Calculation the Energy Levels and Energy Bands (g, β, γ-bands) for ^{154,160}Dy Isotopes

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Abstract:

Interacting boson model (IBM-1) was used in the present work to study some of nuclear structures for selected Dysprosium isotope ^{154}Dy which has transitional motion of the SU(5)-O(6) translation region and ^{160}Dy which has rotational motion of the SU(3) dynamical symmetry.

These isotopes have been classified to be the SU(5)-O(6) translation region (for ^{154}Dy) and SU(3) dynamical symmetry (for ^{160}Dy) by comparing the experimental energy levels which taken from the references with the ideal chart for the three dynamical symmetry U(5) , O(6) , SU(3) , and energy ratios $E(4_1^+)/E(2_1^+)$, $E(6_1^+)/E(2_1^+)$, $E(8_1^+)/E(2_1^+)$ and $(E0_2^+/E2_1^+)$ with the ideal values.

Therefore are calculated the energy levels and gamma transitions for these isotopes depending on the total number of boson (N). The calculated results are compared with the available experimental data and found to be in a good agreement, specially at low-lying states, while at high angular momentum, some theoretical values are somehow larger than the experimental values.

Keywords: interacting boson model, SU(3), translation region, energy levels.

1. Introduction:

The interacting boson model-1 (IBM-1) is an important subjects that is used to study some nuclear properties of all even-mass or odd-mass nuclei. This model has been proposed by Mariscotti . et al .(1969) [1] and Mariscotti (1970) [2] in order to study the energies of ground state rotational band of spherical nucleus. This model is based on the well-known shell model and on geometrical collective models of the atomic nucleus [3-5].The interacting boson model-1 is suitable for describing the collective structure of nuclei with even number of protons and even number of neutrons which have positive parity (π+), and it builds on the interaction valence boson particles outside a nuclear closed shell or boson holes inside a closed shell. The total number of bosons (N) depends on the number of active nucleon (or hole) pairs outside a closed shell and it can be calculated by adding the number of neutrons pairs and protons pairs of (s and d) bosons which can be written as [6-8]:

N=ns+nd

Where: ns= number of s-bosons

nd= number of d-bosons.

The Interacting Boson Model-1 is very successful in studying the properties of many nuclei especially, When the total number of bosons N>>0, but it fails whenever N reaches zero, it completely fails in studying closed shells at 28, 50, 82, and 126 where N=0 because, there is no interacting between proton and neutron bosons (i.e. there is no degree of freedom) [9-11]. The outline of the remaining part of this paper is as follows: starting from an approximate theoretical background of the model, we give the basic formulations defined in the IBM-1 in Section 2. Then, the previous experimental and theoretical data are compared with the calculated values and the general features of even-even $_{66}$Dy isotopes in the range A=154, 160 are reviewed in Section 3. The last section contains some concluding remarks.

2. Theoretical Basis:

2.1 IBM-1 Model:

Hamiltonian operator function according to IBM-1 is written in terms of creation and annihilation operators as follows [12-14]:

$$\hat{H} = \varepsilon\,\hat{n}_d + a_0(\hat{P}^+ . \hat{P}) + a_1(\hat{I} . \hat{I}) + a_2(\hat{Q} . \hat{Q}) + a_3(\hat{T}_3 . \hat{T}_3) + a_4(\hat{T}_4 . \hat{T}_4) \qquad \textbf{(1)}$$

Where ε, a_0, a_1, a_2, a_3 and a_4 are parameters used in IBM-1 to determine the Hamiltonian function, and:

$$\varepsilon = \varepsilon_d - \varepsilon_s \quad \textbf{(2)}$$

Where ε = Boson's energy

ε_d = d- Boson's energy

ε_s = s – Boson's energy

$$\hat{n}_d = (\hat{d}^+ \otimes \tilde{\hat{d}}) \equiv d-bosons \quad operator \quad \textbf{(3)}$$

$$\hat{P} = \frac{1}{2}(\tilde{\hat{d}} \otimes \tilde{\hat{d}}) - \frac{1}{2}(\tilde{\hat{S}} \otimes \tilde{\hat{S}}) \equiv \text{Operator of pairing among bosons} \quad \textbf{(4)}$$

$$\hat{I} = \sqrt{10}\left[\hat{d}^+ \otimes \hat{d}^{\sim}\right] \equiv \text{Angular momentum operator} \quad \textbf{(5)}$$

