

## Fixed Point Theorem Result in Vector Metric Spaces.

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### Abstract

In this paper we give prove some fixed point theorems for contractive conditions in vector metric space, our results are generalized form of some well known results. Our results are motivated by Ishak Altun and Cuneyt Cevik [10]

**Keywords:** - fixed point, vector metric space.

**Theorem 1.1** Let  $X$  be an vector metric space with  $E$  is Archimedes, suppose the mappings  $F, G: X \rightarrow X$  satisfies the following conditions

$$\begin{aligned} & \text{for all } x, y \in X \quad (G_x G_y) \\ & \leq K_1 u(x, y) + K_2 d(F_x, F_y) + K_3 [d(F_x, G_x) + d(F_y, G_y)] + K_4 [d(F_x, G_y) + d(F_y, G_x)] \\ & \quad + K_5 \text{Min} \{d(F_x, G_y), d(F_y, G_x)\} \end{aligned}$$

Where  $K \in [0,1]$  is a constant and

$$u(x, y) \in \{d(F_x, F_y), d(F_x, G_x), d(F_y, G_y), \frac{1}{2}[d(F_x, G_y) + d(F_y, G_x)]\}$$

$$G(X) \subseteq F(X)$$

$F(X)$  or  $G(X)$  is complete subspace of  $X$ ,  $K_1 + K_2 + 2K_3 + 2K_4 < 1$ .

Then  $G$  and  $F$  have a unique point of coincidence in  $X$ , moreover of  $F$  and  $G$  are weakly compatible, then they have a unique common fixed point in  $X$ .

**Proof:** Let  $x_0, x_1 \in X$  define the sequence  $\{x_n\}$  by  $F x_{n+1} = G x_n = y_n$  for  $n \in N$

$$d(y_n, y_{n+1}) \leq K d(y_{n-1}, y_n)$$

$$\text{We have that } d(y_n, y_{n+1}) = d(G x_n, G x_{n+1})$$

$$\leq K_1 u(x_n, x_{n+1}) + K_2 d(F x_n, F x_{n+1}) + K_3 [d(F x_n, G x_n) + d(F x_{n+1}, G x_{n+1})] \\ + K_4 [d(F x_n, G x_{n+1}) + d(F x_{n+1}, G x_n)]$$

$$+ K_5 \text{Min} \{d(F x_n, G x_{n+1}), d(F x_{n+1}, G x_n)\}$$

$$d(y_n, y_{n+1}) \leq d(G x_n, G x_{n+1})$$

$$\leq K_1 u(x_n, x_{n+1}) + K_2 [d(y_{n-1}, y_n)] + K_3 [d(y_{n-1}, y_n) + d(y_n, y_{n+1})]$$

$$+ K_4 [d(y_{n-1}, y_{n+1}) + d(y_n, y_n)] + K_5 \text{Min} \{d(y_{n-1}, y_{n+1}), d(y_n, y_n)\}$$

$$\leq K_1 u(x_n, x_{n+1}) + K_2 d(y_{n-1}, y_n) + K_3 [d(y_{n-1}, y_n) + d(y_n, y_{n+1})] + K_4 [d(y_{n-1}, y_n) + d(y_n, y_{n+1})]$$

$$\leq \frac{K_1}{2} d(y_{n-1}, y_{n+1}) + K_2 d(y_{n-1}, y_n) + K_3 [d(y_{n-1}, y_n) + d(y_n, y_{n+1})] + K_4 [d(y_{n-1}, y_n) + d(y_n, y_{n+1})]$$

$$\leq \frac{K_1}{2} [d(y_{n-1}, y_n) + d(y_n, y_{n+1})] + K_2 d(y_{n-1}, y_n) + K_3 [d(y_{n-1}, y_n) + d(y_n, y_{n+1})]$$

$$+ K_4 [d(y_{n-1}, y_n) + d(y_n, y_{n+1})]$$

$$\leq \left(\frac{K_1}{2} + K_2 + K_3 + K_4\right) d(y_{n-1}, y_n) + \left(\frac{K_1}{2} + K_3 + K_4\right) d(y_n, y_{n+1})$$

$$d(y_n, y_{n+1}) \left(1 - \frac{K_1}{2} - K_3 - K_4\right) \leq d(y_{n-1}, y_n) \left(\frac{K_1}{2} + K_2 + K_3 + K_4\right)$$

$$d(y_n, y_{n+1}) \leq \frac{\left(\frac{K_1}{2} + K_2 + K_3 + K_4\right)}{\left(1 - \frac{K_1}{2} - K_3 - K_4\right)} d(y_{n-1}, y_n)$$

$$d(y_n, y_{n+1}) \leq R \cdot d(y_{n-1}, y_n)$$

Where  $\frac{\left(\frac{K_1}{2} + K_2 + K_3 + K_4\right)}{\left(1 - \frac{K_1}{2} - K_3 - K_4\right)} = R$ ,  $K_1 + K_2 + 2K_3 + 2K_4 < 1$ .

