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Analytical Solution of a Steady Non-Reacting Laminar Fluid Flow in a Channel Filled With Saturated Porous Media with Two Distinct Horizontal Impermeable Wall Conditions

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Abstract

An analysis of the study of momentum and heat transfer characteristics of a non-reacting Newtonian viscous incompressible laminar fluid flow in a channel filled with saturated porous media with both isothermal and isoflux boundary conditions has been carried out. The dimensionless non-linear coupled ordinary differential equations governing the flow and the heat transfer characteristics are solved analytically using the method of undetermined coefficients. Details of velocities and temperature fields for various values of the emerging parameters of the steady solutions of the problems are presented using contour graphs. It was shown that the porous medium may be used to control the flow in a saturated porous media using less permeable materials which offer greater resistance to the boundary-layer momentum development. It was further revealed that increase in porous media permeability and decrease in ratio of viscosities, help to enhance the fluid flow; however the input of the viscous term on the flow behaviour is insignificant for extremely lower values of Darcy number. It was also shown that the fluid temperature is more influenced in isothermal and isoflux processes if the Darcy number, *Da*, or Brinkman number, *Br*, or both are increased and if the ratio of viscosities *M*, is decreased.

Keywords: laminar flow, porous medium, impermeable walls, isothermal, isoflux)

1. Introduction

The study of transport phenomena in a laminar fluid flow in the channel filled with saturated porous media has received considerable attention of scientists, engineers and experimentalists in recent years. This attention is due mainly because this phenomenon is often observed in the field of electronics cooling system, solid matrix heat exchanger, geothermal system, nuclear waste disposal, microelectronics heat transfer equipment, coal and grain storage, petroleum industries, and catalytic converters. Meanwhile, the improvement in thermal systems as well as energy utilization during the convection in any fluid is one of the fundamental problems of the engineering processes, since improved thermal systems will provide better material processing, energy conservation and environmental effects, (Makinde 2004).In recent times porous media models are being applied for simulating more generalized situations such as flow through packed and fluidized beds. The majority of the studies on convection heat transfer in porous media are based on Darcy's law, (Darcy 1856). The accuracy of these results is restricted to specific ranges of Darcy and Reynolds numbers. In the investigation of the importance of Brinkman number and Forchheiwer terms in forced convection over a flat plate (Vafai and Tien 1981), it was presented that the resulting error occurred in heat transfer coefficient when the viscous and inertial terms are negligible. Similarly, a study revealing the wall and inertia effects in mixed convection flow over flat plate was reported (Ranganathan and Viskanta 1984). It was indicated that in natural convection flow, wall effect is negligible for Darcy number below than 10^{-5} (Tong and Subramanian 1985; Lauriat and Prasad 1986). An experiment investigating the free convection from a horizontal circular cylinder embedded in a porous medium, revealed that deviations from the Darcy's law occur when the Reynolds number based on the pore diameter exceeds 1-10, thus, the non-Darcy flow situation is more likely to prevail when the Rayleigh number is sufficiently high and that the boundary layer approximations are relevant (Fand, Steinberg and Cheng 1986). The study of the boundary effects in laminar mixed convection flow through an annular porous medium discussed the conditions under which the Brinkman term can be neglected without producing an unacceptable error (Parang and Keyhani 1987). While the fully developed laminar natural convection flows in vertical annuli in which one of the boundaries is isothermal has also been discussed (Joshi 1987). The combined effects of free and forced convection in vertical cylinder embedded in a porous medium were discussed (Ramanaiah and Malarvizhi 1989), so also was the investigation on momentum transfer based on Brinkman model in a circular cylinder (Pop and Cheng 1992). Investigation of the solution of fully developed laminar mixed convection through a vertical annular duct filled with porous media using the non-Darcian flow model where thermal boundary condition in the inner and out walls are prescribed as isothermal-isothermal, isothermal-isoflux, and isoflux-isothermal separately was carried out (Hong-Sen and Kuen-Tzong 1993). One of the first attempts to highlight this problem was studied and the study presented two different approaches for boundary conditions for constant wall heat flux (Amiri, Vafai and Kuzay 1995). The first approach presented in the work was based on assuming the total heat flux being divided between the two phases depending on the physical values of their effective conductivities and their corresponding temperature gradients at the wall. The second approach also presented assumed that each of the individual phases at the wall receives an equal amount of the total heat flux q_w (Amiri et al. 1995). In this study, good agreements were found between the numerical results using the second approach and the available experimental results. On the other hand, the first approach was used and good agreement was found between their numerical and experimental results (Hwang, Wu and Chao 1995). The transient free convective flow in a vertical channel having a constant temperature and constant heat flux on the channel walls was studied (Paul, Jha and Singh 1996), and the first approach was used to obtain analytical solutions for the temperature profiles, the temperature difference between the two phases, and the Nusselt number (Lee and Vafai 1999) while the solution of mixed convection in a vertical annulus filled with a porous material considering isothermal and isoflux heating at the outer surface of the inner cylinder when the gap between the two cylinders is less, equal and greater than radius of the inner cylinder was obtained (Akhilesh, Jha and Singh 2001). Constant wall heat flux boundary conditions in porous media under local thermal non-equilibrium conditions was investigated (Alazmi and Vafai 2002) and in this work, the effects of variable porosity and thermal dispersion were also analyzed. Different forms of constant heat flux boundary conditions found in the literature were investigated. The effects of pertinent parameters such as porosity, Darcy number, Reynolds number, inertia parameter, and particle diameter and solid-to-fluid conductivity ratio were analyzed. Limiting cases resulting convergence or divergence of the models are also considered. Hitherto, it is not clear what two boundary conditions might be used for the case of constant wall heat flux. In contrast, boundary conditions for constant wall temperature are clear; both phases should have a temperature that equals a prescribed wall temperature. The Brinkman model was used for the theoretical study of the mixed convection boundary layer flow past a horizontal circular cylinder with a constant surface temperature and embedded in a fluid-saturated porous medium in a stream flowing vertically upwards (Nazar, Amin and Pop 2003). Both the cases of a heated (assisting flow) and a cooled (opposing flow) cylinder are considered. It is shown that the two governing dimensionless parameters- Darcy-Brinkman parameter Γ and the mixed convection parameter λ are related to thermal and viscous effects.