$$\hat{Q} = \left[\hat{d}^+ \otimes \hat{s}^{\sim} + \hat{s}^+ \otimes \hat{d}^{\sim}\right] - \frac{\sqrt{7}}{2}\left[\hat{d}^+ \otimes \hat{d}^{\sim}\right] \equiv \text{Quadrupole operator} \quad \textbf{(6)}$$

$$\hat{T}_3 = \left[\hat{d}^+ \otimes \hat{d}^{\sim}\right] \equiv \text{Octupole operator} \quad \textbf{(7)}$$

$$\hat{T}_4 = \left[\hat{d}^+ \otimes \hat{d}^{\sim}\right] \equiv \text{Hexadecapole operator} \quad \textbf{(8)}$$

2.2 Rotational Limit SU(3):

Hamiltonian function operator for dynamical symmetry SU(3) in terms of creation and annihilation operators can be given according to the following equation [14,15-17]:

$$\hat{H} = a_1 \hat{I}^2 + a_2 \hat{Q}^2 \quad \textbf{(9)}$$

The rotation dynamical symmetry represented by sub-group SU(3) and its quantum numbers that make it has diagonal attribute can be described as [14,18,19]:

$$\left| \begin{matrix} U(6) & \supset & SU(3) & \supset & O(3) & \supset & O(2) \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ [N] & & (\lambda,\mu) \quad \tilde{\chi} & & I & & M_I \end{matrix} \right\rangle$$

Where *[N]* is the total number of bosons $(N = N_\pi + N_\nu)$.
The values of (λ, μ) contained in each [N] are given by:
[N] = (2N,0)

$$\oplus(2N-4,2)\oplus(2N-8,4)\oplus\ldots\ldots\oplus\begin{Bmatrix}(0,N)\\2,N-1\end{Bmatrix}\begin{Bmatrix}if \quad N=even\\if \quad N=odd\end{Bmatrix}$$

$$\oplus(2N-6,0)\oplus(2N-10,2)\oplus\ldots\ldots\oplus\begin{Bmatrix}(0,N-3)\\(2,N-4)\end{Bmatrix}\begin{Bmatrix}if \quad N-3=even\\if \quad N-3=add\end{Bmatrix} \quad \textbf{(10)}$$

$$\oplus(2N-12,0)\oplus(2N-16,2)\oplus\ldots\ldots\oplus\begin{Bmatrix}(0,N-6)\\(0,N-7)\end{Bmatrix}\begin{Bmatrix}if \quad N-6=even\\if \quad N-6=odd\end{Bmatrix}$$

μ = 0, 2, 4, …..

$\tilde{X}$ = 0 ,2 ,4 ,…., min (λ, μ) **(11)**

2.3 SU(5) → O(6) Transition Region:

In this region nuclei have transitional properties between (SU(5)) and (O(6)) and the Hamiltonian is give by [3,14,20-22] :

$$\hat{H} = a_0 (\hat{P}^{+} . \hat{P}) + a_1 (\hat{I} . \hat{I}) + a_2 (\hat{Q} . \hat{Q}) \qquad \textbf{(12)}$$

The properties of the nuclei fall in this transitional region depends on the ratio (ε'/a_0), if this ratio is large means nuclei properties are near to U(5) limit and when the ratio is small the properties will be near O(6) limit.

3. Results and Discussions:

The interacting boson approximation version one (IBM-1) has been employed in the present work to study the energy levels for neutron rich even-even ^{154,160}Dyisotopes. The number of protons boson (holes or particles) Nπ =8 for even-even ^{154,160}Dy, while the number of neutrons bosons (particles) Nυ =3 and 6 for ^{154}Dy and ^{160}Dy respectively and the total number of bosons (N) are shown in Table 1.

The examination of the experimental energy levels for the nuclei ^{154,160}Dy shows that ^{154}Dy isotope the belong to the transition region between (SU(5)-O(6)), while ^{160}Dy isotope has been shown their membership to the rotational limit SU(3) .

The best fit for the Hamiltonian parameters equation (1) used in the present work which gives the best agreement between the calculated energy levels in the present work and their corresponding experimental data taken from refs.[23-25] as shown in Table 1.