$$\begin{aligned}
 d(y_n, y_{n-1}) &\leq R^n d(y_0, y_1) \\
 d(y_n, y_{n+p}) &\leq d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) + \dots + d(y_{n+p-1}, y_{n+p}) \\
 &\leq [R^n + R^{n+1} + R^{n+2} \dots + \dots + R^{n+p-1}] (y_0, y_1) \\
 d(y_n, y_{n+p}) &\leq \frac{R^n}{(1-R)} d(y_0, y_1)
 \end{aligned}$$

Now since E is Archimedean then  $\{y_n\}$  is an E-cauchy sequence since the range of F contains the range of T and the range of at least one is E-complete. There exists  $z \in F(X)$  such that  $Fx_n \rightarrow z$  Hence there exists a sequence  $\{a_n\}$  in E such that  $d(Fx_n, z) \leq a_n$ , on the other hand, we can find  $w \in x$ , such that  $Fw = z$

$$\begin{aligned}
 \text{Let us show that } G_w = z \text{ we have } d(G_w, z) &\leq d(G_w, Gx_n) + d(Gx_n, z) \\
 &\leq k_1 u(x_n, w) + K_2 d(Fx_n, Fw) + K_3 [d(Fx_n, Gx_n) + d(Fw, Gw)] \\
 &\quad + K_4 [d(Fx_n, Gw) + d(Fw, Gx_n)] + K_5 \text{Min}\{d(Fx_n, Gw) d(Fw, Fx_n)\} + a_{n+1} \\
 d(G_w, z) &\leq K_1 u(x_n, w) + K_2 d(Fx_n, z) + K_3 [d(Fx_n, Gx_n) + d(G_w, z)] + K_4 [d(Fx_n, z) + d(Gx_n, z)] \\
 &\quad + K_5 \text{Min}\{d(Fx_n, z) d(Gx_n, z)\} + a_{n+1} \\
 &\leq K_1 u(x_n, w) + K_2 d(Fx_n, z) + K_3 [d(Fx_n, z) + d(z, Gx_n) + d(G_w, z)] \\
 &\quad + K_4 [d(Fx_n, z) + d(Gx_n, z)] + K_5 \text{Min}\{d(Fx_n, z), d(Gx_n, z)\} + a_{n+1}
 \end{aligned}$$

**Case-I** when  $\text{Min}\{d(Fx_n, z), d(Gx_n, z)\} = d(Fx_n, z)$

$$\begin{aligned}
 &\leq K_1 u(x_n, w) + (K_2 + K_3 + K_4 + K_5) d(Fx_n, z) + K_3 d(G_w, z) + (k_3 + K_4) d(Gx_n, z) + a_{n+1} \\
 (1 - K_3) d(G_w, z) &\leq K_1 u(x_n, w) + (K_1 + K_3 + K_4 + K_5) a_n + (K_3 + K_4 + 1) a_{n+1} \\
 d(G_w, z) &\leq \left(\frac{K_1}{1 - K_3}\right) u(x_n, w) + \frac{K_1 + K_3 + K_4 + K_5}{(1 - K_3)} a_n + \frac{(K_3 + K_4 + 1)}{(1 - K_3)} a_{n+1}
 \end{aligned}$$

$$d(G_w, z) \leq R_1 u(x_n, w) + R_2 a_n + R_3 a_{n+1}$$

For all n, since.

$$u(x_n, w) \in \{d(Fx_n, Fw) d(Fx_n, Gx_n), d(Fw, Gw)\} \frac{1}{2} [d(Fx_n, Gw) + d(Fw, Gx_n)]$$

