The reliability of the instrument was determined using inter-rater approach and kendall's coefficient of concordance statistic gave .84. Data was analysed using t-test and analysis of variance. The findings revealed among others that; age and class levels determine students' logical reasoning in mathematics. It was recommended that age and class level should be given serious recognition in planning and organizing the mathematics curriculum.

INTRODUCTION

Mathematics is recognized as an instrument of education which strengthens the power of attention, develop the sense of order and of construction. It is an activity which requires all arms on desk for desired outcome. It also requires problem solving which is the most important strand in mathematics. Some problems in mathematics involve single step, two step, and multi-step towards getting their solution. For instance, the translation of words problem into mathematics and the skill of solving such problem require multi-step which is crucial and involved critical thinking. Developing critical thinking is an essential goal of mathematics education. The logical thinking and mathematical reasoning used to solve multi-step mathematics problems develops the critical thinking necessary to face life's more complex situations. Logical reasoning holds everything together in most science subjects and mathematics in particular. Thus, logical reasoning facilitates students' ability towards solving most problems in mathematics correctly. The ability to manipulate these mathematics concepts has been identified with Piaget's stage of concrete operation, which theoretically emerges from ages 11 and above years.

Studies have shown that age has positive effect on students' achievement and that, the use of abstraction increase with increasing age (MarryCross, 2004; Ohuche & Otada 2002). Also, Fajemidagha and Copeland (2003) assert that, the chronological age is designated and specified for each of the students, is an interaction of cognitive skills and experiences used in the performance of logical reasoning of thought (Adebe, 2001). To better understand student learning obstacles and cultivate creative thinking ability, teachers need to assess student solution procedures in detail, especially the multiple representations for their solutions, including formulas, graphs and language. This will help the teachers to determine if students misunderstand a certain concept or are stuck at a specific point. Teachers can then provide more effective guidance to students taking their age into consideration.

Another variable that seems to determine students' logical reasoning is gender. Brandy and Eisten (1995) had shown that there is a considerable inconsistency in the literature as to the nature, extent and sources of bias in the differential performances between boys and girls in mathematics. The authors noted that with the inconsistent findings and significant methodological flaws observed, more empirical researches are needed to investigate the existence of gender bias in the classroom. This is probably why Kadiri (2004) noted that available literatures have not been able to identify a single direction of difference in performance in mathematics between male and female students subject to the inequalities in their physiological structures. Findings of some research studies have reported no statistical difference in the performance of boys and girls in mathematical tasks, while some reported differences in favour of the female and vice-versa. For example Jahun (1989), Smith and Walker (1988) indicated differences in performance in favour of the females, while Salman (1997), Aiyedun (2000), Jahun and Momoh (2001), Abiam and Odok (2006) reported non-significant statistical difference in the performance of male and female students in mathematical tasks. Adeleke (2007) equally observed that, problem solving performance of students is not sensitive to gender. Also, Reyes and Stanic (1988) reported that male students achieved a higher level in mathematics. However, there is still need to investigate the way male and

female students' reason mathematically. This study therefore aimed to explore the interplay between the determinants of students' logical reasoning and mathematics achievement.

Purpose of the Study

The main purpose of this study was to ascertain the determinants of students' logical reasoning in mathematics. Specifically this study hopes to:

1. Determine the extent age influences students' logical reasoning in mathematics.
2. Determine how male logical reasoning differs from their female counterparts in mathematics.
3. Determine the extent class level influences students' logical reasoning in mathematics.

Research Questions

1. What is the achievement mean score of students based on their age in mathematical reasoning test?
2. What is the achievement mean score of students based on their sex in mathematical reasoning test?
3. What is the achievement mean score of students based on their class level in mathematical reasoning test?

Hypothesis

H_{o1} There is no significant differences among the achievement mean score of students based on their age in mathematical reasoning test.

.**H_{o2}**. There is no significant difference between the achievement mean score of male and female students in mathematical reasoning test.

H_{o3}. There is no significant differences among the achievement mean score of students based on their class level in mathematical reasoning test.

