

Information Criteria and Log Linear Models Selection for Contingency Tables

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Abstract

In this paper, Log linear model analysis was used to test the significance of interactions of three categorical variables as well as determining the most parsimonious model that best fit the data. Many approaches exist in model selection; backward elimination method is however used in this work. Likelihood ratio statistic (G^2), Akaike information criteria (AIC) and Bayesian information criteria (BIC) were used to check the adequacy of the model of the best fit. The data used is secondary data and SPSS version 16.0 statistical software was used for the analysis. It was discovered from the result of the analysis that all two-factor interactions were significant except breed*chick loss. This implies that breed is independent of chick loss. We also observed that the best model fitted has a generating class: breed*age, age*chick loss. The final model in harmony with the hierarchy principle is $\text{Log } m_{ijk} = \mu + \mu_{B(i)} + \mu_{A(j)} + \mu_{C(k)} + \mu_{BA(ij)} + \mu_{AC(jk)}$.

Keywords: categorical data, hierarchical log linear models, Likelihood ratio test statistic, AIC, BIC, interaction.

1. INTRODUCTION

Log linear models are used for qualitative data in contingency tables. Log linear provide a powerful tool for testing out associations among categorical variables. It is a specialized case of generalized linear model for Poisson distributed data and is more commonly used for analyzing multi-dimensional contingency tables that involves more than two variables, although, it can also be used to analyze two-way contingency table (Knoke and Burke, 2002). There are many approaches in model selection viz: stepwise selection, backward elimination, selection based on standardized estimates of parameter of saturated models and selection based on marginal and partial association due to Brown (1978) in selecting model and/or models of best fit. In log linear analysis, expected values of the observations are given by linear combination of number of parameters. Akaike information criteria and Bayesian information criteria are used for model fit. The comparison of the model using information criteria can be viewed as equivalent to likelihood ratio test and understanding the differences among criteria may make it easier to compare the results and used them to make informed decisions (Akaike, 1973). This research work aimed at developing an appropriate log linear model for examining associations/interactions among breed, age and chick loss of chickens collected from Sambo feeds, Awka, Nigeria. This work will also examine the adequacy of the model fit using AIC and BIC information criterion.

2. Review of Related Literature

Goodman (1971) provided both forward and backward stepping procedures in obtaining suitable model that will suit the data for the hierarchical log linear model in contingency table. The results shown declared that backward stepping procedure is more reliable to be used in obtaining suitable model that fit the data.

Andersen (1974) presented a survey on the theory of multidimensional contingency tables and showed that estimation and performance of asymptotic as well as exact tests are simple if only the decomposable models are considered.

Benedetti and Brown (1978) proposed strategies in building log-linear models in multidimensional contingency tables using either stepwise method or standardized estimates of the parameters of the saturated model.

Onder and Adiguzel (2010) applied hierarchical log linear analysis method to occupational fatalities in the underground coal mines of Turkish Hardcoal Enterprises. The accidents records were evaluated and the main factors affecting the accidents were defined. The results found showed that the mostly affected job group by the fatality accidents was the production workers and these workers were mostly exposed to roof collapses and methane explosions.

Olmus (2012) presented a modeling effort in estimating the relationships between driver's fault and carelessness and the traffic variables. The result of the analysis showed that the best fit model regarding these variables was log-linear model. Also, the associations of the factors with the accident severity and the contributions of the various factors and interactions between these variables were assessed. The obtained results provided valuable information in regard to preventing undesired consequences of traffic accidents.

3. Research Methodology

The log linear modeling has been recommended as a statistical analysis method when the dependent and independent variables are categorical (Lacobucci, et al (1990)). A log linear model describes the association/ interaction pattern among a set of categorical variables. We try to fit a model so as to avoid using a saturated model. The saturated model in log linear model analysis is a model that incorporates all possible effects, such as one-way effect, two way interactions effect, and three-way interactions e.t.c. A saturated model imposes no constraints on the data and always reproduced the observed counts. The parsimonious model in log linear model analysis is incomplete model that achieves satisfactory level of goodness of fit. The log linear model is called hierarchical whenever the model contains higher-order effects incorporates lower-order effects composed of the variables. The reason for including lower-order terms is that the statistical significance and practical interpretation of higher –order terms depend on how the variables are coded. This is undesirable, but with hierarchical models, the same results are obtained, irrespective of how the variables are coded.(Ageresti, 2002). The log linear model for the contingency table is expressed as follows:

Log (expected cell frequency) is grand mean, main effects parameters, and second and higher order interactions. For instance, the model with only three main effects, that is independent model, is:

$$\log m_{ijk} = \mu + \mu_{B(i)} + \mu_{A(j)} + \mu_{C(k)} \quad i=1, 2; j=1,2,3; k=1,2.$$