At least one of the following four cases hold for all n,

**Case-II**  $d(G_w, z) \leq d(Fx_n, Fw) + R_2 a_n + R_3 a_{n+1}$

$$\leq d(Fx_n, z) + R_2 a_n + R_3 a_{n+1}$$

$$\leq a_n + R_1 a_n + R_3 a_{n+1}$$

$$d(G_w, z) \leq (1 + R_1) a_n + R_3 a_{n+1} \leq 2a_n$$

**Case-III**  $d(G_w, z) \leq d(Fx_n, Gx_n) + R_2 a_n + R_3 a_{n+1}$

$$d(G_w, z) \leq d(Fx_n, z) + d(Gx_n, z) + R_2 a_n + R_3 a_{n+1}$$

$$\leq d(Fx_n, z) + a_{n+1} + R_2 a_n + R_3 a_{n+1}$$

$$\leq d(Fx_n, z) + (1 + R_3) a_{n+1} + R_2 a_n$$

$$\leq (1 + R_2) a_n + (1 + R_3) a_{n+1} \leq 3a_n$$

**Case-IV**  $d(G_w, z) \leq k_1 d(Fw, Gw) + R_2 a_n + R_3 a_{n+1}$

$$\leq k_1 [d(Fw, z) + d(z, Gw) + R_2 a_n + R_3 a_n]$$

$$d(G_w, z) \leq k_1 d(G_w, z) + (R_2 + R_3) a_n$$

$$d(G_w, z) \leq \frac{1}{(1 - k_1)} (R_2 + R_3) a_n$$

**Case-V**  $d(G_w, z) \leq \frac{1}{2} [d(Fx_n, Gw) + d(Fw, Gx_n) + R_2 a_n + R_3 a_{n+1}]$

$$\leq \frac{1}{2} [d(Fx_n, Gw) + d(Fw, z) + d(z, Gx_n)] + R_2 a_n + R_3 a_{n+1}$$

$$\leq \frac{1}{2} [d(Fx_n, Gw)] + \frac{1}{2} a_{n+1} + R_3 a_{n+1} + R_2 a_n$$

$$\leq \frac{1}{2} [d(Fx_n, Gw)] + \frac{1}{2} a_{n+1} + R_3 a_{n+1} + R_2 a_n$$

$$\leq \frac{1}{2} [d(Fx_n, Gw)] + \left(\frac{1}{2} + R_3\right) a_{n+1} + R_2 a_n$$

$$\leq \frac{1}{2} [d(Fx_n, z) + d(z, Gw)] + \left(\frac{1}{2} + R_3\right) a_{n+1} + R_2 a_n$$

$$\begin{aligned} &\leq \frac{1}{2} [d(G_w, z)] + \frac{1}{2} a_n + R_2 a_n + \left(\frac{1}{2} + R_3\right) a_{n+1} \\ &\leq \frac{1}{2} d(G_w, z) + \left(\frac{1}{2} + R_2\right) a_n + \left(\frac{1}{2} + R_3\right) a_{n+1} \\ d(G_w, z) &\leq (1 + 2R_2) a_n + (1 + 2R_3) a_{n+1} \end{aligned}$$

Since the infimum of sequence on the right side of last inequality are zero then

$$d(G_w, z) = 0 \text{ i. e. } G_w = z$$

Therefore z is a point of coincidence of F and G if  $Z_1$  is another point of coincidence then

there is  $w_1 \in X$  with  $Z_1 = Gw_1 = Fw_1$  now

$$\begin{aligned} d(z, z_1) &= d(G_w, Gw_1) \\ &\leq k_1 u(w, w_1) + K_2 d(F_w, Fw_1) + K_3 [d(F_w, G_w) + d(Fw_1, Gw_1)] + K_4 [d(F_w, Gw_1) + d(Fw_1, Gw)] \\ &\quad + K_5 \text{Min} \{d(F_w, G_w) + d(F_w, Gw_1)\} \\ &\leq K_1 u(w, w_1) + K_2 d(z, z_1) + K_3 [d(z, z) + d(z, z_1)] + K_4 [d(z, z_1) + d(z_1, z)] \\ &\quad + K_5 \text{Min} \{d(z, z) + d(z_1, z)\} \\ &\leq K_1 u(w, w_1) + K_2 d(z, z_1) + K_3 d(z, z_1) + 2K_4 d(z, z_1) \\ &\leq k_1 u(w, w_1) + (k_2 + K_3 + 2K_4) d(z, z_1) \end{aligned}$$

Where

$$\begin{aligned} u(w, w_1) &\in \left\{ d(F_w, Fw_1) d(F_w, G_w) d(Fw_1, Gw_1) + \frac{1}{2} [d(F_w, Gw_1) + d(F_w, G_w)] \right\} \\ &= 0, d(z, z_1) \end{aligned}$$

Hence  $d(z, z_1) = 0$  that is  $z = z_1$

If F and G are weakly compatible then it is obvious, that Z is unique common fixed point of F and G.