A comparison between theoretical and experimental energy levels taken from refs.[23-25] are shown in figures (1,2). In these figures we notice that a very good agreement between our calculation for the g-band in comparison with the experimental data for all nuclei under study, and a reasonable agreements for the other bands. Our calculations are consistent with the previous theoretical studies using IBM-1 in different mass regions.

The energy ratios $\frac{E4_1^+}{E2_1^+}, \frac{E6_1^+}{E2_1^+}, \frac{E8_1^+}{E2_1^+}, \frac{E0_2^+}{E2_1^+}$ has been calculated theoretically for the even-even ^{154,160}Dy isotopes and compared with their corresponding experimental values taken from refs. [23-25] and with the typical values for each limit [3,11] as shown in Table 2 and figures (3-6).

4. Conclusion:

The even-even Dysprosium ^{154,160}Dy isotopes have (66) protons and (88, 94) neutrons respectively. The core is taken at major closed shell (82) for protons and (126) for neutrons. Therefore, the number of bosons was determined for ^{154}Dy and ^{160}Dy to be (11) and (14) respectively.

From Table 1, which shows the Hamiltonian parameters used in the present work for the IBM-code, it has been noticed that the parameters (EPS) & (P.P) were appeared for ^{154}Dy isotope, i.e., it has the most SU(5)-O(6) properties, while the parameters (Q.Q) & (CHI) were appeared for ^{160}Dy isotope, i.e., it has more SU(3) properties.

From Figures (1 & 2), that show the comparison between experiment [23-25] and calculated energy levels, it has been noticed that our theoretical calculations are in excellent agreement for the g-band (low) and in reasonable agreement with β-band (middle) and γ-band (high).

From Table 2 and with the comparison between the calculated and experimental energy ratios $\frac{E4_1^+}{E2_1^+}, \frac{E6_1^+}{E2_1^+}, \frac{E8_1^+}{E2_1^+}, \frac{E0_2^+}{E2_1^+}$ ((see figures (3-6)) it has been confirmed that even-even ^{154}Dy isotope falls in the transition region SU(5)-O(6), while even-even ^{160}Dy isotope membership to the rotational limit SU(3).

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Table 1: The Hamiltonian Parameters Used in the IBM-Code for ^{154,160}Dy Isotopes

Nucleus	N	EPS	P.P.	L.L.	Q.Q.	$T_3.T_3$	$T_4.T_4$	CHI	SO6
^{154}Dy	11	0.8440	0.1660	0.0055	0.0000	0.005 5	0.0000	-0.6322	1.0000
^{160}Dy	14	0.0000	0.0000	0.0122	-0.0044	0.0000	0.0000	-1.3220	1.0000

