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Calculation Energy Levels, B(E2), Electric Quadrupole Moment and (P.E.S.) of the Even-Even ^{156,158}Dy Isotopes Using IBA-1

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Abstract :
Nuclear structure of ^{156,158}Dy isotopes have been studied in the frame work of the interacting boson approximation model (IBM-1). The contour plot of the potential energy surfaces, V(β,γ), shows that the nucleus ^{156}Dy is deformed and has transitional characters between SU(3) and O(6) limits, and the nucleus ^{158}Dy is deformed and has rotational characters, SU(3). Levels energy spectra belonging to the g,β,γ bands, electromagnetic transition rates B(E2), electric quadrupole moment (Q) are calculated. The calculated values are compared with the available theoretical and experimental data and show reasonable agreement.

Keywords: potential energy surfaces; nuclear structure; ^{156,158}Dy; B(E2).

1. Introduction :

Arima and Iachello (1979) [1] have developed the interacting boson model (IBM), which is based on the well-known shell model and on geometrical collective model of the atomic nucleus. This model is to describe nuclear properties such as spins, energies of the levels, decay probabilities for the emission of gamma quanta, probabilities of electromagnetic transition and their reduced matrix elements for different transitions multipole moment and mixing ratios [1,2].

The (IBM-1) is used in the present work, this model represents very important step formed in the description of collective nuclear excitations. The underlying U(6) group structure of model basis leads to a simple Hamiltonian which is capable of describing the three specific limits of collective structure vibrational SU(5), rotational SU(3) and gamma unstable O(6) [1,3].

In the simplest version of the interacting boson model (IBM-1), its assumed that low-lying collective states in even-even nuclei away from closed shells are dominated by excitation of the valence protons and the valence neutrons (particles outside the major closed shell) while the closed shell core is inert. Furthermore, its assumed that the particle configurations which are most important in shaping the properties of the low-lying states are these in which identical particles are coupled together forming pairs of angular momentum 0 and 2 [1,4,5].

2. (IBA-1) Model

The IBA-1 model was applied to the positive parity low-lying states in even-even ^{156,158}Dy isotopes. The proton, π, and neutron, υ, bosons are treated as one boson and the system is considered as an interaction between s-bosons and d-bosons. Creation (s^+,d^+) and annihilation (s,d) operators are for s and d bosons.

2.1. The Hamiltonian Operator of The (IBM-1) :

The Hamiltonian employed for the present calculation is given as [6,7]:

$$\hat{H} = \varepsilon \hat{n}_d + a_o \hat{P}.\hat{P} + a_1 \hat{L}.\hat{L} + a_2 \hat{Q}.\hat{Q} + a_3 \hat{T}_3.\hat{T}_3 + a_4 \hat{T}_4.\hat{T}_4 \qquad (1)$$

Where;

n_d is the number of boson; P.P, L.L, Q.Q, $T_3.T_3$ and $T_4.T_4$ represent pairing, angular momentum, quadrupole, octupole and hexadecupole interactions between the bosons respectively; ε is the boson energy; and a_o, a_1, a_2, a_3, a_4 is the strengths of the pairing, angular momentum, quadrupole, octupole and hexadecupole interactions respectively.

2.2. O(6)→SU(3) Transition Region :

In this region nuclei have transitional properties between (SU(3)) and (O(6)) and the Hamiltonian is give by [8] :

$$\hat{H}^{(II+III)} = a_o \hat{P}.\hat{P} + a_1 \hat{L}.\hat{L} + a_2 \hat{Q}.\hat{Q} \qquad (2)$$

Nuclei fall in this region depends on the ratio (a_o/a_2). When this ratio is large the properties will be near to O(6) limit and when this ratio is small the properties will be near to SU(3).

2.3. Rotational Limit SU(3) :

Hamiltonian function operator for dynamical symmetry SU(3) in terms of creation and annihilation operators can be given according to the following equation [9-12] :

$$\hat{H} = a_1 \hat{I}^2 + a_2 \hat{Q}^2 \quad \textbf{(3)}$$

The rotation dynamical symmetry represented by sub-group SU(3) and its quantum numbers that make it has diagonal attribute can be described as [9,13,14] :

$$\left| \begin{array}{ccccccc} U(6) & \supset & SU(3) & \supset & O(3) & \supset & O(2) \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ [N] & & (\lambda,\mu) & \tilde{\chi} & I & & M_I \end{array} \right\rangle$$

Where *[N]* is the total number of bosons *($N = N_\pi + N_\nu$)*.
The values of (λ, μ) contained in each [N] are given by :
[N] = (2N,0)

$$\oplus(2N-4,2)\oplus(2N-8,4)\oplus.....\oplus\left\{\begin{matrix}(0,N)\\2,N-1\end{matrix}\right\}\left\{\begin{matrix}if & N=even\\if & N=odd\end{matrix}\right\}$$

$$\oplus(2N-6,0)\oplus(2N-10,2)\oplus.....\oplus\left\{\begin{matrix}(0,N-3)\\(2,N-4)\end{matrix}\right\}\left\{\begin{matrix}if & N-3=even\\if & N-3=add\end{matrix}\right\} \quad \textbf{(4)}$$

$$\oplus(2N-12,0)\oplus(2N-16,2)\oplus.....\oplus\left\{\begin{matrix}(0,N-6)\\(0,N-7)\end{matrix}\right\}\left\{\begin{matrix}if & N-6=even\\if & N-6=odd\end{matrix}\right\}$$

μ = 0, 2, 4, …..

