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Optimal tilt angles for solar collectors facing south at Fez city (Morocco)

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Abstract

The main objective of this paper is to determine the optimal tilt angles for south-facing solar collectors at Fez city in Morocco. This is this would allow collectors to receive the maximum of the incident solar energy. Thirteen models were used to calculate the global solar radiation reaching an inclined surface facing south. To determine the optimal tilt angle we varied the inclination angle from -20° to 90° by a step of 5°. The performances of each model are evaluated by comparing the calculated and measured global solar radiation on a surface facing south and tilted by 34°. Results of the best found model show that the optimum tilted angle is varying between 62° (December) and -6° (June). Also, the yearly-average optimal tilt angle is found to be 32° for Fez.

Keywords: Optimum tilt angle; global solar radiation; diffuse solar radiation; Fez (Morocco).

Nomenclature

H_g	daily global solar radiation on horizontal surface (kWh/m² day).
H_d	daily diffuse solar radiation (kWh/m² day).
H_t	daily global solar radiation on tilted surface (kWh/m² day).
H_b	daily beam solar radiation on tilted surface (kWh/m² day).
H_0	daily extraterrestrial solar irradiation (kWh/m² day).
I_0	solar constant 1 367 W/m².
β_{opt}	optimum tilt of surface from horizontal (°).
φ	latitude of the place (°).
δ	solar declination (°).
R^2	correlation coefficient (%).
MSE	mean square error (kWh/m²day).
rRMSE	relative root mean square error (%).
rMBE	relative mean bias error (%).

1. Introduction

The performances of solar systems depend on several parameters. Among these parameters are the inclination and orientation of these systems. It is well known that the best orientation for a solar collector in the north hemisphere is the south direction but the inclination is changing every time.

A number of studies have already been done on optimal tilt and orientation angles determination for solar surfaces (Bakirci (2012), Jafarkazemi *et al.* (2012), Ghosh et al. (2010), Jafarkazemi & Saadabadi (2012), Demain *et al.* (2013), Gunerhan & Hepbasli (2007), El-Kassaby (1988),Tang & Wu (2004), Noorian et al. (2008), Kamali et al. (2006), and Jamil Ahmad & Tiwari (2009)). Liu & Jordan (1962) developed a simple method to estimate the monthly average daily radiation for each calendar month on surfaces facing directly towards the equator. Jafarkazemi et al. (2012) proposed the optimum tilt angle's maps for all regions in Iran. They applied an anisotropic method to estimate solar radiation on tilted surface by using 20-years of average cloud factor. El-Kassaby (1988) suggested the relation $\beta opt=\varphi+3.5$ for yearly optimum tilt angle for Egypt (φ is the latitude for the considered site). Tang & Wu (2004) obtained the yearly optimum tilt angles for different regions in China as $\beta opt=\varphi+$ (4 to -10).

The main objective of this study is to determine the optimal tilt angles for solar collectors facing south in the city of Fez (Morocco). The daily horizontal diffuse and global solar radiations, collected over the12 months of 2012, were used to estimate the total solar radiation on a south-facing surface of different inclination angles (-20°≤β≤90°). The measurements of the global solar radiation on a 34° tilted surface were used for evaluating all the thirteen models tested in this study.

2. Data and measurements

The data used in this work have been collected in the city of Fez in Morocco (latitude 33°56'N, longitude 4°99' W, altitude: 579 m) during the period between 1st January 2012 to December 2012. Fez is characterized by seasonal climate, shifting from cool and rain in winter to dry and hot in summer, but the weather is mild, especially during spring and autumn. The devices used for collected the data used in this work are: a Kip & Zonen (CM-11 model) pyranometer for measuring global solar irradiance on horizontal surface and on a surface tilted with 34° and facing south. Another identical pyranometer equipped with a shadow-band is used for measuring diffuse solar irradiance. Figure 1 shows the daily global solar radiation on horizontal surface and on a surface tilted with an angle of 34° and facing south and the daily diffuse solar radiation at Fez during the considered year.