The study of free-convection flow through an annular porous medium addresses the Brinkman-extended Darcy model (Brinkman flow) of a laminar free-convective flow in an annular porous region (Jha 2005). Closed form expressions for velocity field, Temperature field, Skin-friction and Mass flow rate are given under a thermal boundary condition of mixed kind at the outer surface of the inner cylinder while the inner surface of the outer cylinder is isothermal. The investigation of the entropy generation rate in a laminar flow through a channel filled with saturated porous media (Makinde and Osalusi 2005) where the upper surface of the channel was adiabatic and the lower wall was assumed to have a constant heat flux was studied. The velocity and temperature profiles were obtained for large Darcy number (Da) and used to obtain the entropy generation number and the irreversibility ratio. Their result showed that heat transfer irreversibility dominates over fluid friction irreversibility, and viscous dissipation has no effect on the entropy generation rate at the center-line of the channel. The study of convective heat transfer in unsteady laminar parallel flows (Brereton and Jiang 2006) revealed that, in many practical and experimental circumstances, free convection flows are generated adjacent to surfaces dissipating heat at a prescribed heat flux rate. In the investigation of hydromagnetic mixed convection flow of an incompressible viscous electrically conducting fluid and mass transfer over a vertical porous plate with constant heat flux embedded in a porous medium (Makinde 2009), It was found that for positive values of the buoyancy parameters, the skin friction increased with increasing values of both the Eckert number (Ec) and the magnetic field intensity parameter (M) and decreased with increasing values of both the Schmidt number (Sc) and the permeability parameter (K). Boundary layer flow over a flat plate with slip flow and constant heat flux surface condition was studied and it was shown that the slip parameters is a function of the local Reynolds number, the local Knudsen number, and the tangential momentum accommodation coefficient representing the fraction of the molecules reflected diffusively at the surface (Aziz 2010). As the slip parameter increases, the slip velocity increases and the wall shear stress decreases. Recently, the study of flow of a Maxwell fluid through a porous medium induced by a constantly accelerating plate was carried out and the result showed that when there is porous medium like in sand (Hassan and Fenuga 2011), the relaxation time behaviour changes as time increases. Also it is clearly seen that velocity increases in both cases as time increases which satisfies the initial and boundary conditions. Thus, it implies that the porous medium has effect over the flow. They further show that the results exist and unique.

The main objective of this thesis is to make an investigation of convection heat transfer of laminar flow in a channel filled with saturated porous media with isothermal and isoflux heating walls and to solve the second order ordinary differential equations for the dependent variables u and θ as functions of y and, in particular to obtain the steady-state analytical solutions.