$\tilde{X}$ = 0 ,2 ,4 ,…., min (λ, μ) **(5)**

2.4. Transition Rates :

The electric quadrupole transition operator employed in this study is given by [15,16] :

$T_m^{(E2)}=\alpha_2[d^\dagger s+s^\dagger d]_m^{(2)}+\beta_2[d^\dagger d]_m^{(2)}$ **(6)**

Where;

α_2 and β_2 are the parameters which is used to description the different terms in operator. The reduced electric quadrupole transition rates between $L_i \rightarrow L_f$ states are given by [1] :

$$B(E2, L_i \rightarrow L_f) = \frac{1}{2Li+1}\left|\left\langle L_f \left\| \hat{T}(E2) \right\| L_i \right\rangle\right| \quad \textbf{(7)}$$

And the electric quadrupole moment ($Q2_1^+$) is [17] :

$$Q_{2_1^+} = -\alpha_2 \sqrt{\frac{16\pi}{40}}\frac{2}{7}(4N+3) \quad \textbf{(8)}$$

As to the potential energy surface operator is given by [18] :

$$V(N,\beta,\gamma)=\frac{N}{1+\beta^2}\left(\varepsilon_s+\varepsilon_d\beta^2\right)+\frac{N(N-1)}{\left(1+\beta^2\right)}\left(A_1\beta^2+A_2\beta^3\cos 3\gamma+A_3\beta^2+A_4\right) \quad \textbf{(9)}$$

Where;

β is the magnitude of nuclear deformation taken the values (0-2.4);

γ asymmetry angle taken the values (0^o-60^o);A_1,A_2,A_3,A_4 parameters relationship with the function of the surface potential.

3. Results and Discussion

3.1. Energy Levels :

In this work we have studied the nuclear structure of even-even Dy (A=156) isotope which is classified to O(6)→SU(3) transition region and even-even Dy (A=158) isotope which is classified to rotational dynamical

symmetry SU(3) by comparing the energy ratios $\frac{E0_2^+}{E2_1^+}, \frac{E8_1^+}{E2_1^+}, \frac{E6_1^+}{E2_1^+}, \frac{E4_1^+}{E2_1^+}$ with ideal values [19,20] for three dynamical symmetries SU(5), O(6) and SU(3) of IBM-1 (shown in figures (1) to (4)).

Table (1) presents the isotopes used in the present work according to its atomic mass number, total number of boson and the corresponding Hamiltonian parameters used in the IBM-Code according to O(6)→SU(3) transition region (in ^{156}Dy) and rotational dynamical symmetry SU(3) (in ^{158}Dy).

Tables (2) and (3) and figures (5) and (6) present values of the energy levels (present work), according to energy bands (g, β, and γ bands) in comparison with available experimental data.

This table list the new energy levels belong to, β_1, β_2, γ_1 and γ_2 bands with their spins and parties. The results show that, the β-band is a large extent emerge than the γ-band for the dynamical symmetry SU(3), while the emergence of γ-band is increasing for the isotopes having the transitional dynamical symmetry SU(5)-O(6).

The β-band is not difficult to see it in the dynamical symmetry SU(3), in the low spin states, while the γ-band is difficult to find it due to the high spin state.

3.2. B(E2) and Electric Quadrupole Moments (Q) :

More information can be obtained by studying the reduced transition probabilities B(E2). The (IBMT-code) have been employed. The parameters E2SD and E2DD which are used in the present calculations have been determined.

where;

E2SD=α_2 and E2DD=$\sqrt{5}\,\beta_2$

Table (4) presents the values of (E2SD) and (E2DD) used in the present work with the experimental values of B(E2) taken from ref. [24].

The quadrupole moment (Q) is an important property for nuclei. It is defined as the deviation from the spherical charge distribution inside the nucleus . From the quadrupole moment, we can determine if the nucleus is spherical, deformed oblate or prolate shapes.

A comparison between our theoretical calculations and the recent available experimental data for B(E2) taken from refs. [25-28] and the previous theoretical work [25,29,30] are presented in Tables (5) and (6).

The branching ratios (*R*, *R*' and *R*") for the three dynamical limits are defined as [1,31] :

$$R=R'=R''=2[(N-1)/N]=2\rightarrow SU(5) \qquad (10)$$

$$\approx 1.4,\ R'=R''=0\rightarrow SU(3) \qquad (11)\quad \frac{10}{7}\frac{(N-1)(2N+5)}{N(2N+3)}\ R=$$

$$\approx 1.4,\ \frac{10}{7}\frac{(N-1)(N+5)}{N(N+4)}\ R=R'=R''=0\rightarrow O(6) \qquad (12)$$

The calculated branching ratios and the typical values [1,31] for the limits SU(5), SU(3) and O(6) are presented in Table (7).

3.3. Potential Energy Surface (P.E.S.) :

One of methods to knowledge the deformation of nuclear structure, calculation the potential energy surface.

In the present work, we were used the IBM-1 analysis for the set of the plots potential energy surface function V(N,β,γ) calculate by using the parameters (A's) infer from (IBMP-Code) program, as shown in table (8).
Figures (7) and (8) show the potential energy surface as a function of deformed parameters (β,γ).

4. Conclusions :

The interacting boson model version one (IBM-1) gives us a very closing value with the experiment.

Since the energy levels depends on the total boson number so that only the ground state band will appear.

The even-even ^{156,158}Dy isotopes have (66) protons and (89,90) neutrons respectively. The core is taken at major closed shell (82) for protons and neutrons. Therefore, the number of bosons were determined for ^{156}Dy and ^{158}Dy, is equal (12) and (13) bosons respectively.

Hamiltonian parameters in table (1) are very small so that these parameters vary to any change may occurs in any one of these parameters, so that it is difficult to get the coincidence values between the energy levels in high energy states.

In case of quadruple electrical transitions B(E2) for even–even nuclei, we find that the values of α_2 and β_2 parameters increase whenever the number of bosons increases in one element isotopes.

The study of the reduced transition probability B(E2:$2_1^+ \rightarrow 0_1^+$) that it decreases as the mass number increase, and this is a key signature that the nuclei evolve from O(6) to SU(3) limits.

The electric quadruple moment increase as the mass number increase.

The nuclear deformation increases with the increasing of valance boson number.