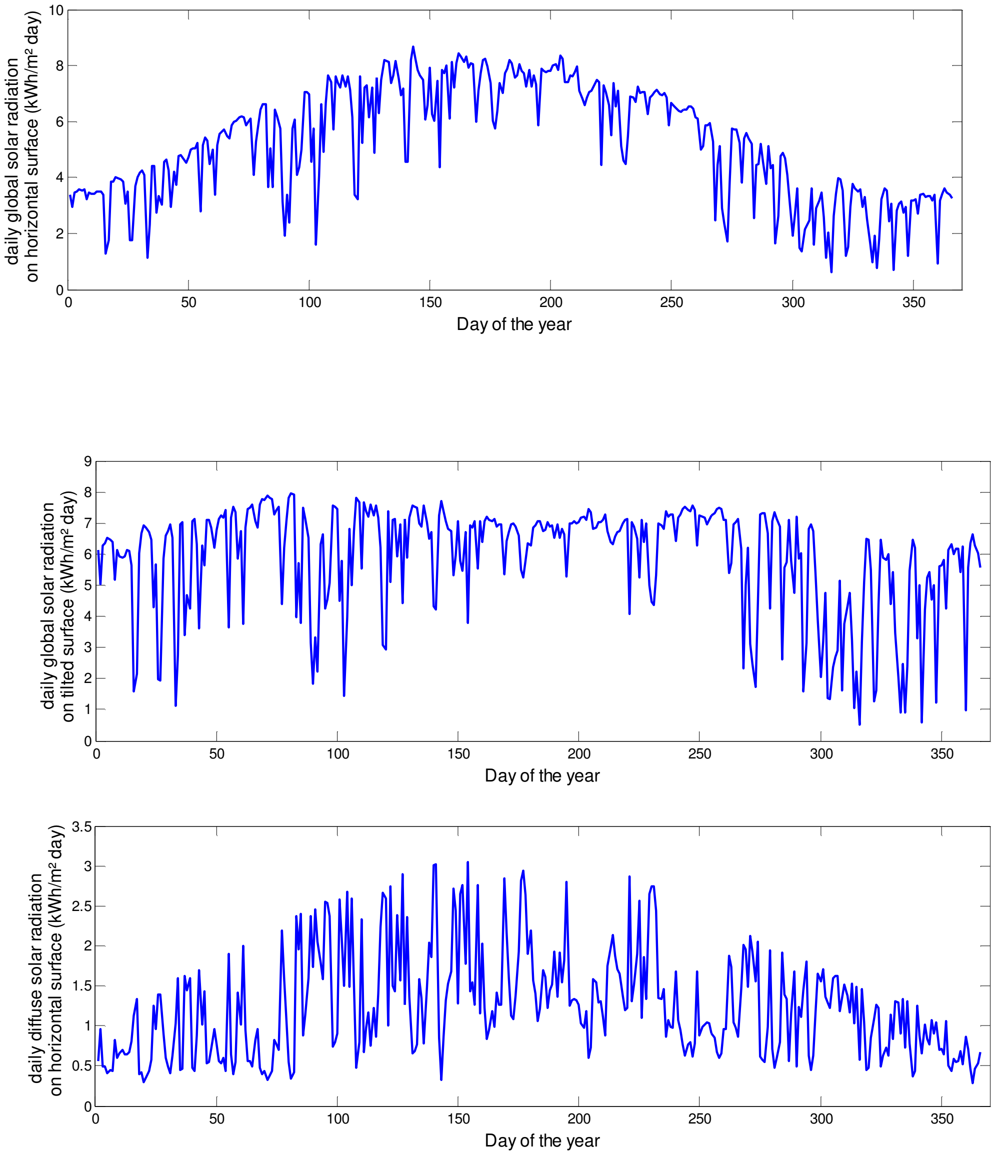

Figure 1. Daily variations of global solar radiation on horizontal surface, global solar radiation on 34° tilted south-facing surface and diffuse solar radiation at Fez.