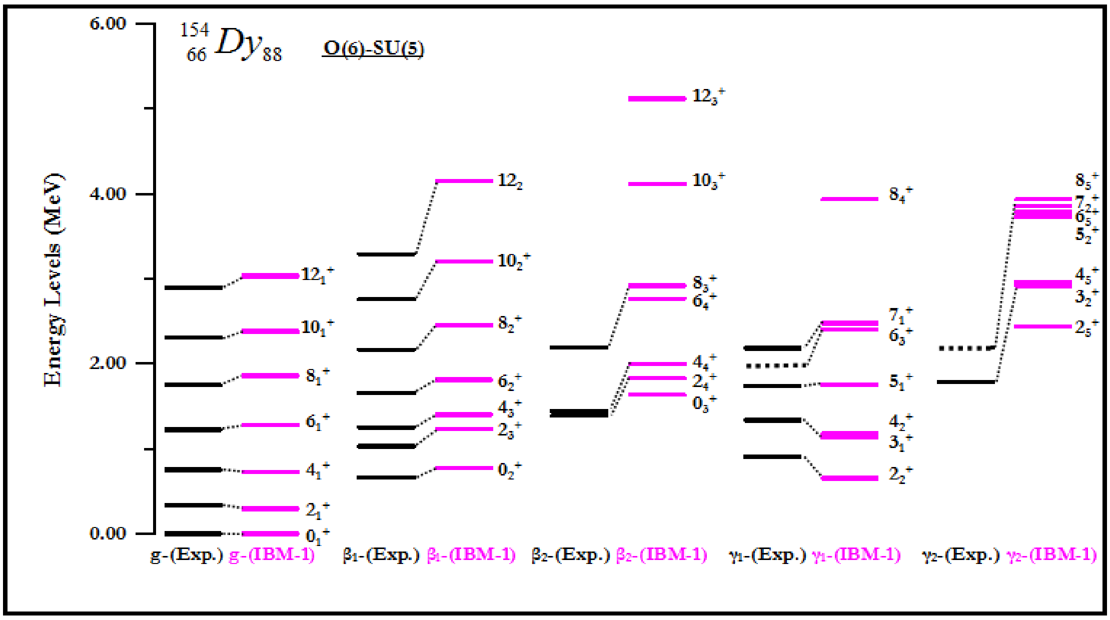

Figure 1: Comparison Between Experiment [23-25] and Calculated Energy Levels for ^{154}Dy Isotope

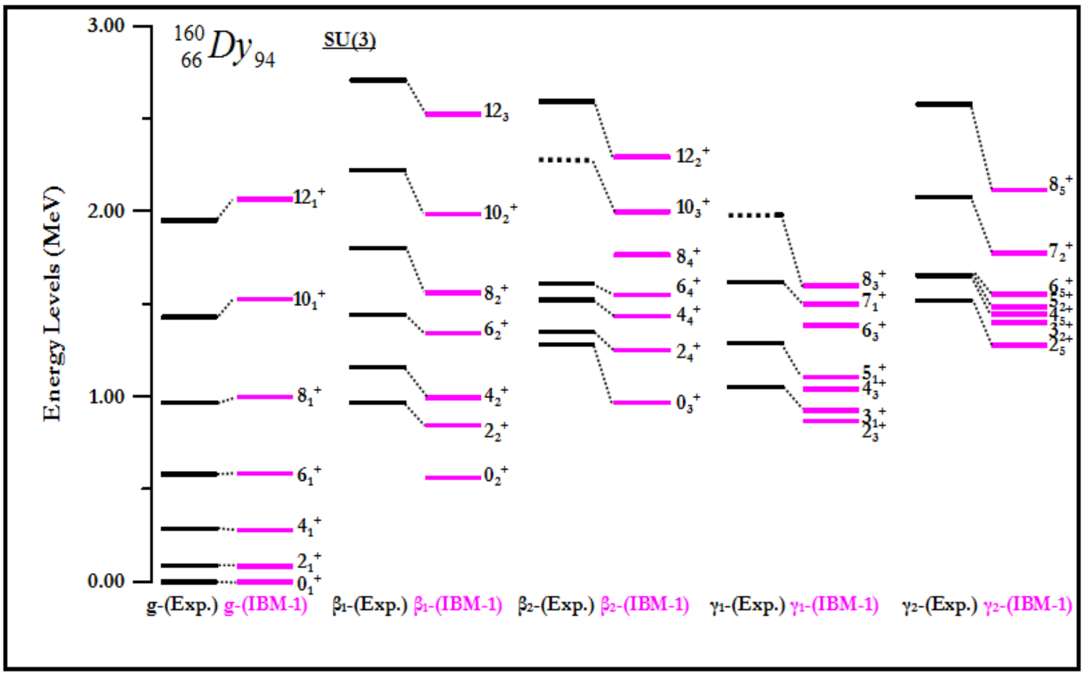

Figure 2: Comparison Between Experiment [23-25] and Calculated Energy Levels for ^{160}Dy Isotope

Journal of Natural Sciences Research
ISSN 2224-3186 (Paper) ISSN 2225-0921 (Online)
Vol.4, No.3, 2014

Table 2: Typical Energy Levels Ratios for Each Limits [3,11]

Limit	$\frac{E4_1^+}{E2_1^+}$	$\frac{E6_1^+}{E2_1^+}$	$\frac{E8_1^+}{E2_1^+}$	$\frac{E0_2^+}{E2_1^+}$
SU(5)	**2**	**3**	**4**	**2**
SU(3)	**3.33**	**7**	**12**	**>>2**
O(6)	**2.5**	**4.5**	**7**	**4.5**