Methods

The research design for this study is ex-post-facto research design. This design is appropriate because the researcher is only linking age, sex, and class level to logical reasoning and achievement in mathematics. The study was carried in Olamaboro Local Government of Kogi State. The sample for this study was 420 senior secondary students, 140 each of SSI, II and III. Simple random sampling technique was used to draw seven (7) schools from 21 schools and four hundred and twenty (420) SS students from the 7 schools. The instrument for data collection was Mathematical Reasoning Test (MRT) designed by the researchers. The reason for using MRT is to prevent guess work and to see logical and illogical sequence followed by the students to solve mathematical problems. The instrument is made up of ten (10) essay questions with five steps each to arrive at the correct answer. The Mathematical Reasoning Test (MRT) was designed by the researchers in line with the students' curriculum in senior secondary schools. The instrument was validated by two mathematics Educators and one measurement and evaluation lecturer. Test-retest method was used to determine the reliability of the instrument. The reliability coefficient was found to be 0.84 using Pearson product moment correlation method.

The researchers personally administered the test to senior secondary students in all seven (7) public secondary schools that were randomly drawn. All the papers were retrieved at the spot and there was 100% return rate. In the analysis, students' correct solving step (CSS) was taken as their logical reasoning while incorrect solving steps (ICSS) was taken as illogical reasoning. Each question in the instrument has five steps to arrive at the final solution, and carries five (5) marks. When a step is correctly solved, a mark is assigned to that step otherwise the mark is award to incorrect solving. The data collected were analyzed using descriptive statistics, t-test statistic and Analysis of variance (ANOVA). The descriptive statistics was used to answer the research questions while t-test and ANOVA were used to test the null hypothesis at 0.05 level of significance.

Results

Table 1: Mean ($\overline{X}$) and standard deviation (SD) of students' logical reasoning in mathematics according to age.

	Variables	**N**	$\overline{X}$	**SD**
TCSS or logical reasoning	14 years and below	80	23.25	6.26
	(15-17) years	177	28.59	7.65
	Above 17 years	163	28.24	7.97
	Total	420	27.44	7.79
TICSS or illogical reasoning	14 years and below	80	26.94	6.42
	(15-17) years	177	21.37	7.45
	Above 17 years	163	21.83	7.88
	Total	420	22.61	7.72

Where TCSS = Total correct solving steps or (logical reasoning);
TICSS = Total incorrect solving steps or (illogical reasoning)

With regard to correct solving steps (logical reasoning), table 1 shows that, 80 students whose age is 14 years and below had a mean ($\overline{X}$) of 23.25, 177 students whose age is between (15-17) years had a mean of 28.59 and

163 students whose age is above 17 years has a mean of 28.24. However, in the students total incorrect solving steps (illogical reasoning) the mean ($\overline{X}$) for the three age categories are; $\overline{X}$ = 26.94, $\overline{X}$ = 21.37and $\overline{X}$ = 21.83 respectively. From this result, students whose ages are (15-17) years and above 17 years have similar logical reasoning in mathematics. The students in these age categories have higher logical reasoning in maths than those whose age is 14 years and below.

Table 2: Mean and Standard Deviation of Students' logical reasoning in Mathematics according to sex.

	Sex	**N**	$\overline{X}$	**SD**
TCSS or logical reasoning	Male	210	27.48	7.98
	Female	210	27.39	7.62
TICSS or (illogical reasoning)	Male	210	22.65	7.95
	Female	210	22.57	7.49

From table 2, the mean score of male students was 27.48 while that of the female students was 27.39 in their total correct solving steps (logical reasoning). However, in the students total incorrect solving steps, (illogical reasoning) the mean of the male students is $\overline{X}$ 22.65 whereas that of their female counterparts is $\overline{X}$ = 22.57. From this result, both male and female students have similar logical and illogical reasoning in mathematics.