3. Models for estimation of solar radiation on a tilted surface

The daily global solar irradiation received on a tilted surface consists of three components including: beam, diffuse and reflected from the ground (Liu & Jordan (1962)):

$$H_t = H_{TB} + H_{TR} + H_{TD} \quad (1)$$

Where H_{TB}, H_{TR} and H_{TD} are respectively the daily beam, reflected and diffuse radiation on a tilted surface in (kWh/m²day).

3.1 Daily reflected component on a tilted surface

The daily ground reflected radiation incident on an inclined surface in (kWh/m²day) is given by the relation (Liu & Jordan (1962)):

$$H_{TR} = R_r H_g \quad (2)$$

$$R_r = \rho\left(\frac{1+\cos(\beta)}{2}\right) \quad (3)$$

Where R_r is the ratio of the average reflected radiation on the tilted surface and ρ (0.2) is the foreground's albedo.

3.2 Daily beam component on a tilted surface

The daily beam radiation incident on an inclined surface in (kWh/m²day) is given by the relation:

$$H_{TB} = R_b\left(H_g - H_d\right) \quad (4)$$

Where, R_b, is the ratio of the average beam radiation on the tilted surface to that on a horizontal surface for each day. R_b is given by (Liu & Jordan (1962)):

$$R_b = \frac{\cos(\varphi-\beta)\cos(\delta)\sin(\omega_s') + \omega_s'(\frac{\pi}{180})\sin(\varphi-\beta)\sin(\delta)}{\cos(\varphi)\cos(\delta)\sin(\omega_s) + \omega_s(\frac{\pi}{180})\sin(\varphi)\sin(\delta)} \quad (5)$$

Where φ is the latitude, δ is the solar declination, ω_s' is the sunset hour angle for the tilted surface and ω_s is the hour angle.

The solar declination for any day (n) of the year can be obtained as follows:

$$\delta = 23.45\sin(\frac{360(n+284)}{365}) \quad (6)$$

The sunset hour angle ω_s and the sunset hour angle ω_s' for the tilted surface are given by:

$$\omega_s = ar\cos[-\tan(\delta)\tan(\varphi)] \quad (7)$$

$$\omega_s' = \min\{\omega_s, ar\cos[-\tan(\delta-\beta)\tan(\varphi)]\} \quad (8)$$

3.3 Daily diffuse solar component on a tilted surface

The daily diffuse radiation incident on an inclined surface in (kWh/m²day) is given by the relation:

$$H_{TD} = R_d H_d \quad (9)$$

The methods used to calculate the ratio of the average diffuse radiation on the tilted surface to that on a horizontal surface R_d are classified as isotropic and anisotropic models. In this work we tested four isotropic models (table 1) and nine anisotropic models (table 2).

For the model M11 (Klucher (1979) the parameters θ and θ_z are calculated by the following equations:

$$\cos(\theta) = \sin(\delta)\sin(\phi-\beta) + \cos(\delta)\cos(\phi-\beta)\cos(\omega) \quad (10)$$

$$\cos(\theta_z) = \sqrt{1-\sin(\delta)\sin(\varphi-\beta) + \cos(\delta)\cos(\varphi-\beta)\cos(\omega)} \quad (11)$$

Table 1. Isotropic models for calculating R_d

Model n°	Model formulas	Source
M1	$R_d = \frac{[1+\cos(\beta)]}{2}$	Liu & Jordan (1962)
M2	$R_d = \frac{[2+\cos(\beta)]}{3}$	Koronakis (1986)
M3	$R_d = \frac{[3+\cos(2\beta)]}{4}$	Badescu (2002)
M4	$R_d = 1 - \frac{\beta}{180}$	Tian *et al.* (2001)