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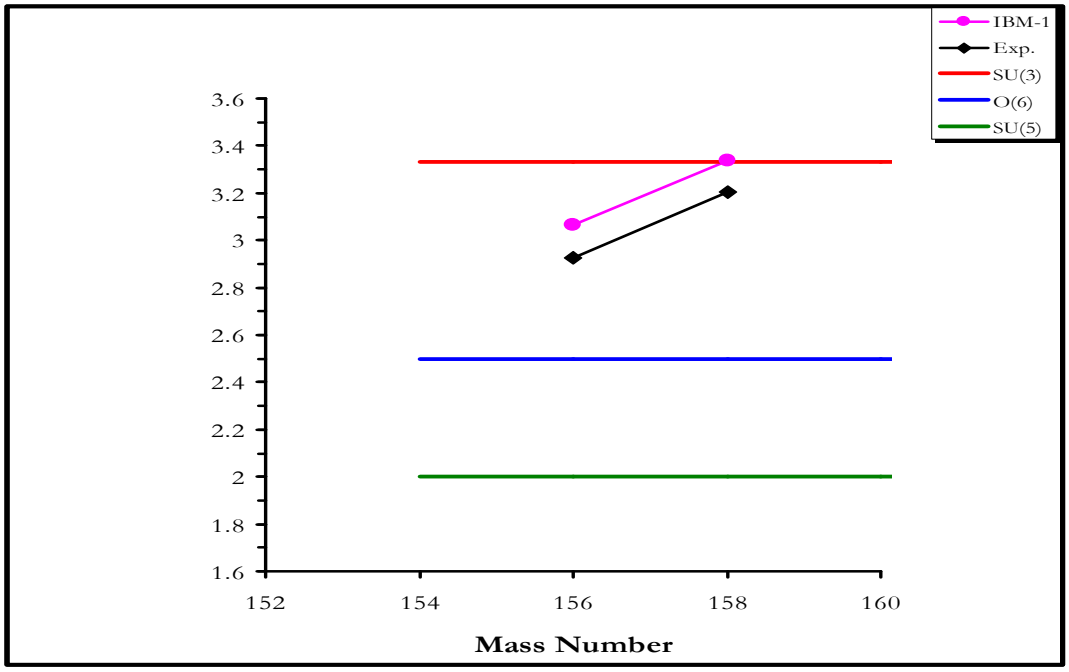


Figure (1): The Comparison of $E4_1^+/E2_1^+$ Theoretically, Experimentally [21-23] and with the Typical Values [19,20] for Each Limit.

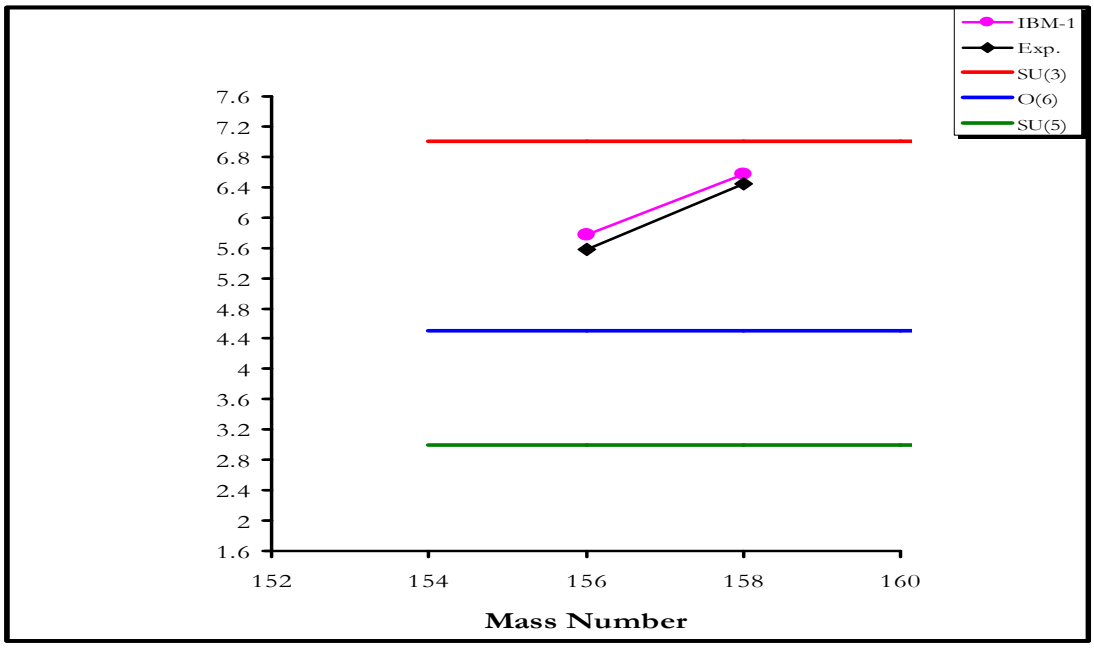


Figure (2): The Comparison of $E6_1^+/E2_1^+$ Theoretically, Experimentally [21-23] and with the Typical Values [19,20] for Each Limit.