Table 3: Mean and Standard Deviation of students' logical reasoning in mathematics according to class level

	Class level	**N**	$\overline{X}$	**SD**
TCSS or Logical Reasoning (LR)	SS I	140	22.48	5.85
	SS II	140	27.78	7.12
	SS III	140	32.05	7.12
	Total	420	27.44	7.79
TICSS or Illogical Reasoning	SS I	140	27.49	5.95
	SS II	140	22.29	6.79
	SS III	140	18.05	7.27
	Total	420	22.61	7.72

From table 3, SSI students had a mean ($\overline{X}$) = 22.48, SSII students had mean ($\overline{X}$) = 27.78 and SSIII students had mean ($\overline{X}$) = 32.05 in their total correct solving steps (TCSS) or Logical Reasoning (LR) From this result students whose class level are SSIII have higher logical reasoning in mathematics than those whose class level are SSII and SSI. However, in the students total incorrect solving steps (TICSS) or Illogical Reasoning (ILR), the mean ($\overline{X}$) for the three class levels (SSI, SSII and SSIII) are: SSI, $\overline{X}$ = 27.49; SSI, $\overline{X}$ = 22.29; and SSIII $\overline{X}$ = 18.05 respectively.

Hypothesis 1

Table 4: Analysis of variance (ANOVA) of students' logical reasoning based on age

	Group	**Sum of Square**	**df**	**Mean Square**	**F**	**Sig**	**Decision**
TCSS or Logical Reasoning (LR)	Between groups	1741.70	2	870.85	15.32	0.000	S
	Within groups	23701.56	417	56.84			
	Total	25443.26	419				
TICSS or Illogical Reasoning	Between groups	1869.62	2	934.81	16.89	0.000	S
	Within groups	23076.35	417	55.34			
	Total	24945.96	419				

The result presented in Table 4 shows students logical reasoning based on age. The total correct solving step points and total incorrect solving step point were used for the analysis. As indicated in the table, the f-ratio for total correct solving steps was obtained as 15.32 with 0.000 level of significance. Also, for the total incorrect solving steps, the f-ratio was 16.89 with 0.000 level of significance. In both cases, the probability level (0.000) obtained was less than the level of significance (0.05) set by the researcher. Therefore, the hypothesis that age will not have significant influence among students in their logical reasoning in mathematics was not upheld. This implies age have significance on students logical reasoning in mathematics.

Hypotheses 2

Table 5: t-test comparison of mean scores of male and female students' logical and illogical reasoning.

	Sex	N	$\overline{X}$	SD	df	t_{cal}	t_{crit}	Decision
TCSS or Logical Reasoning (LR)	Male	210	27.48	7.98	418	.12	.91	NS
	Female	210	27.39	7.62				
TICSS or Illogical Reasoning (ILR)	Male	210	22.65	7.95	418	.10	.92	NS
	Female	210	22.57	7.49				

Table 6 shows that independent t-test analysis of students logical reasoning in mathematics based on sex. The table indicated that for total correct solving steps (TCSS) or Logical Reasoning (LR) points the t-calculated is 0.12 with 0.910 probability level, also for total incorrect solving step (TICSS) or Illogical Reasoning (ILR) the value of t=0.10 with 0.92 probability level was obtained. The probability value for TCSS Logical Reasoning (LR) and TICSS Illogical Reasoning (ILR) was then compared with 0.05 level of significance set by the researchers and the result was found not significant. The hypothesis that, sex will not have significant influence on student's logical reasoning in mathematics was upheld.

Hypothesis 3

Table 6: Analysis of variance (ANOVA) of students' logical Reasoning in mathematics based on class level.

	Class	Sum of Square	df	Mean Square	F	Sig	Decision
TCSS or Logical Reasoning (LR)	Between Groups	.6437.54	2	3218.77	70.62	0.000	S
	Within groups	19005.72	417	45.58			
	Total	25443.25	419				
TICSS or Illogical Reasoning (ILR)	Between Group	6252.35	2	3126.67	69.75	0.000	S
	Within Group	186992.61	417	44.83			
	Total	24945.96	419				

$\alpha = 0.05$

Analysis of data in table 10 revealed that, F-computed from TCSS or Logical Reasoning (LR) and TICSS or Illogical Reasoning (ILR) are 70.62 and 69.75 respectively. The associated probability levels are 0.000 and 0.000 respectively. These probability levels were compared with 0.05 level of significance set by the researchers and there were found to be less than 0.05. Based on this, the null hypothesis which states that, class level has no significance influence among students in their logical reasoning in mathematics was not upheld. This implies that class level significantly influence students' logical reasoning in mathematics.