2. Mathematical Formulation

We present in this work, the set of dimensional coupled non-linear ordinary differential equations describing a Newtonian viscous incompressible laminar flow with two distinct boundary conditions in the mathematical models. We consider momentum and coupled heat transfer equations by laminar flow for the steady state hydro-dynamically and thermally developed situations which have unidirectional flow of a viscous combustible non-reacting Newtonian fluid in the x-direction between impermeable boundaries at $\bar{y}=0$ *and* $\bar{y}=a$ as in figure 1 below. The channel is composed of a lower heated wall with surface constant temperature (isothermal) or constant heat flux (isoflux) while the upper wall is fixed and isothermal. We follow closely and modify the models presented in Lamidi and Olanrewaju (2010). The additional viscous dissipation term in our model, is due to Al-Hadhrami, Elliot and Ingham (2003), (Makinde 2006), and is valid in the limit of very small and very large porous medium permeability.

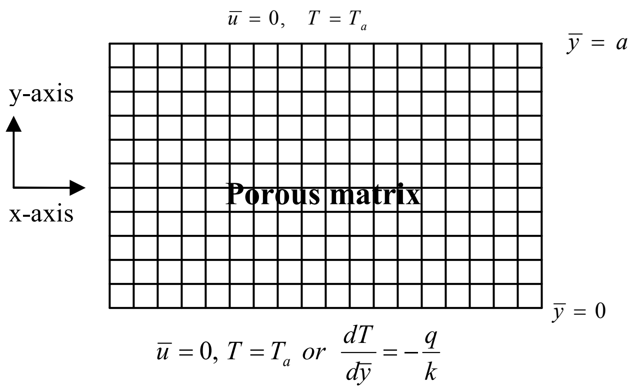

Figure 1: Problem geometry

In the following sections, the dimensionless non-linear coupled steady-state momentum and energy balance equations, which govern the problems, are obtained and subsequently, the resulting boundary-value problem are solved.

The Brinkman momentum equation with its boundary conditions are

$$\mu_e \frac{d^2\bar{u}}{d\bar{y}^2} - \frac{\mu}{K}\bar{u} + G = 0, \quad \bar{u}(0)=0, \quad \bar{u}(a)=0 \qquad (1)$$

The dimensionless form of equation (1) is

$$M\frac{d^2u}{dy^2} - \frac{u}{Da} = -1, \qquad u(0)=0,\ u(1)=0 \qquad (2)$$

The steady-state thermal energy equation for the problem is given as

$$k\frac{d^2T}{d\bar{y}^2} + \mu\left(\frac{d\bar{u}}{d\bar{y}}\right)^2 = 0 \qquad (3)$$

with the following boundary conditions

$$T=T_a \text{ or } \frac{dT}{d\bar{y}} = -\frac{q}{k} \text{ at } \bar{y}=0 \text{ and } T=T_a \text{ at } \bar{y}=a,\ 0\le\bar{y}\le a, \qquad (4)$$

where in the case described above, $a, Da, E, G, k, K, q, T_0, T, \bar{u}, \bar{y}, \mu, \mu_e, \rho$ are defined in the nomenclature.

The dimensionless energy equation with its boundary conditions are respectively given as

$$\frac{d^2\theta}{dy^2} + Br\left(\frac{du}{dy}\right)^2 = 0, \qquad (5)$$

and

$$\theta = 0 \text{ or } \frac{d\theta}{dy} = -1 \text{ at } y=0 \qquad (6)$$

$$\theta = 0 \text{ at } y = \lambda, \quad 0 \le \lambda \le 1 \tag{7}$$

In order to unify the isothermal heating and isoflux heating at one condition, we may write

$$\theta = 0 \text{ or } \frac{d\theta}{dy} = -1 \text{ to } A\frac{d\theta}{dy} + B\theta = C \quad \text{at } y=0 \tag{8}$$

and

$$\theta = 0 \text{ at } y = \lambda, \quad 0 \le \lambda \le 1 \tag{9}$$

where *A, B* and *C* are constants depending on the isothermal heating or isoflux heating.
For the isothermal heating: $A = 0$, $B = 1$ and $C = 0$
while for isoflux heating: $A = 1$, $B = 0$ and C = -1

3. Method of Solution

We shall use extensively the method of undetermined coefficients in solving the formulated problems, and the summary of the result with the corresponding contour graphical representation which we use the symbolic algebraic computer programming software packages namely MAPLE and MATLAB programming packages to get shall be presented.
Now, using the method of undetermined coefficients, the solution of (2) could be easily obtained as

$$u(y) = c_1 e^{n_1 y} + c_2 e^{n_2 y} + Da \,, \tag{10}$$

where

$$n_1 = \frac{1}{\sqrt{DaM}}, \quad n_2 = -\frac{1}{\sqrt{DaM}}, \quad c_1 = \frac{Da(e^{\lambda n_1} - 1)}{e^{\lambda n_2} - e^{\lambda n_1}}, \quad c_2 = -\frac{Da(e^{\lambda n_2} - 1)}{e^{\lambda n_2} - e^{\lambda n_1}}$$