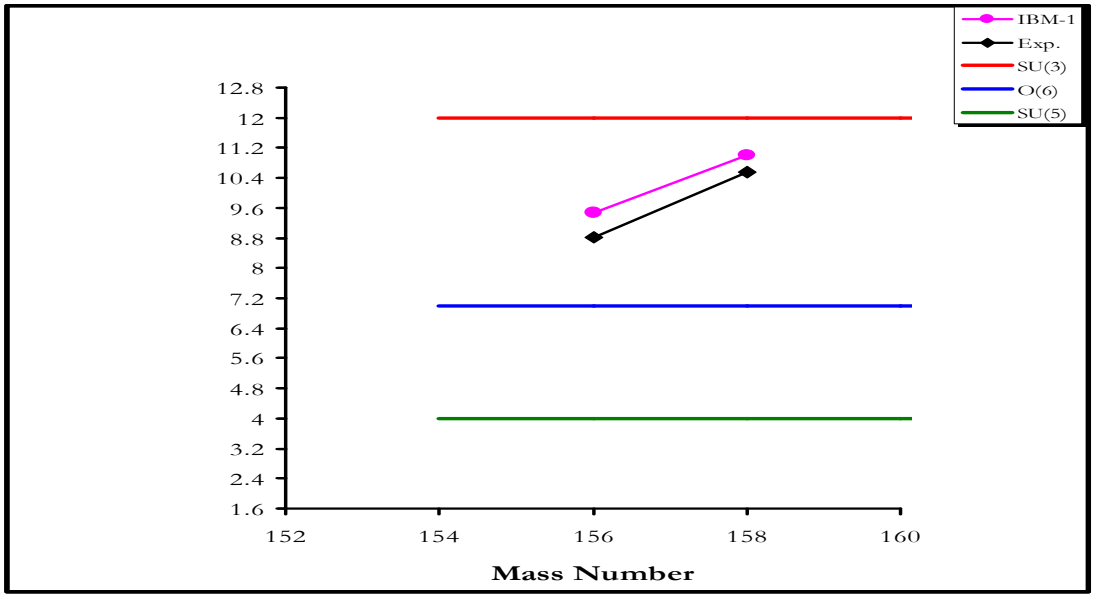


Figure (3): The Comparison of $E8_1^+/E2_1^+$ Theoretically, Experimentally [21-23] and with the Typical Values [19,20] for Each Limit.

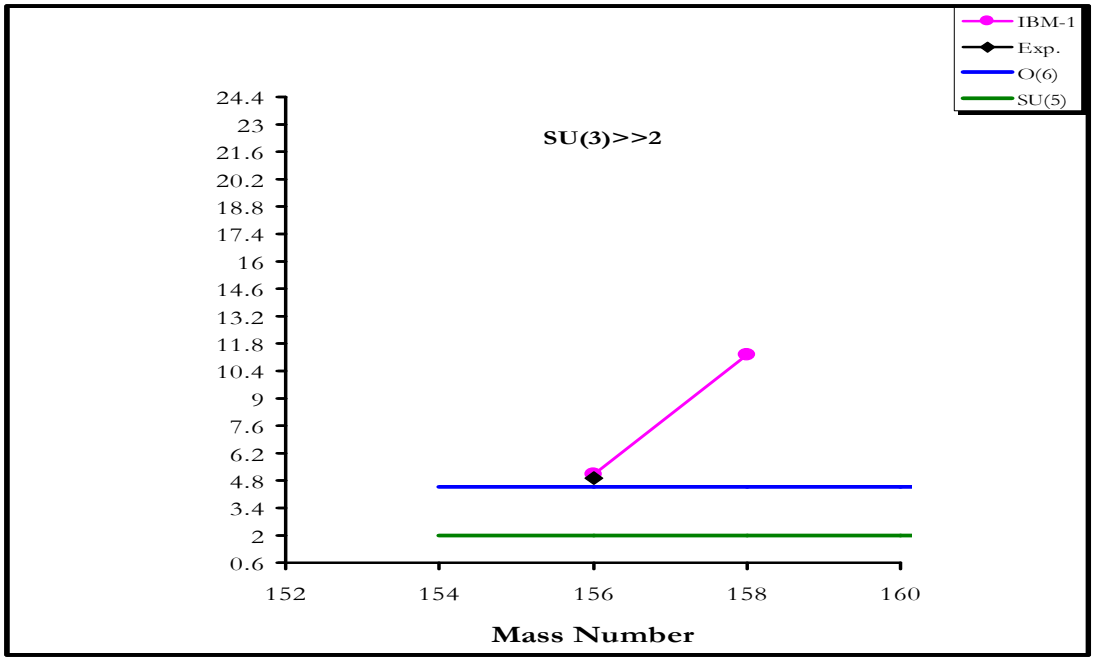


Figure (4): The Comparison of $E0_2^+/E2_1^+$ Theoretically, Experimentally [21-23] and with the Typical Values [19,20] for Each Limit.

Table (1): The Hamiltonian Parameters Used in the IBM-Code for ^{156,158}Dy Isotopes.

Isotope	EPS MeV)(	P.P. (MeV)	L.L. (MeV)	Q.Q. (MeV)	$T_3.T_3$ (MeV)	$T_4.T_4$ (MeV)	CHI	SO6
^{156}Dy	0.0000	0.2220	0.0158	-0.0082	0.0000	0.0000	-1.1422	1.0000
^{158}Dy	0.0000	0.0000	0.0100	-0.0138	0.0000	0.0000	-1.3220	1.0000

Table (2): Comparison between Experiment [21,22] and Calculated Energy Levels in MeV for ^{156}Dy Isotope.

Band	g-band or β-band (γ-band) [21,22] (MeV)						
	$0^+(2^+)$	$2^+(3^+)$	$4^+(4^+)$	$6^+(5^+)$	$8^+(6^+)$	$10^+(7^+)$	$12^+(8^+)$
g-(Exp.)	0.000	0.138	0.404	0.770	1.216	1.725	2.286
g-(IBM-1)	0.000	0.110	0.368	0.774	1.227	1.829	2.481
β_1-(Exp.)	0.676	0.890	1.168	1.437	1.859	2.316	2.707
β_1-(IBM-1)	0.741	0.953	1.215	1.628	1.798	2.504	3.160
β_2-(Exp.)	----	1.382	----	1.525	1.959	2.448	2.997
β_2-(IBM-1)	1.378	1.504	1.527	1.631	2.190	2.700	3.561
γ_1-(Exp.)	0.829	1.022	1.088	1.336	----	1.729	----
γ_1-(IBM-1)	0.577	0.687	0.835	1.020	1.242	1.501	2.192
γ_2-(Exp.)	----	----	1.627	----	----	----	2.787
γ_2-(IBM-1)	1.259	1.375	1.757	1.960	2.195	2.220	2.949

Table (3): Comparison between Experiment [21,23] and Calculated Energy Levels in MeV for ^{158}Dy Isotope.