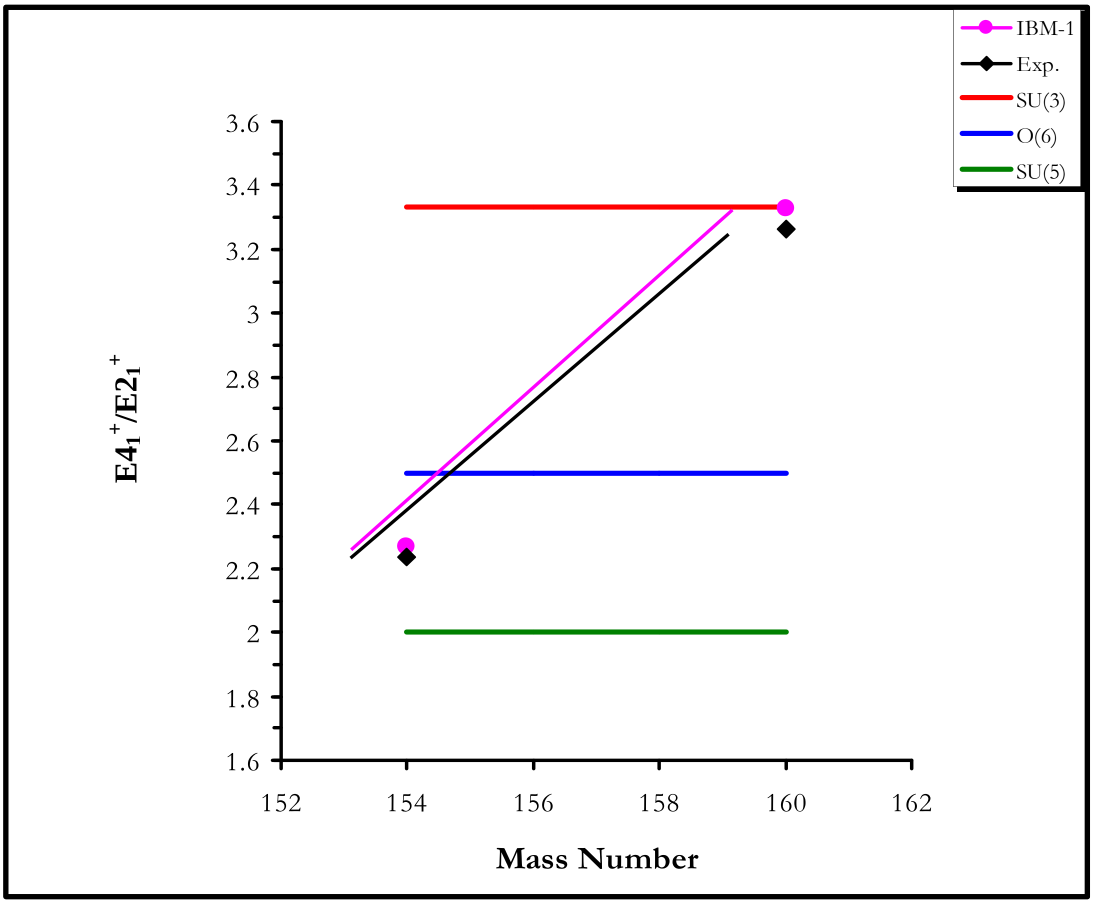

Figure 3: The Comparison of ($E4_1^+/E2_1^+$) Theoretically, Experimentally for ^{154,160}Dy Isotopes [23-25] and with the Typical Values [3,11] for Each Limit