Discussion

The finding of this study revealed that; age and class level have significant influence on students in their logical reasoning in mathematics while sex, have no significant influence on students logical reasoning in mathematics. The testing of null hypothesis one established that age has a significant influence in students' logical reasoning in mathematics. The analysis showed that students of age 15 and above displayed identical logical reasoning faculty which was significantly different from those of age 14 and below. Although this study was not design to establish the stages of students' logical reasoning, it seems to support Piaget idea that human mental structure development are in stages and it's a function of age. The findings of this study is in line with that of MarryCross (2004) and Ohuche and Otada (2002) who noted that age has positive effect on students' achievement with abstraction increasing as age increases.

Table 3 and 4 revealed that sex has no significant influence on students' logical reasoning in mathematics. The finding is in sharp contrast to the works of Jahun (1989), Smith and Walker (1988) who found difference in performance in favour of female. The finding of the study is also in disagreement with that of Reyer and Stanic (1988) who reported that male students achieve higher in mathematics than female. The finding of this study is in consonant with that of Jahun and Momoh (2001), and Abiam and Odok (2004) who reported non-significant difference between male and female mathematical task

However, rejecting null hypothesis five which indicates that class level significantly influence students logical reasoning in mathematics can be said to be in agreement to the work of MaryCross (2004) who studied the effect of mastery learning strategy in secondary school students' cognitive achievement in mathematics. Her study revealed that the mastery learning strategy has a positive effect on students' achievement in mathematics. These findings are in compromise because the various class levels seem to describe the student level of mastery. Students don't cross from one class level to another without showing some evidence of mastery of the content of

the previous levels.

The findings of this study call on the curriculum planner and teachers to bear in mind the influence of age and class level on the content of the mathematics curriculum. The organisation of this content into syllabus, scheme of work and unit of work for the various class levels should consider the age limit of those at that level. The finding of this study has shown that logical reasoning is not gender sensitive as such, students are expected to perform equally irrespective of their gender.To the teachers, the findings of this study calls on the mathematics teachers to adopt instructional methods and strategies that will give students equal learning attention rather than focusing on a particular gender or sets of students with certain attractive background. Special attention should be given to students based on individual weaknesses rather than on general factors as sex and parental background or status.

The findings of this study seem to be drawing the attention of the school administrators that much cares is required in the organisation of learning activities into school levels and the implementation of policies in this regard. Student should not be promoted unnecessary from one level to the other without meeting the necessary requirements. Student should not be allowed to jump to a higher class without considering their ages and mastery of the content of previous class.

Conclusion and Recommendations

Based on the findings of this study, it is recommended that:.

- Students' age should be given serious recognition in planning and organising the mathematics curriculum and proper attention should be taken to ensure that the implementation of the mathematics curriculum is in conformity with the expected age range of the students to make teaching and learning of mathematics a work wise exercise.
- Conscious effort should be made to discourage sex stereotyping in the teaching and learning of mathematics. All form of bias associated with sex stereotyping in mathematics or mathematics related courses should be dissuaded and discouraged.
- The rigidity of the mathematics curriculum across classes should be adhered to. It is of no essence making it flexible for easy modification by teachers and local education authority. Therefore, mathematics teachers should innovatively teach mathematics according to stipulated classes.
- Teachers should make student to acquire problem solving and creative thinking abilities that can help all levels of students to learn math well. Under this premise, teachers need to adopt multiple materials and instructional strategies to encourage students to do discussion, interpretation and innovation. In this way, students will be trained to think about math instead of memorizing math.

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