Band	g-band or β-band (γ-band) [21,23] (MeV)						
	$0^+(2^+)$	$2^+(3^+)$	$4^+(4^+)$	$6^+(5^+)$	$8^+(6^+)$	$10^+(7^+)$	$12^+(8^+)$
g-(Exp.)	0.000	0.099	0.317	0.638	1.044	1.520	2.049
g-(IBM-1)	0.000	0.091	0.304	0.638	1.094	1.672	2.271
β_1-(Exp.)	----	0.946	1.164	1.486	1.893	2.478	2.807
β_1-(IBM-1)	1.036	1.126	1.338	1.573	2.129	2.707	3.206
β_2-(Exp.)	0.991	1.086	----	----	----	----	----
β_2-(IBM-1)	1.505	1.595	2.208	2.543	2.998	3.577	4.276
γ_1-(Exp.)	----	1.045	1.280	1.315	1.547	1.676	----
γ_1-(IBM-1)	1.127	1.218	1.339	1.491	1.674	1.886	2.130
γ_2-(Exp.)	1.362	1.513	1.514	1.920	2.154	----	2.528
γ_2-(IBM-1)	1.996	2.088	2.209	2.261	2.544	2.756	2.999

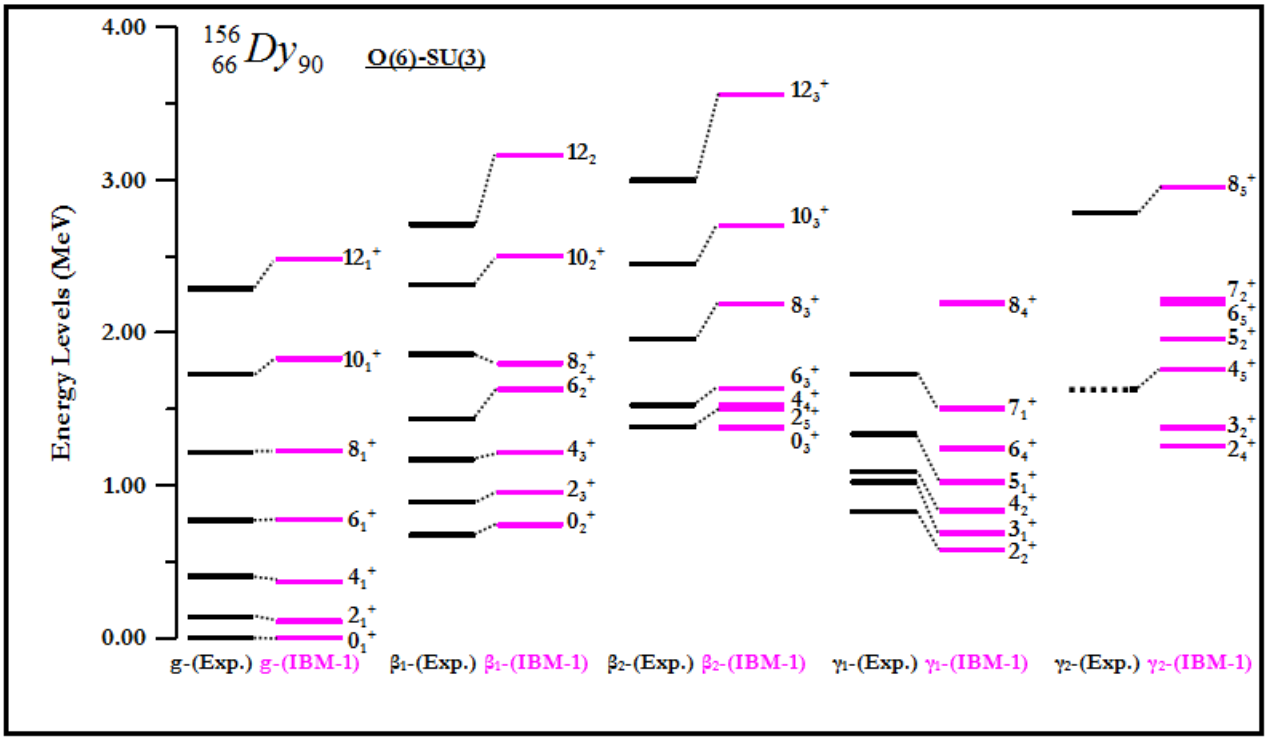


Figure (5): Comparison between Experiment [21,22] and Calculated Energy Levels for ^{156}Dy Isotope.