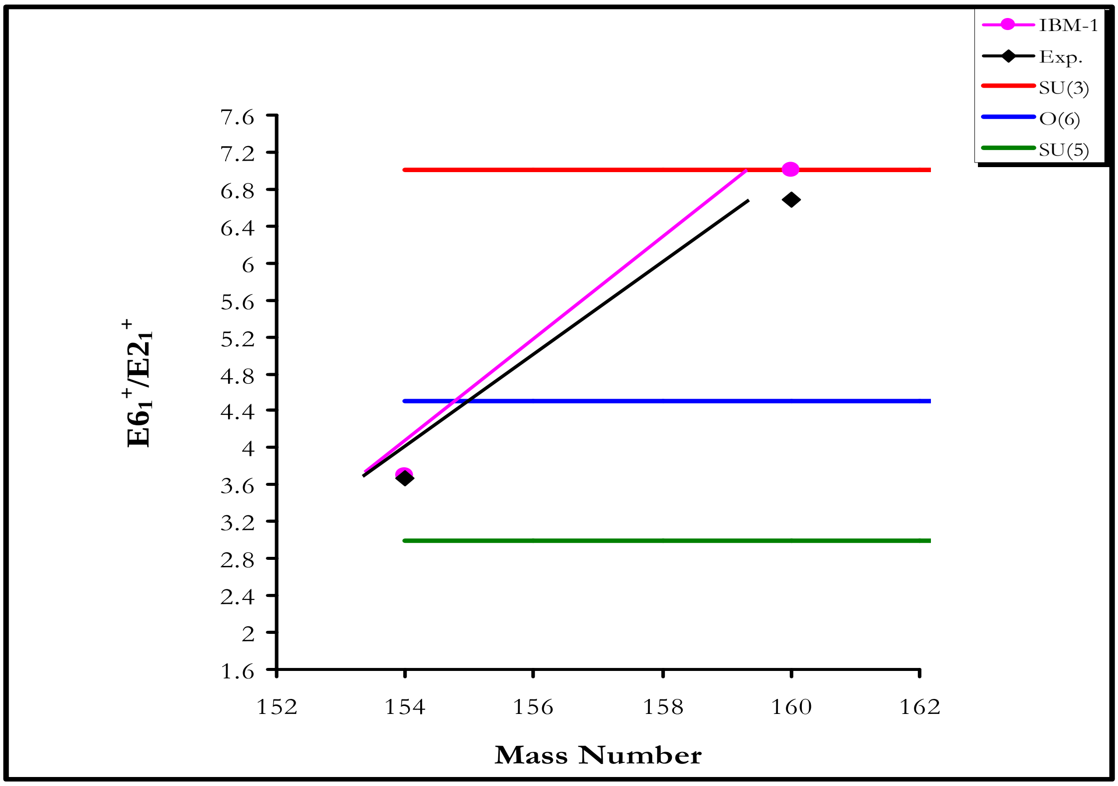

Figure 4: The Comparison of ($E6_1^+/E2_1^+$) Theoretically, Experimentally for ^{154,160}Dy Isotopes [23-25] and with the Typical Values [3,11] for Each Limit