Table 2. Anisotropic models for calculating R_d

Model n°	Model formulas	Source
M5	$R_d = \frac{H_b}{H_0} R_b + \left(1 - \frac{H_b}{H_0}\right)\frac{[1+\cos(\beta)]}{2}$	Hay (1979)
M6	$R_d = \frac{H_b}{H_0} R_b + \Omega\cos(\beta) + \left(1 - \frac{H_b}{H_0} - \Omega\right)\frac{[1+\cos(\beta)]}{2}$; where $\Omega = \left\{\max\left[0, \left(0.3 - 2\frac{H_b}{H_0}\right)\right]\right\}$	Skartveit & Olseth (1986)
M7	$R_d = 0.51 R_b + \frac{[1+\cos(\beta)]}{2} - \frac{1.74}{1.26\pi}\left[\sin(\beta) - \left(\beta\frac{\pi}{180}\right)\cos(\beta) - \pi\sin^2\left(\frac{\beta}{2}\right)\right]$	Steven & Unsworth (1980)
M8	$R_d = \frac{H_b}{H_0} R_b + \left(1 - \frac{H_b}{H_0}\right)\frac{[1+\cos(\beta)]}{2}\left[1 + \sqrt{\frac{H_b}{H_0}}\sin^3\left(\frac{\beta}{2}\right)\right]$	Reindel *et al.* (1990)
M9	$R_d = \left[\frac{1+\cos(\beta)}{2}\right]\left[1 + \left(1 - \left(\frac{H_d}{H_0}\right)^2\right)\sin^3\left(\frac{\theta}{2}\right)\right]\left[1 + \left(1 - \left(\frac{H_d}{H_0}\right)^2\right)\cos^2(\theta)\cos^3(\theta_z)\right]$	Klucher (1979)
M10	$R_d = \cos^2\left(\frac{\beta}{2}\right)\left[1 + \sin^3\left(\frac{\beta}{2}\right)\right]\left[\left(1 + \cos^2(\theta)\right)\sin^3(\theta_z)\right]$	Temps & coulson (1977)
M11	$R_d = \frac{1+\cos(\beta)}{2} + 0.05\frac{H_b R_b}{H_d}\left(\cos(\theta) - \frac{1}{\cos(\theta_z)}\left(\frac{1+\cos(\beta)}{2}\right)\right)$	Bugler (1977)
M12	$R_d = \frac{H_g}{H_0} R_b + \left(1 - \frac{H_g}{H_0}\right)\left(\frac{1+\cos(\beta)}{2}\right)$	Iqbal (1983)
M13	$R_d = \frac{H_b}{H_0} R_b + \left(1.0115 - 0.20293\beta - 0.080823\beta^2\right)\left(1 - \frac{H_b}{I_0}\right)$	Willmot (1982)

4. Performances evaluation

The accuracy of different models is assessed by means of four widely used statistic parameters: mean square error (MSE), relative root mean square error (RRMSE), mean absolute percentage error (MAPE) and the correlation coefficient (R^2) which is used to test the linear regression between predicted and observed data.

The following expressions were used:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(H_{ci} - H_{mi})^2 \quad (12)$$

$$RRMSE = 100\frac{\sqrt{n\sum_{i=1}^{n}(H_{ci} - H_{mi})^2}}{\sum_{i}^{n} H_{mi}} \quad (13)$$

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{H_{ci} - H_{mi}}{H_{mi}} \times 100\right| \quad (14)$$

$$R^2 = \frac{\sum_{i}^{n}(H_{ci} - \overline{H}_c)(H_{mi} - \overline{H}_m)}{\left\{\left[\sum_{i}^{n}(H_{ci} - \overline{H}_c)^2\right]\left[\sum_{i}^{n}(H_{mi} - \overline{H}_m)^2\right]\right\}^{0.5}} \quad (15)$$

Where n is the number of data, H_{ci} is the i^{th} predicted value, H_{mi} is the i^{th} measured value and $\bar{H}_m$ and $\bar{H}_c$ are respectively the measured and calculated mean values.