Where μ is the overall effect, $\mu_{B(i)}$ is effect due to i^{th} level of B, $\mu_{A(j)}$ is effect due to j^{th} level of A, $\mu_{C(k)}$ is effect due to k^{th} level of C. We impose the sum- to- zero identifiability conditions.

$$\sum_i \mu_{B(i)} = \sum_j \mu_{A(j)} = \sum_k \mu_{C(k)} = 0$$

Also, the model with three main effects and three terms of two-way interactions, that is second order full model is

$\log m_{ijk} = \mu + \mu_{B(i)} + \mu_{A(j)} + \mu_{C(k)} + \mu_{BA(ij)} + \mu_{BC(ik)} + \mu_{AC(jk)}$. We impose the sum-to-zero identifiability conditions

$$\sum_i \mu_{BA(ij)} = \sum_j \mu_{BA(ij)} = \sum_i \mu_{BC(ik)} = \sum_k \mu_{BC(ik)} = 0$$

4. The mathematical log linear model for 3 dimensional contingency table

We fit the log-linear model with their possible interactions. The log-linear model containing the μ - terms for 3-variables is given by

$$\log m_{ijk} = \mu + \mu_{1(i)} + \mu_{2(j)} + \mu_{3(k)} + \mu_{12(ij)} + \mu_{13(ik)} + \mu_{23(jk)} + \mu_{123(ijk)}$$

The underlying distribution for log- linear model in any multi dimensional table is multinomial, product multinomial or independent Poisson (Fienberg, 1970).Without loss of generality; consider the multinomial underlying distribution for n_{ijk} , Jolayemi (1989), then

$$P(n, \pi_{ijk}) = \binom{n}{(n_{ijk})} \prod_{ijk} (\pi_{ijk})^{n_{ijk}}$$

$$L = \log p = \log \binom{n}{(n_{ijk})} + \sum_{ijk} n_{ijk} \log \pi_{ijk}$$

$$= \log \binom{n}{(n_{ijk})} + \sum_{ijk} n_{ijk} \log \left(\frac{m_{ijk}}{n} \right)$$

$$= \log \binom{n}{\underline{n}_{ijk}} + \sum_{ijk} n_{ijk} \log(m_{ijk}) - n \log n$$

Making use of the langragian multiplier we have

$$F = \log \binom{n}{\underline{n}_{ijk}} + \sum_{ijk} n_{ijk} \log(m_{ijk}) - n \log n - \lambda \left(\sum_{ijk} m_{ijk} - n \right)$$

$$F = \log \binom{n}{\underline{n}_{ijk}} + \sum_{ijk} n_{ijk} \left(\mu + \mu_{1(i)} + \mu_{2(j)} + \mu_{3(k)} + \mu_{12(ij)} + \mu_{13(ik)} + \mu_{23(jk)} \right) - \lambda \left[\sum_{ijk} e^{\mu + \mu_{1(i)} + \mu_{2(j)} + \mu_{3(k)} + \mu_{12(ij)} + \mu_{13(ik)} + \mu_{23(jk)}} - n \right]$$

Taking derivations, we have

$$\frac{\partial F}{\partial \mu} = \sum_{ijk} n_{ijk} - \lambda \sum_{ijk} e^{\mu + \mu_{1(i)} + \mu_{2(j)} + \mu_{3(k)} + \mu_{12(ij)} + \mu_{13(ik)} + \mu_{23(jk)}} = 0$$

$$\frac{\partial F}{\partial \mu_{1(i)}} = \sum_{jk} n_{ijk} - \lambda \sum_{jk} e^{\mu + \mu_{1(i)} + \mu_{2(j)} + \mu_{3(k)} + \mu_{12(ij)} + \mu_{13(ik)} + \mu_{23(jk)}} = 0$$

$$\frac{\partial F}{\partial \mu_{2(j)}} = \sum_{ik} n_{ijk} - \lambda \sum_{ik} e^{\mu + \mu_{1(i)} + \mu_{2(j)} + \mu_{3(k)} + \mu_{12(ij)} + \mu_{13(ik)} + \mu_{23(jk)}} = 0$$

$$\frac{\partial F}{\partial \mu_{3(k)}} = \sum_{ij} n_{ijk} - \lambda \sum_{ij} e^{\mu + \mu_{1(i)} + \mu_{2(j)} + \mu_{3(k)} + \mu_{12(ij)} + \mu_{13(ik)} + \mu_{23(jk)}} = 0$$