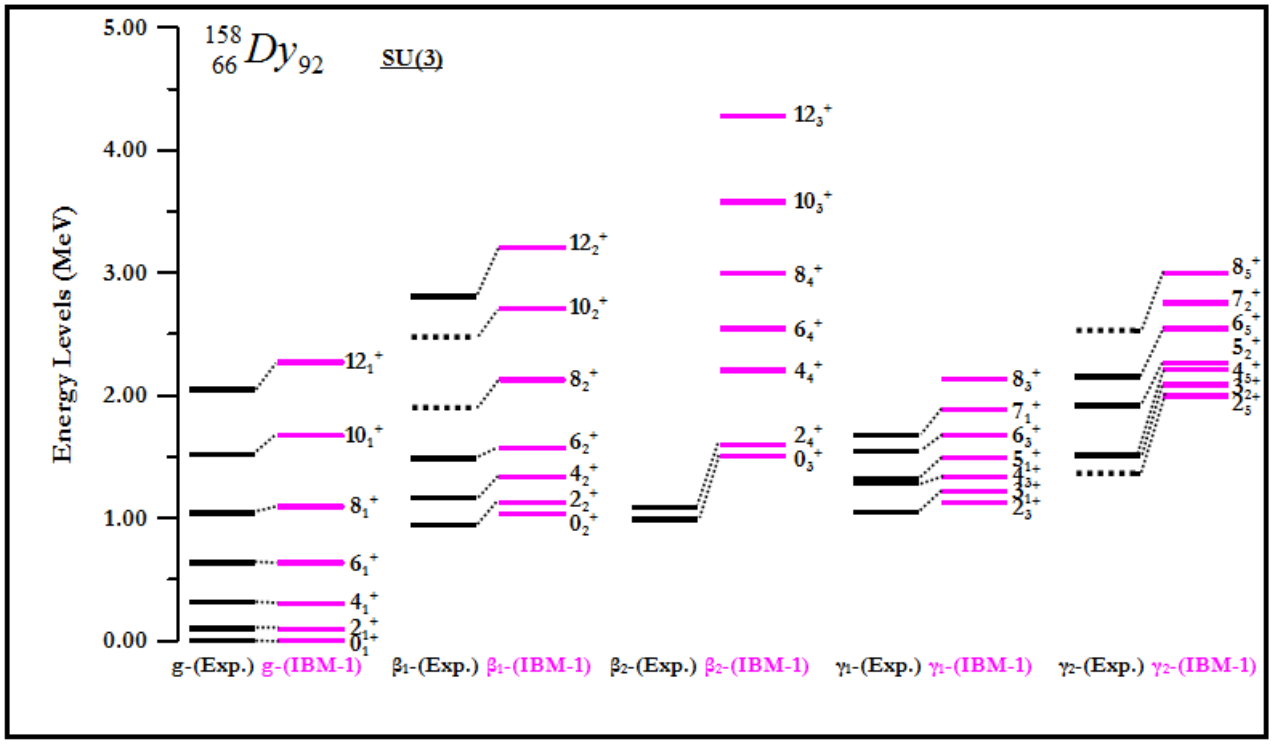


Figure (6): Comparison between Experiment [21,23] and Calculated Energy Levels for ^{158}Dy Isotope.

Table (4): The Experimental Values [24] of B(E2) and the Coefficients (E2SD, E2DD) for ^{156,158}Dy Used in the Present Work.

Isotope	B(E2:$2_1^+ \rightarrow 0_1^+$)e^2b^2 [24]	E2SD (eb)	E2DD (eb)
^{156}Dy	0.7420	0.0831	-0.2599
^{158}Dy	0.9320	0.0996	-0.2711

Table (5): The Experimental and Calculated B(E2)↓ Using IBMT-Code and the Quadrupole Moment $Q2_1^+$ for ^{156}Dy Isotope.

$J_i^\pi \rightarrow J_f^\pi$	$B(E2)\downarrow e^2b^2$		
	Exp.	**IBM-1**	**Previous Work**
$2_1^+ \rightarrow 0_1^+$	0.7420 [27]	0.7424	0.6260 [30]
$2_1^+ \rightarrow 0_2^+$	----	0.0003	----
$2_1^+ \rightarrow 0_3^+$	----	0.0000	0.0000 [29]
$2_2^+ \rightarrow 0_1^+$	----	0.0000	0.0000 [29]
$2_2^+ \rightarrow 0_2^+$	----	0.0085	----
$2_2^+ \rightarrow 2_1^+$	----	0.0432	----
$2_3^+ \rightarrow 0_1^+$	----	0.0044	0.0063 [29]
$2_3^+ \rightarrow 0_2^+$	----	0.5912	----
$2_3^+ \rightarrow 0_3^+$	----	0.0025	----
$2_4^+ \rightarrow 0_2^+$	----	0.0035	----
$2_4^+ \rightarrow 0_3^+$	----	0.3537	----
$2_1^+ \rightarrow 2_2^+$	----	0.0432	----
$4_1^+ \rightarrow 2_1^+$	1.3240 [26,28]	1.2000	1.0900 [29]
$4_1^+ \rightarrow 2_3^+$	----	0.0103	----
$4_2^+ \rightarrow 2_1^+$	----	0.0000	----
$4_2^+ \rightarrow 2_2^+$	----	0.2959	----
$4_2^+ \rightarrow 2_3^+$	----	0.0115	----
$Q2_1^+$	----	-1.7760	----

Table (6): The Experimental and Calculated B(E2)↓ Using IBMT-Code and the Quadrupole Moment $Q2_1^+$ for ^{158}Dy Isotope.