5. Results and discussions

Based on the analysis presented above, the daily solar radiation at different tilt angles was calculated by using four isotropic and nine anisotropic models presented in tables 1 and 2.

Table 3. The performances obtained for estimated daily global solar radiation on a surface tilted by 34° by using the isotropic and anisotropic models.

Models	**R^2**	**MSE**	**RRMSE**	**MAPE**
M1	99.55	0.0304	2.9626	2.6103
M2	**99.59**	**0.0256**	**2.7173**	**2.5472**
M3	99.41	0.0563	4.0309	3.5514
M4	99.33	0.0748	4.6469	4.2488
M5	98.73	0.0790	4.7754	4.3728
M6	98.72	0.0788	4.7712	4.3013
M7	98.41	0.8064	15.2592	19.7073
M8	98.75	0.0788	4.7712	4.3861
M9	99.06	0.1876	7.3604	6.7388
M10	98.22	0.3036	9.3623	9.4577
M11	99.51	0.1145	5.7489	4.7940
M12	99.36	0.0442	3.5707	3.8735
M13	99.45	0.0474	3.6984	3.1988

All models were evaluated by using the statistical performances obtained by comparing the measured and the calculated global solar irradiances on a 34° tilted surface. The statistical performances are summarized in Table 3. It can be observed from this table that the M2 model gives the highest correlation coefficient (99.59%) and the smaller values of MSE (0.0256 kWh/m²day), RRMSE(2.7173%) and MAPE(2.5472%).

All other models give the best results, however, acceptable results except the model of M7 (Steven &Unsworth (1980)) for which the RRMSE (15.2592%) and the MAPE (19.7073%) are relatively large.

Figure 2 shows the comparison between measured daily global solar radiation on a surface facing south and tilted by 34° and the calculated values by using M2 model for a surface with same orientation.

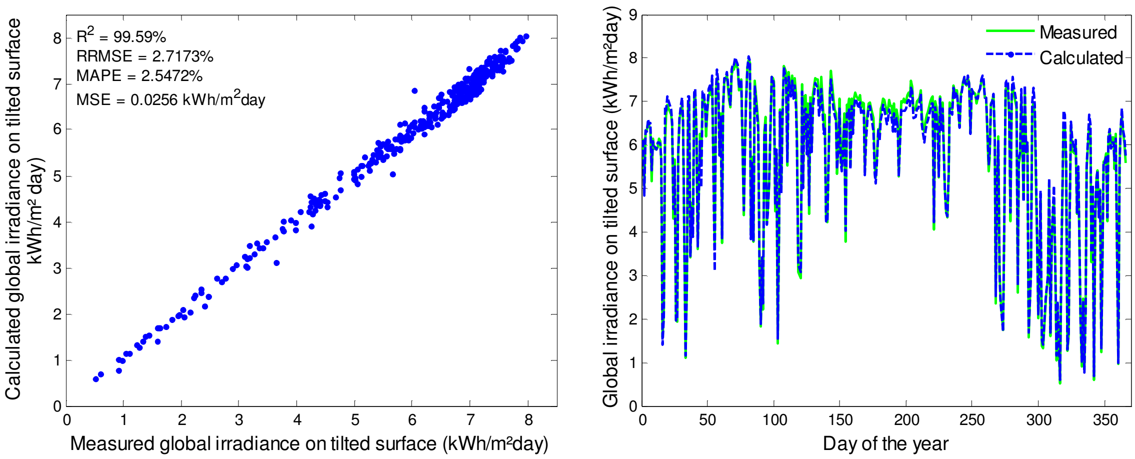

Figure 2. Comparison between measured and calculated daily global solar radiations on tilted south-facing surface (34°) by using M2 model.