$$\frac{\partial F}{\partial \mu_{12(ij)}} = \sum_k n_{ijk} - \lambda \sum_k e^{\mu + \mu_{1(i)} + \mu_{2(j)} + \mu_{3(k)} + \mu_{12(ij)} + \mu_{13(ik)} + \mu_{23(jk)}} = 0$$

$$\frac{\partial F}{\partial \mu_{13(ik)}} = \sum_j n_{ijk} - \lambda \sum_j e^{\mu + \mu_{1(i)} + \mu_{2(j)} + \mu_{3(k)} + \mu_{12(ij)} + \mu_{13(ik)} + \mu_{23(jk)}} = 0$$

$$\frac{\partial F}{\partial \mu_{23(jk)}} = \sum_i n_{ijk} - \lambda \sum_i e^{\mu + \mu_{1(i)} + \mu_{2(j)} + \mu_{3(k)} + \mu_{12(ij)} + \mu_{13(ik)} + \mu_{23(jk)}} = 0$$

The equation above may be written as

$$n - \lambda \sum_{ijk} e^{\mu + \mu_{1(i)} + \mu_{2(j)} + \mu_{3(k)} + \mu_{12(ij)} + \mu_{13(ik)} + \mu_{23(jk)}} = 0 \tag{1.0}$$

$$n_{i..} - \lambda \sum_{jk} e^{\mu + \mu_{1(i)} + \mu_{2(j)} + \mu_{3(k)} + \mu_{12(ij)} + \mu_{13(ik)} + \mu_{23(jk)}} = 0 \quad (1.1)$$

$$n_{.j.} - \lambda \sum_{ik} e^{\mu + \mu_{1(i)} + \mu_{2(j)} + \mu_{3(k)} + \mu_{12(ij)} + \mu_{13(ik)} + \mu_{23(jk)}} = 0 \quad (1.2)$$

$$n_{..k} - \lambda \sum_{ij} e^{\mu + \mu_{1(i)} + \mu_{2(j)} + \mu_{3(k)} + \mu_{12(ij)} + \mu_{13(ik)} + \mu_{23(jk)}} = 0 \quad (1.3)$$

$$n_{ij.} - \lambda \sum_k e^{\mu + \mu_{1(i)} + \mu_{2(j)} + \mu_{3(k)} + \mu_{12(ij)} + \mu_{13(ik)} + \mu_{23(jk)}} = 0 \quad (1.4)$$

$$n_{i.k} - \lambda \sum_j e^{\mu + \mu_{1(i)} + \mu_{2(j)} + \mu_{3(k)} + \mu_{12(ij)} + \mu_{13(ik)} + \mu_{23(jk)}} = 0 \quad (1.5)$$

$$n_{.jk} - \lambda \sum_i e^{\mu + \mu_{1(i)} + \mu_{2(j)} + \mu_{3(k)} + \mu_{12(ij)} + \mu_{13(ik)} + \mu_{23(jk)}} = 0 \quad (1.6)$$

From equation (19) above, it can be show that $\lambda = 1$

$$\text{Since } n - \lambda \sum m_{ijk} = n - \lambda n$$

Where

$$m_{ijk} = e^{\mu + \mu_{1(i)} + \mu_{2(j)} + \mu_{3(k)} + \mu_{12(ij)} + \mu_{13(ik)} + \mu_{23(jk)}}$$

$$\therefore \lambda = 1$$

From the above equations we have the estimates of the μ -terms as

$$\sum_{ijk} n_{ijk} - \sum_{ijk} e^{\mu + \mu_{1(i)} + \mu_{2(j)} + \mu_{3(k)} + \mu_{12(ij)} + \mu_{13(ik)} + \mu_{23(jk)}} = 0$$

$$\sum_{ijk} n_{ijk} - e^{\mu} \sum_{ijk} e^{\mu_{1(i)} + \mu_{2(j)} + \mu_{3(k)} + \mu_{12(ij)} + \mu_{13(ik)} + \mu_{23(jk)}} = 0$$

$$\sum_{ijk} n_{ijk} - e^{\mu} IJK = 0$$

$$e^{\mu} = \frac{\sum_{ijk} n_{ijk}}{IJK}$$

$$\hat{\mu} = \log \left(\frac{\sum_{ijk} n_{ijk}}{IJK} \right) \quad (1.7)$$

Similarly

$$\hat{\mu}_{1(i)} = \log \left(\frac{\sum_{jk} n_{i..}}{JK} \right) - \mu \quad (1.8)$$

$$\hat{\mu}_{2(j)} = \log \left(\frac{\sum_{ik} n_{.j.}}{IK} \right) - \mu \quad (1.9)$$