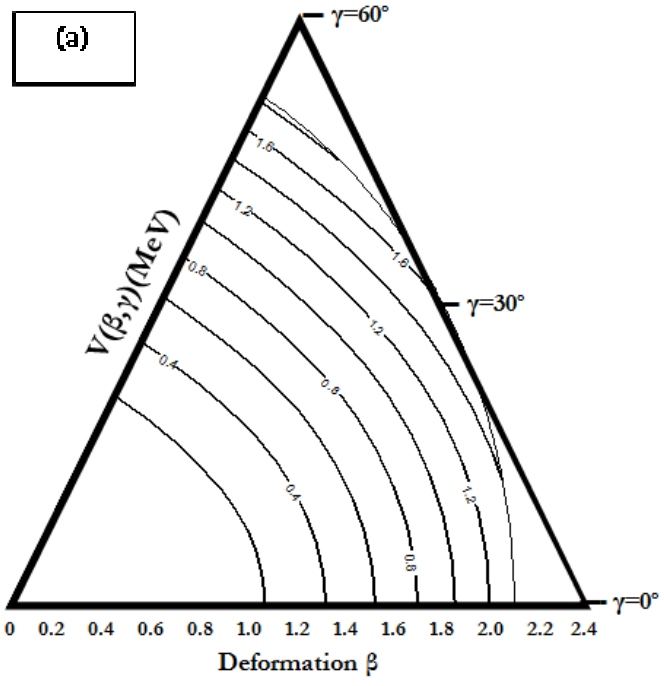
$J_i^\pi \rightarrow J_f^\pi$	$B(E2)\downarrow e^2b^2$		
	Exp.	**IBM-1**	**Previous Work**
$2_1^+ \rightarrow 0_1^+$	0.9320 [27]	0.9333	1.0362 [25]
$2_1^+ \rightarrow 0_2^+$	----	0.0002	0.0000 [29]
$2_1^+ \rightarrow 0_3^+$	----	0.0000	0.0000 [29]
$2_2^+ \rightarrow 0_1^+$	----	0.0000	----
$2_2^+ \rightarrow 0_2^+$	----	0.3200	0.5040 [25]
$2_2^+ \rightarrow 2_1^+$	----	0.0020	----
$2_3^+ \rightarrow 0_1^+$	0.0300 [25]	0.0200	0.0218 [25]
$2_3^+ \rightarrow 0_2^+$	----	0.7366	----
$2_3^+ \rightarrow 0_3^+$	----	0.0026	----
$2_4^+ \rightarrow 0_2^+$	----	0.0033	----
$2_4^+ \rightarrow 0_3^+$	----	0.4585	----
$2_1^+ \rightarrow 2_2^+$	----	0.0020	----
$4_1^+ \rightarrow 2_1^+$	1.3780 [26,28]	1.3091	1.4650 [20]
$4_1^+ \rightarrow 2_3^+$	0.0056 [25]	0.0098	0.0006 [20]
$4_2^+ \rightarrow 2_1^+$	----	0.0000	----
$4_2^+ \rightarrow 2_2^+$	----	0.9766	1.1080 [20]
$4_2^+ \rightarrow 2_3^+$	----	0.0018	0.0010 [20]
$Q2_1^+$	----	-2.0550	----

Table (7): The Comparison between the Calculated B(E2) Branching Ratios and the Typical Values [1,31] for the Limits SU(5), SU(3) and O(6) for ^{156,158}Dy Isotopes.

B(E2) Ratios	^{156}Dy	^{158}Dy	SU(5) Limit	SU(3) Limit	O(6) Limit
$R = \frac{B(E2: 4_1^+ \rightarrow 2_1^+)}{B(E2: 2_1^+ \rightarrow 0_1^+)}$	1.616	1.403	2	1.4	1.4
$R' = \frac{B(E2: 2_1^+ \rightarrow 2_2^+)}{B(E2: 2_1^+ \rightarrow 0_1^+)}$	0.058	0.002	2	0	1.4
$R'' = \frac{B(E2: 0_2^+ \rightarrow 2_1^+)}{B(E2: 2_1^+ \rightarrow 0_1^+)}$	0.002	0.001	2	0	0
$R''' = \frac{B(E2: 2_2^+ \rightarrow 0_1^+)}{B(E2: 2_2^+ \rightarrow 2_1^+)}$	0.000	0.000	0	0	0
$R'''' = \frac{B(E2: 4_2^+ \rightarrow 2_1^+)}{B(E2: 4_2^+ \rightarrow 2_2^+)}$	0.000	0.000	0	0	0

Table (8): Parameter Used for Potential Energy Surface Calculations in (IBMP-Code) Program for ^{156,158}Dy Isotopes.

Isotope	N	EPS	EPD	A_1	A_2	A_3	A_4
^{156}Dy	12	-0.041	0.076	0.052	-0.020	-0.022	0.000
^{158}Dy	13	-0.069	0.872	-0.007	-0.088	-0.002	0.000