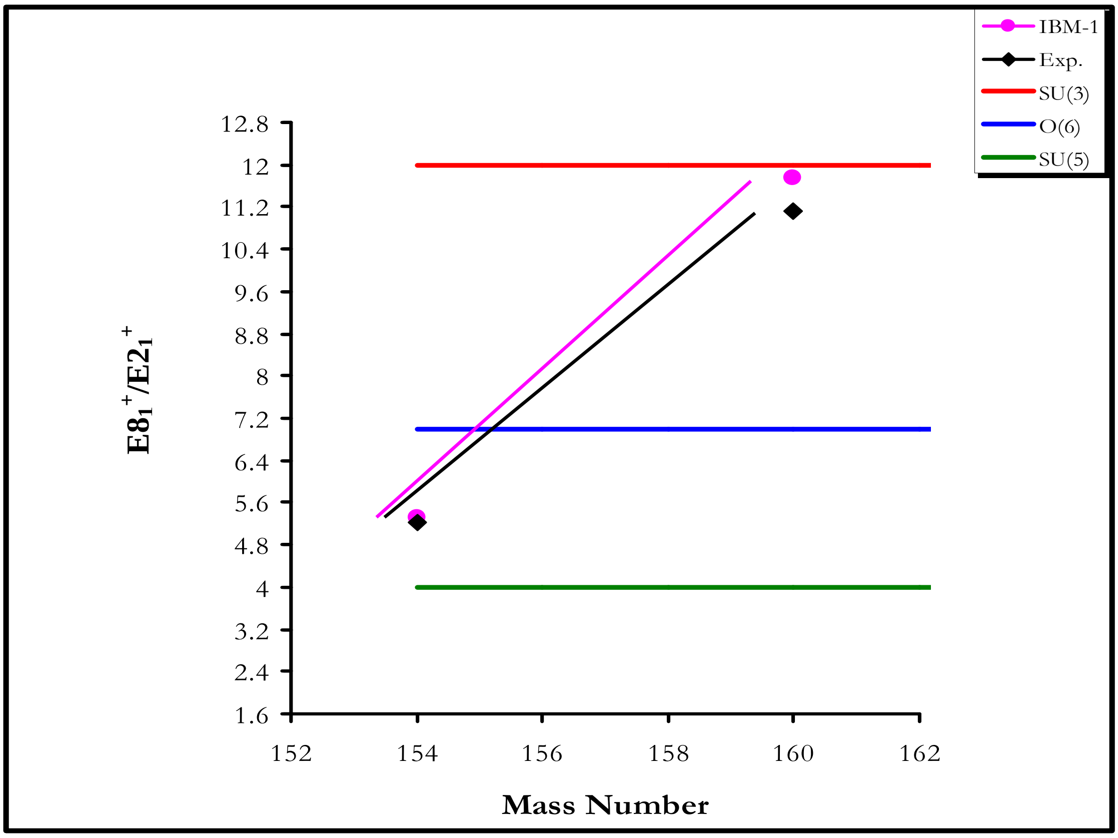

Figure 5: The Comparison of ($E8_1^+/E2_1^+$) Theoretically, Experimentally for ^{154,160}Dy Isotopes [23-25] and with the Typical Values [3,11] for Each Limit

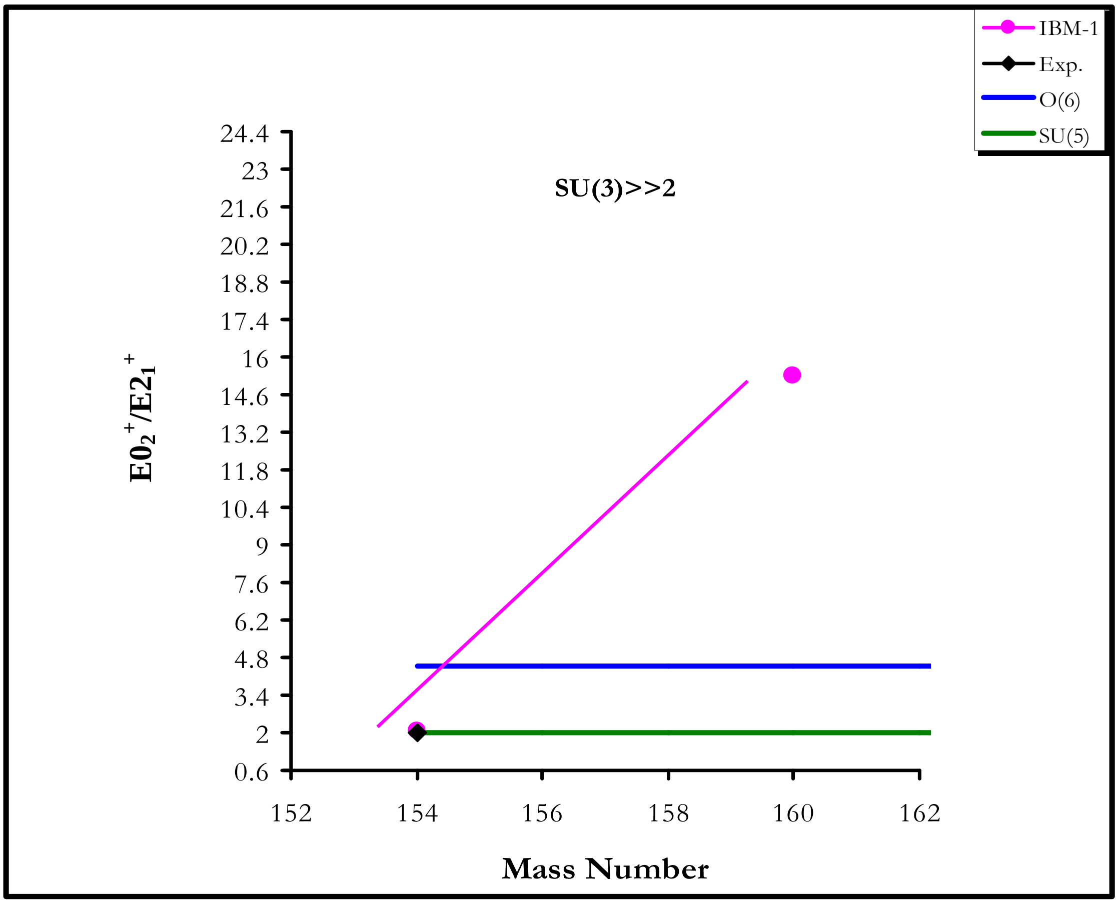

Figure 6: The Comparison of ($E0_2^+/E2_1^+$) Theoretically, Experimentally for ^{154,160}Dy Isotopes [23-25] and with the Typical Values [3,11] for Each Limit