$$\hat{\mu}_{3(k)} = \log \left(\frac{\sum_{ij} n_{..k}}{IJ} \right) - \mu \quad (1.91)$$

$$\hat{\mu}_{12(ij)} = \log \left(\frac{\sum_k n_{ij.}}{K} \right) - \mu - \hat{\mu}_{1(i)} - \hat{\mu}_{2(j)} \quad (1.92)$$

$$\hat{\mu}_{13(ik)} = \log \left(\frac{\sum_j n_{i.k}}{J} \right) - \mu - \hat{\mu}_{1(i)} - \hat{\mu}_{3(k)} \quad (1.93)$$

$$\hat{\mu}_{23(jk)} = \log \left(\frac{\sum_i n_{.jk}}{I} \right) - \mu - \hat{\mu}_{2(j)} - \hat{\mu}_{3(k)} \quad (1.94)$$

5. Data Presentation and Analysis

Data for 510 chickens on breed, age and chick loss were collected from poultry record book of Sambo feeds, Awka. The breed, age, and chick loss were classified into two, three and two categories of levels respectively.

Table (1): Description of data (variables)

Number	Variables	Coding/values	Abbreviations
1	Breed	1= broiler 2= old layer	Breed (B)
2	Age	1= one month 2= two months 3= three months	Age(A)
3	Chick loss	Yes = Loss No = No Loss	Chick loss(C)

Table (2): Data Presentation

Breed	Age	Chick loss	
		Yes	No
Broiler	1	55	67
	2	16	44
	3	8	45
Old layer	1	48	66
	2	20	52
	3	18	71

6. Model of this Study

The log linear model to be considered is the three dimensional case since three variables or factors are involved. The saturated log linear model for this contingency table with three variables is given as

$$\log m_{ijk} = \mu + \mu_{B(i)} + \mu_{A(j)} + \mu_{C(k)} + \mu_{BA(ij)} + \mu_{BC(ik)} + \mu_{AC(jk)} + \mu_{BAC(ijk)} \quad (2.1)$$

Where

$\log m_{ijk}$ = log of expected count

μ = the grand mean or overall mean

$\mu_{B(i)}$ = i^{th} level of breed

$\mu_{A(j)}$ = j^{th} level of age

$\mu_{C(k)}$ = k^{th} level of chick loss

$\mu_{BA(ij)}$ = interaction between i^{th} level of breed and j^{th} level of age

$\mu_{BC(ik)}$ = Interaction between i^{th} level of breed and k^{th} level of chick loss

$\mu_{AC(jk)}$ = Interaction between j^{th} level of age and k^{th} level of chick loss

$\mu_{BAC(ijk)}$ = Interaction between i^{th} level of breed, j^{th} level of age and k^{th} level of chick loss

7. Model Selection

The main goal is to find the fewest model that fits the data. The method adopted for this research is backward elimination. In backward elimination, we usually start with the most complex model. Term is then sequentially deleted from the model. G^2 is computed for each of the current and reduced model (model resulting from deletion) and using a cut off of predetermined α , say 0.05, we delete the term for which p-value is significant (term with highest p-value). The process continues further deletion which leads to a significantly poorer fit.

8. Comparison of Models and Goodness of Fit Measure

The goodness of fit can be tested using either the Pearson chi-square test statistic or likelihood ratio statistic (G^2) to find the best model from possible models. The G^2 statistic is given as

$$G^2 = 2 \sum_{ijk} n_{ijk} \ln \left(\frac{n_{ijk}}{m_{ijk}} \right) \quad (2.2)$$

Where G^2 is chi-square distributed with degree of freedom (d.f) equal to:
 d.f = number of cells in the table – number independent parameters estimated.

9. Information Theory measures of Model Fit

Information criteria should be considered also for testing for the adequacy of the model fit. Akaike information criteria (Akaike, 1973) is given as :

$$AIC = G^2 - 2 \text{ d.f} \quad (2.3)$$

Another information criteria is Bayesian Information (Raftery, 1986) is given as

$$BIC = G^2 - \ln n \text{ d.f} \quad (2.4)$$

Where n and d.f is the total sample size and degree of freedom respectively.

10. Results and Discussion of the Analysis

Table (3): Tests whose k-way and higher order effects are zero

K	d.f	G^2 (likelihood ratio)	p- value
1	11	142.126	0.000
2	7	37.138	0.000
3	2	0.826	0.662

In Table 3, k=3 gives the G^2 for the model without the three- factor interactions

$B^* A^* C^*$. That is, the line tests for hypothesis that $BAC=0$. From this result, there is no sufficient reason not accept this hypothesis for $k=3$ ($p > 0.05$). Similarly, $k=2$ indicates the model without the third and second order effects. That is, the line tests the hypothesis that $BA=BC=AC=0$. For $k=1$, corresponding to a model that has no effects. The first and second effects were significant ($p < 0.05$). Finally, a model with the first and second order effects can be seen to be adequate to represent our data.