**Theorem 1.2 :-** Let X be an vector metric space with E is Archimedean, suppose the mapping  $F; G; X \rightarrow X$  satisfies the following conditions

$$\begin{aligned} \text{for all } x, y, \in X \quad &d(G_x, G_y) \\ &\leq k u(x, y) + K_2 d(F_x, F_y) + K_3 [d(F_x, G_x) + d(F_y, G_y)] + K_4 [d(F_x, G_y) + d(F_y, G_x)] \\ &+ K_5 \text{min} \{d(F_x, G_y) d(F_y, G_x)\} \end{aligned}$$

where  $k \in [0,1]$  is constant and

$$u(x, y) \in \left\{ d(F_x, G_y), \frac{1}{2} [d(F_x, G_x) + d(F_y, G_y)], \frac{1}{2} [d(F_x, G_y) + d(F_y, G_x)] \right\}$$

1.  $G(x) \subseteq F(x)$ ,  $k_1 + K_2 + 2K_3 + 2K_4 < 1$ .
2.  $F(X)$  or  $G(X)$  is E-complete subspace of X then F and G have a unique point of coincidence in X moreover if F and G are weakly compatible then they have a unique common fixed point is X

**Proof:-** Let us define the sequence  $\{x_n\}$  and  $\{y_n\}$  as in the proof of theorem we show that

$$d(y_n, y_{n+1}) \leq K d(y_{n-1}, y_n)$$

$$\begin{aligned} d(y_n, y_{n+1}) &= d(Gx_n, Gx_{n+1}) \\ &\leq k_1 u(x_n, x_{n+1}) + K_2 d(Fx_n, Fx_{n+1}) + K_3 [d(Fx_n, Gx_n) + d(Gx_{n+1}, Gx_{n+1})] \\ &\quad + K_4 [d(Fx_n, Gx_{n+1}) + d(Fx_{n+1}, Gx_n)] + K_5 \text{Min} \{d(Fx_n, Gx_{n+1}) d(Fx_{n+1}, Gx_n)\} \end{aligned}$$

We have to consider three case  $u(x_n, x_{n+1}) = d(y_{n-1}, y_n)$

$$u(x_n, x_{n+1}) = \frac{1}{2} [d(y_{n-1}, y_n) + d(y_n, y_{n+1})] \text{ and } u(x_n, x_{n+1}) = \frac{1}{2} [d(y_{n-1}, y_{n+1})]$$

Only second case if  $u(x_n, x_{n+1}) = \frac{1}{2} [d(y_{n-1}, y_n) + d(y_n, y_{n+1})]$

$$\begin{aligned} d(y_n, y_{n+1}) &\leq d(Gx_n, Gx_{n+1}) \leq K_1 u(x_n, x_{n+1}) + K_2 d(y_{n-1}, y_n) \\ &\quad + K_3 [d(y_{n-1}, y_n) + d(y_n, y_{n+1})] + K_4 [d(y_{n-1}, y_{n+1}) + d(y_n, y_n) + K_5 \text{Min} \{d(y_{n-1}, y_{n+1}), d(y_n, y_n)\}] \\ &\leq K_1 u(x_n, x_{n+1}) + K_2 d(y_{n-1}, y_n) + K_3 [d(y_{n-1}, y_n) + d(y_n, y_{n+1})] + K_4 [d(y_{n+1}, y_n) + d(y_n, y_{n+1})] \\ &\leq \frac{K_1}{2} d(y_{n-1}, y_n) + \frac{1}{2} d(y_n, y_{n+1}) + K_2 d(y_{n-1}, y_n) + K_3 [d(y_{n-1}, y_n) + d(y_n, y_{n+1})] \end{aligned}$$

$$\begin{aligned}
 &+K_4[d(y_{n-1}, y_n) + d(y_n, y_{n+1})] \\
 d(y_n, y_{n+1}) &\leq \left(\frac{K_1}{2} + K_2 + K_3 + K_4\right) d(y_{n-1}, y_n) + \left(\frac{1}{2} + K_3 + K_4\right) d(y_n, y_{n+1}) \\
 \left(1 - \frac{1}{2} - K_3 - K_4\right) d(y_n, y_{n+1}) &\leq \left(\frac{K_1}{2} + K_2 + K_3 + K_4\right) d(y_{n-1}, y_n) \\
 d(y_n, y_{n+1}) &\leq \frac{\left(\frac{K_1}{2} + K_2 + K_3 + K_4\right)}{\left(\frac{1}{2} - K_3 - K_4\right)} d(y_{n-1}, y_n)
 \end{aligned}$$