To determine the optimum angle of inclination by using M2 model (Koronakis (1986)), we calculated the solar radiation by changing the angle of reception from -20° to 90°with a step of 5° for each month of the year. Figure 3 and 4 show the calculated monthly average daily global solar radiations at Fez city. It is clear from these graphs that it exists, for each month of the year, a unique β_{opt} for which the solar radiation is at its peak for the given month.

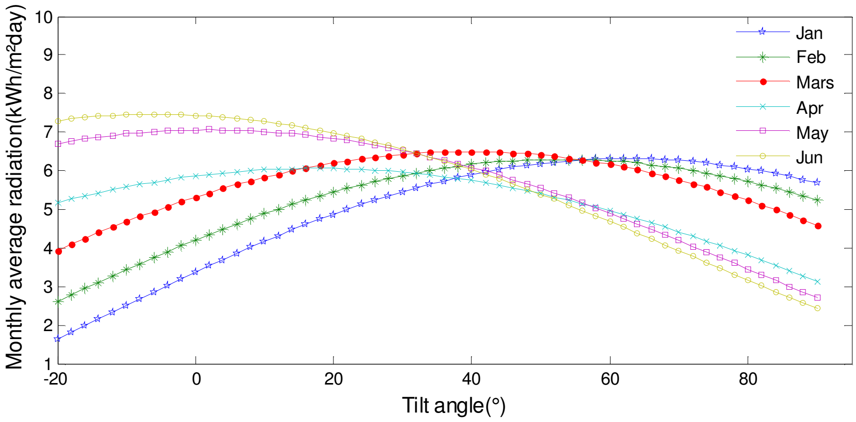

Figure 3. Monthly average daily solar radiations versus tilt angle for a south-facing surface for winter and spring months.

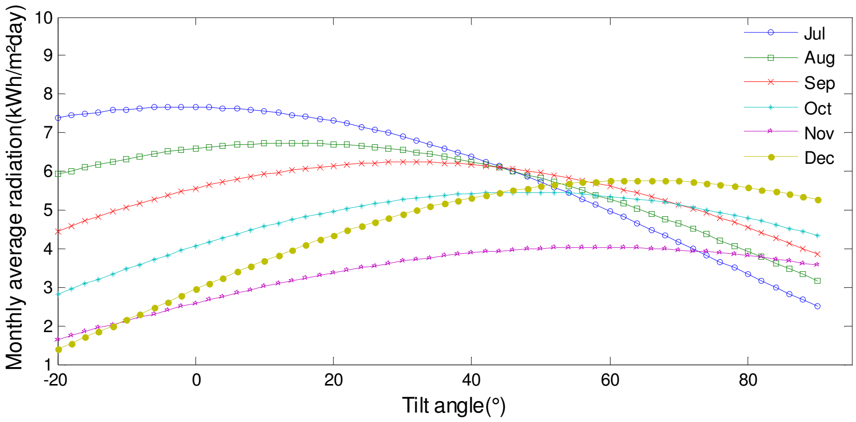

Figure 4. Monthly average daily solar radiations versus tilt angle for a south-facing solar surface for summer and autumn months.

We have then found that, for the city of Fez, the optimum tilted angle changed between -6° (June) and 64° (December). Table 4 shows the measured global solar radiations on horizontal and on 34° tilted surfaces and the global solar radiation calculated by M2 model by using the optimum tilt angle.

Table 4. Monthly average global solar radiations measured on horizontal surface H_g, on 34° inclined surface H_t and calculated $H_t(34°)$ by M2 model by using the optimum tilt angle β_{opt} (last column).