Table 4: Test of partial Associations

Effect name	Abbreviations	d.f	Partial chi-square	p-value
breed*age	BA	2	7.136	0.028
breed*chick loss	BC	1	0.004	0.951
age*chick loss	AC	2	28.543	0.000
Breed	B	1	3.141	0.076
Age	A	2	36.929	0.000
Chick loss	C	1	64.919	0.000

In Table 4, the model run showed that breed*age and age*chick loss of the second order interaction parameters were significant ($p\text{-value} < 0.05$) while breed*chick loss of the second order interaction was not significant ($p\text{-value} > 0.05$). Also, the main effects terms age and chick loss were significant ($p\text{-value} < 0.05$), while the main effect term breed was not significant ($p\text{-value} > 0.05$).

The result of backward elimination for the selection of the best fit model can be seen in Table 5

Table 5: The results of the backward elimination search for the best fit of the model.

Step in the modeling process	Deleted effect	Chi-square	Degree of freedom	p-value
1	breed*age*chick loss	0.826	2	0.662
2	breed*age breed*chick loss age*chick loss	7.136 0.004 28.543	2 1 2	0.028 0.951 0.000
3	breed*age age*chick loss	7.450 28.857	2 2	0.024 0.000
4 Generating class	breed*age age*chick loss	0.830	3	0.842

In Table 5, the ‘‘deleted effect’’ is the change in the chi-square after the effect is deleted from the model. At each step the effect with the highest significance level for the likelihood ratio is deleted provided the significance level is higher than 0.05. In Table 5, the process begins with the saturated model, the 3-way interaction term is removed but this does not have a significant effect ($p\text{-value} > 0.05$). The two-way interaction term are eliminated one at time as the next steps ($p > 0.05$). Thus, non-significant interaction terms are removed one at time until all those left are significant. The process then ends and concludes that the best fit for this data set has the generating class

$$\text{Log } m_{ijk} = \mu + \mu_{B(i)} + \mu_{A(j)} + \mu_{C(k)} + \mu_{BA(ij)} + \mu_{AC(jk)}$$

According to this model, it is observed that breed*chick loss of second order interaction of the variables breed (B) and chick loss(C) is insignificant. The remaining model bilateral interaction terms namely breed*age and age*chick loss has been included in the model.

Table7: Summary of the results of the test statistics used for checking the adequacy of the best model fit (AIC, BIC, G^2 and χ^2)

Model	AIC	BIC	G^2	χ^2	d.f	p-value
(B,A,C)	24.14	-6.50	37.14	36.33	7	<0.001
(C,BA)	19.69	-1.48	29.69	28.76	5	<0.001
(A,BC)	24.82	-0.59	36.82	35.21	6	<0.001
(BA,BC)	21.37	4.43	29.37	28.32	4	<0.001
(BA,AC)	-5.17	-17.87	0.83	0.82	3	0.84
(BC,AC)	-0.04	-16.98	7.96	7.85	4	0.09
(BA,BC,AC)	-3.17	11.64	0.83	0.82	2	0.67
(BAC)	0	0	0.00	0.00	0	-

4) CONCLUSION

This work has demonstrated that the log linear models can help to identify the detailed patterns of interaction between variables in multidimensional contingency tables as well as finding the most parsimonious model that best fit to the data. The log linear modeling performed on this study is based on the three factors: Breed, age and chick loss. The result of the findings showed that all two-factors are significant except breed*chick loss. This implies that breed is independent of chick loss.

We also discovered that the best model fit to the data which the final model has generating class: breed*age, age*chick loss. The final model in harmony with the hierarchy principle is written as:

$$\text{Log } m_{ijk} = \mu + \mu_{B(i)} + \mu_{A(j)} + \mu_{C(k)} + \mu_{BA(ij)} + \mu_{AC(jk)}$$

The findings also revealed that the results of the goodness of the fit statistics showed that the final model adequately fit the data set.

The values of goodness of fit statistics are:

Likelihood ratio (G^2) = 0.83; d.f=3; p-value = 0.84

Pearson Chi-square (χ^2) = 0.82; d.f=3; p-value = 0.85

AIC = -5.17; BIC = -17.87

The above results of the test statistics: AIC, BIC, G^2 and χ^2 confirmed model adequacy.

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