Where

$$R = \frac{\left(\frac{K_1}{2} + K_2 + K_3 + K_4\right)}{\left(\frac{1}{2} - K_3 - K_4\right)} \quad \text{since } k_1 + K_2 + 2K_3 + 2K_4 < 1.$$

$$d(y_n, y_{n+1}) \leq R d(y_{n-1}, y_n) \text{ Hence } (4)$$

In the proof of theorem, that  $\{y_n\}$  is an E-cauchy sequence then there exist  $z \in F(X)$ ,  $w \in X$  is  $(a_n)E$  such that  $F_w = z$ ,  $d(Fx_n, z) \leq a_n$

Now we have to show that  $G_w = z$  we have

$$\begin{aligned}
 d(G_w, z) &\leq d(G_w, Gx_n) + d(Gx_n, z) \leq u(x_n, w) + a_{n+1} \\
 &\leq K_1 u(x_n, w) + K_2 d(Fx_n, F_w) + K_3 [d(Fx_n, Gx_n) + d(F_w, G_w)] + K_4 [d(Fx_n, G_w) + d(F_w, Gx_n)] \\
 &+ K_5 \text{Min} \{d(Fx_n, G_w) + d(F_w, Gx_n) + a_{n+1}\} \\
 d(G_w, z) &\leq K_1 u(x_n, w) + K_2 d(Fx_n, z) + K_3 [d(Fx_n, z) + d(z, Gx_n) + d(G_w, z)] \\
 &+ K_4 [d(Fx_n, z) + d(Gx_n, z)] + K_5 \text{Min} \{d(Fx_n, z) + d(Gx_n, z)\} + a_{n+1}
 \end{aligned}$$

**Case-I** when  $\text{Min} \{d(Fx_n, z) + d(Gx_n, z)\} = d(Fx_n, z)$

$$\begin{aligned}
 &\leq K_1 u(x_n, w) + K_2 d(Fx_n, z) + K_3 [d(Fx_n, z) + d(z, Gx_n)] + d(G_w, z) + K_4 [d(Fx_n, z) + d(Gx_n, z)] \\
 &+ K_5 \{d(Fx_n, z)\} \\
 (1 - K_3) d(G_w, z) &\leq K_1 u(x_n, w) + (K_1 + K_3 + K_4 + K_5) a_n + (K_3 + K_4 + 1) a_{n+1} \\
 d(G_w, z) &\leq \frac{K_1}{1 - K_3} u(x_n, w) + \frac{(K_1 + K_3 + K_4 + K_5)}{1 - K_3} a_n + \frac{(K_3 + K_4 + 1)}{1 - K_3} a_{n+1}
 \end{aligned}$$

$$d(G_w, z) \leq R_1 u(x_n, w) + R_2 a_n + R_3 a_{n+1}$$

$$u(x_n, w) \in \{d(Fx_n, F_w), \frac{1}{2} [d(Fx_n, Gx_n) + d(F_w, G_w)] + \frac{1}{2} [d(Fx_n, G_w) + d(F_w, Gx_n)]\}$$

At least one of three cases holds for all n, consider only the case of

$u(x_n, w) = \frac{1}{2} [d(Fx_n, Gx_n) + d(F_w, G_w)]$  because the other two cases have shown that the proof of theorem it is satisfied

$$\begin{aligned}
 d(G_w, z) &\leq \frac{R}{2} [d(Fx_n, Gx_n) + d(F_w, G_w)] + R_2 a_n + R_3 a_{n+1} \\
 &\leq \frac{R}{2} [d(Fx_n, z) + d(Gx_n, z) + d(F_w, z) + d(z, G_w)] + R_2 a_n + R_3 a_{n+1} \\
 &\leq \frac{R}{2} [a_n + a_{n+1}] + \frac{R}{2} d(G_w, z) + R_2 a_n + R_3 a_{n+1} \\
 d(G_w, z) &\leq \frac{R}{2} d(G_w, z) + \left(\frac{R}{2} + 1\right) a_n + \left(\frac{R}{2} + R_3\right) a_{n+1}
 \end{aligned}$$