Month	H_g(kWh/m^{2}day)	H_t(kWh/m^{2}day)	H_t(34°)(kWh/m^{2}day)	β_{opt}(°)
Jan.	3.3602	6.3119	5.5981	62
Feb.	4.1738	6.2763	5.9705	53
Mar.	5.3658	6.6073	6.5634	40
Apr.	5.7190	5.9194	5.8075	18
May.	7.1074	7.1105	6.5843	2
Jun.	7.3708	7.3992	6.4709	-6
Jul.	7.6997	7.7045	6.9096	-2
Aug.	6.5842	6.7039	6.4980	13
Sep.	5.6660	6.3447	6.3183	30
Oct.	4.1151	5.5212	5.2719	47
Nov.	2.6339	4.0863	3.7142	58
Dec.	2.8602	5.5800	4.8900	64

Monthly, seasonal and yearly optimum tilt angles are shown in Figure 5. Optimum tilt angles in summer and autumn months have minimum values and the highest optimum tilt angles values belong to winter and spring. The yearly average tilt angle is about 32°.

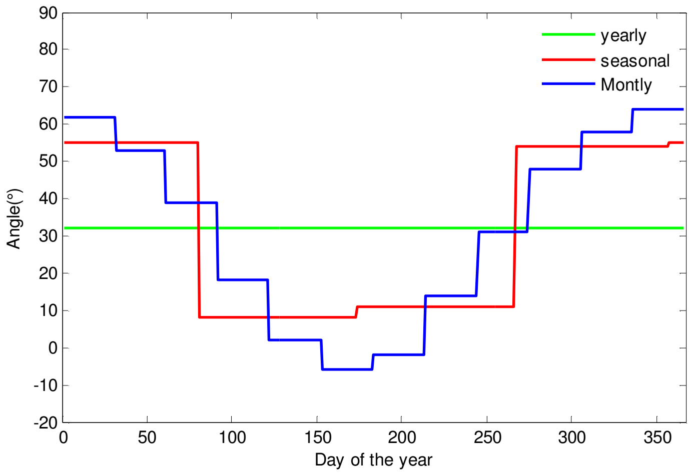

Figure 5. Monthly, seasonal and yearly optimum tilt angles.

Table 5. The optimum tilt angles for each month obtained by applying the thirteen tested models.

Models	Jan.	Feb.	Mar.	Apr.	May.	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.	Yearly
M1	61	52	39	17	2	-6	-2	12	29	46	55	63	30
M2	62	53	40	18	2	-6	-2	13	30	47	58	64	32
M3	61	51	37	14	2	-5	-2	11	27	43	54	63	28
M4	62	53	39	12	-4	-13	-7	8	28	46	57	64	29
M5	63	53	40	18	2	-6	-2	13	31	47	58	64	32
M6	62	53	40	18	2	-6	-2	13	30	47	57	64	32
M7	62	54	41	21	3	-7	-3	15	32	49	59	64	34
M8	63	54	40	18	2	-6	-2	13	31	48	59	65	32
M9	60	51	38	18	6	-2	1	15	29	44	53	62	32
M10	60	50	37	19	7	-1	1	16	28	44	52	62	32
M11	61	52	39	17	2	-4	-1	12	29	46	55	63	31
M12	62	53	39	18	2	-7	-3	13	30	47	58	64	31
M13	62	52	39	14	-2	-10	-5	10	29	46	56	64	30

In table 5, we report the optimum tilt angles obtained by applying the thirteen tested models. In this table, we can see that the difference between results vary between 12° in June and 3° in January.

Conclusion

In this work, thirteen methods were tested to determine the optimum tilt angles for south-facing solar collectors for obtaining the maximum solar radiation at Fez city in Morocco. The comparison between measured and calculated global solar radiation on 34° inclined surface shows that M2 model (Koronakis (1986)) gives the highest correlation coefficient (99.59%) and the smaller values of MSE (0.0256 kWh/m²day), RRMSE (2.7173%) and MAPE(2.5472%) than the other models.

The obtained results by applying M2 show that: The optimum tilt angle in June go to a minimum of -6° then it increases during the winter months and reaches a maximum of 64°in December. Also, the average optimum tilt angles for the summer and winter month's are12°and 55° respectively. Finally, the yearly average optimum tilt angle is found to be 32°. This angle can be used for fixed installations or for building applications.

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