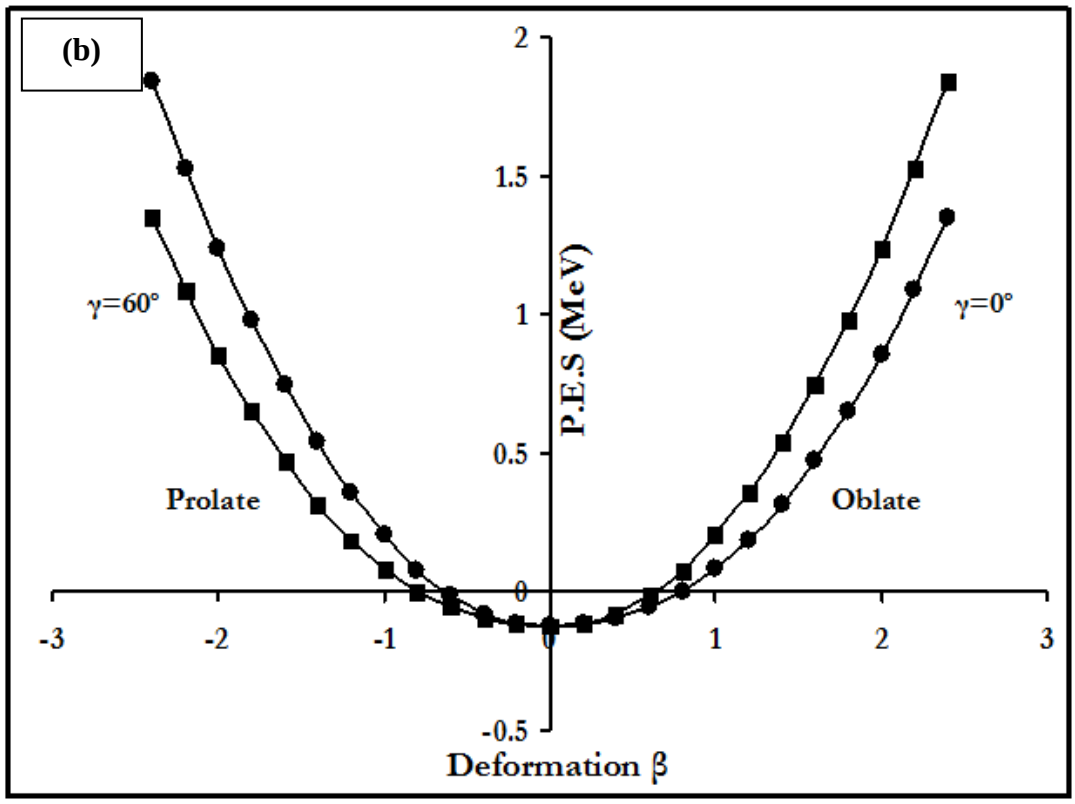
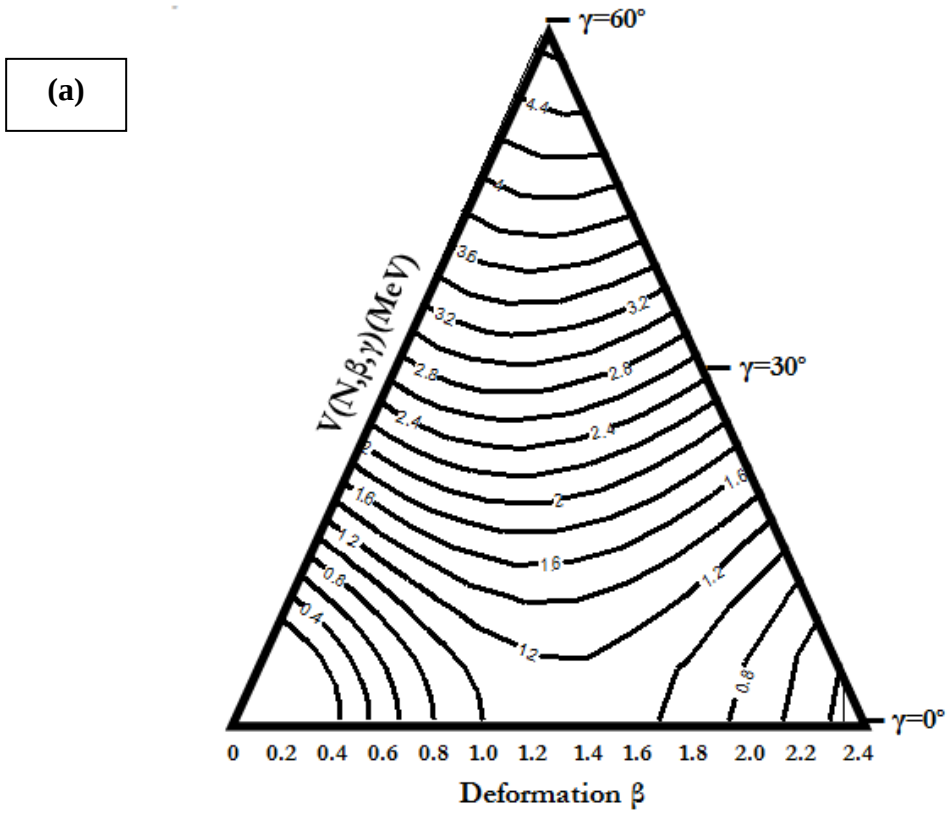


Figure (7) :
(a): The Contour Plot for the ^{156}Dy Isotope at γ=60°.
(b): The Axial Symmetric for the ^{156}Dy Isotope.



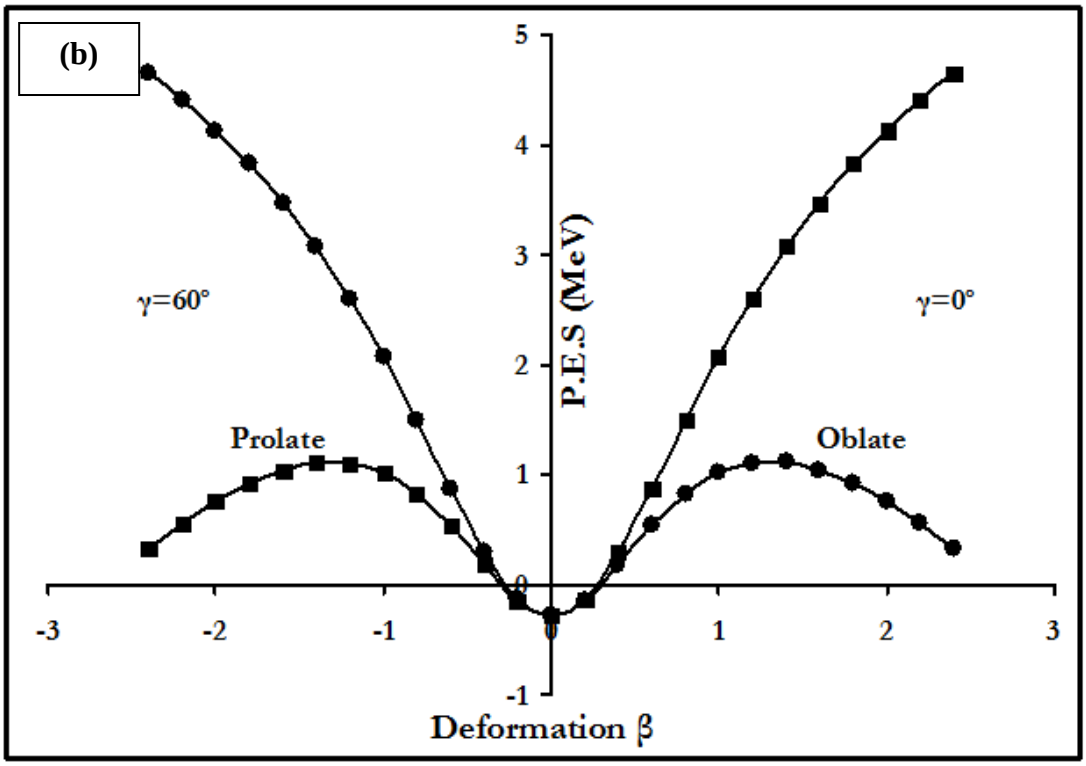


Figure (8) :
(a): The Contour Plot for the ^{158}Dy Isotope at γ=60^{o}.
(b): The Axial Symmetric for the ^{158}Dy Isotope.