This is  $d(G_w, z) \leq 4a_n$

Since  $4a_n \leq 0$  then  $G_w = z$  hence z is a point of coincidence of F and G the uniqueness of z as in the proof of theorem 1.1 also if F and G are weakly compatible then it is obvious that z is unique common fixed point of G and F by [1.1]

**Theorem (1.3)** Let X be an vector metric space with E is Archimedean suppose the mapping  $F, G: X \rightarrow X$  satisfies the following conditions

1. For all  $x, y \in X$

$$\begin{aligned}
 d(Gx, Gy) &\leq b. d(Fx, Gx) + c. d(Fy, Gy) + e. d(Fx, Gy) + f. d(Fy, Gx) \\
 &+ g. d(Fx, Fy) + h. d(Fx, Fy) + i. [d(Fx, Gx) + d(Fy, Gy)]
 \end{aligned}$$

$$+j[d(Fx, Gy) + d(Fy, Gx)] \\ +R \text{Min} \{d(Fx, Gy), d(Fy, Gx)\}$$

Where b,c,d,e,f,g,h,i,j and R are nonnegative and  $b+c+d+e+g+h+i+j+R < 1$

2.  $G(X) \subseteq F(X)$
3. F(x) or G(x) is E-complete subspace of X then F and G have a unique point of coincidence in X moreover if F and G are weakly compatible then have a unique common fixed point in X

**Proof:-**Let us define the sequence  $\{x_n\}$  and  $\{y_n\}$  as in the proof of theorem (1.1) we have to show that

$d(y_n, y_{n+1}) \leq kd(y_{n-1}, y_n)$  for some  $k \in [0,1]$  and all n consider  $Fx_{n+1} = Gx_n = y_n$  for all n then.

$$d(y_n, y_{n+1}) \leq (b + g + h + i + \theta + j)d(y_{n-1}, y_n) + (c + i + \theta + j)d(y_n, y_{n+1})$$

and

$$d(y_{n+1}, y_n) \leq (b + i + j + f)d(y_n, y_{n+1}) + (c + g + h + i + f + j)d(y_{n-1}, y_n)$$

Hence

$$[2 - (b + 2i + 2j + c + \theta)]d(y_n, y_{n+1}) \leq (b + 2g + 2i + 2h + \theta + j + c)d(y_{n-1}, y_n)$$

$$d(y_n, y_{n+1}) \leq \frac{b + 2g + 2h + 2i + \theta + j + c}{[2 - (b + c + \theta + 2i + 2j)]} d(y_{n-1}, y_n)$$

$$\text{If we choose } K_1 = \frac{(b + \theta + j + c + 2g + 2h + 2i)}{[2 - (b + c + \theta + 2i + 2j)]}$$

Then

$$K_1 \in [0,1]$$

In the proof of theorem (1.1) we illustrate that  $\{y_n\}$  is an E couchy sequence, then there exist  $Z \in F(X)$   $w, \in X$  and  $(a_n)$  in E such that  $Fw = Z$ ,  $d(Fx_n, Z) \leq a_n$  and  $a_n \downarrow 0$

Let us show that  $Gw = Z$  we have  $d(Gw, z) \leq d(Gw, Gx_n) + d(Gx_n, z)$

$$d(Gw, z) \leq (b + f + i + j)d(Gw, z) + (c + f + g + h + i + j)d(Fx_n, z) + (c + \theta + i + j + 1)d(Gx_n, z)$$

$$+R \text{Min}\{d(z, Gx_n)d(Fx_n, Gw)\}$$

$$d(Gw, z) \leq (b + f + i + j + R)d(Gw, z) + (c + f + g + h + i + j + R)a_n \\ + (c + f + g + h + i + j + R)a_{n+1}$$

$$\leq (b + f + i + j + R)d(Gw, z) + (2c + 2f + 2g + 2h + 2i + j + R)a_n$$

$$d(Gw, z) \leq \frac{(2c + 2f + 2g + 2h + 2i + j + R)}{1 - (b + f + i + j + R)} a_n.$$

For all n. then  $d(Gw, z) = 0$  i. e.  $Gw = z$ . Hence Z is a point of coincidence of F and G. Then uniqueness of z is easily seen. Also if F and G are weakly compatible, then it is obvious that z is unique common fixed point of G and F by [1.1].

**Acknowledgement:** Author [Rks] , thankful to Ishak Altun and Cuneyt Cevik for motivation by their results for extension.

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