Now, using (10) in (5) we have

$$\frac{d^2\theta}{dy^2} = -Br\left(n_1 c_1 e^{n_1 y} + n_2 c_2 e^{n_2 y}\right)^2 \tag{11}$$

Solving for homogeneous and particular parts to get the general solution using the method of undetermined coefficients, we have

$$\theta(y) = \theta_c(y) + \theta_p(y)$$

i.e

$$\theta(y) = c_3 + c_4 y - (c_5 e^{2n_1 y} + c_6 e^{2n_2 y} + c_7 y^2)\, Br\, c_8 \,, \tag{12}$$

where

$$c_3 = c_9(-c_{10} + c_{11} + c_{12}e^{2\lambda n_1} - c_{13}e^{2\lambda n_2}), \quad c_4 = -c_9(c_{14} - c_{15}e^{2\lambda n_2} - c_{16}e^{2\lambda n_1}),$$

$$c_5 = \frac{1}{4}c_2^2 DaM\,, \quad c_6 = \frac{1}{4}c_1^2 DaM\,, \quad c_7 = c_1 c_2\,, \quad c_8 = \frac{1}{Da^2 M^2},$$

$$c_9 = \frac{1}{4Da^2 M^2(B\lambda - A)}, \quad c_{10} = 4Brc_1 c_2 \lambda^2 A$$

$$c_{11} = \lambda Br(2A\sqrt{DaM} + BDaM)(c_2^2 + c_1^2) + 4\lambda CDa^2 M^2,$$

$$c_{12} = -Br\, c_2^2 DaMA\,, \quad c_{13} = Brc_1^2 DaMA\,,$$

$$c_{14} = Br(2A\sqrt{DaM} + BDaM)(c_2^2 + c_1^2) + 4CDa^2 M^2 - 4BBrc_1 c_2 \lambda^2,$$

$$c_{15} = B\, c_1^2 DaMBr\,, \quad c_{16} = Bc_2^2 DaMBr\,,$$

4. Results and Discussion

Velocity Profile

The velocity profiles for various values of *Da* and *M* are shown in figures 2 to 4 for equation (3). As expected the figures demonstrate that the flow is of parabolic type. It is revealed in figure 2 that the velocity profile of the fluid increases as Darcy number, *Da*, increases. This is due to the physical fact that for a large Darcy number, *Da*, the permeability of the medium is large and as a result of it, less resistance is offered by the medium on the flow field. In figure 3, the change in velocity is negligible for all values of *M* when *Da* = 0.001(very small) hence it is shown clearly that the input of viscous term on the flow behaviour is insignificant for lower values of Darcy number, whereas in figure 4, the velocity decreases as *M* increases when *Da* = 0.1. This implies that a decrease in ratio of viscosities *M* indicates that the effective viscosity is less than the fluid viscosity. In general, we observe that the flow is parabolic and symmetrical about the y = 0.5. Thus, increase in porous media permeability and decrease in ratio of viscosities help to enhance the fluid flow.

Temperature Profile

Figures 5 to 13 show the influences of the emerging parameters on the fluid temperature distribution within the channel. With the isothermal boundary conditions, it is observed in figure 5 that the temperature of the fluid increases as porous media permeability (Darcy number) increases. In figure 6, the temperature profile is also seen to be reducing as the ratio of viscosities, *M*, increases; this is due largely to the fact that, as ratio of viscosities, *M*, increases, the effective viscosity increases which results in a corresponding decrease in the fluid temperature within the channel. In figure 7, the simultaneous effects of the Darcy number, *Da* and ratio of viscosities, *M* were shown; this confirms that the effects of Darcy number and the ratio of viscosities, *M* are to increase and decrease respectively the temperature of the fluid.

In the case of an isoflux heating condition in figure 8- 10, the fluid temperature profiles increase as *Da* increases. In figure 9, it was deduced that as the fluid temperature reduces, the viscosities ratio, *M* increases. Figure 10 displayed the simultaneous effects of the Darcy number, *Da* and ratio of viscosities, *M*, this confirms that the temperature increases with increase in Darcy number and decreases with increase in ratio of viscosities, *M*. We emphasize in figure 11 to 13, the variation in *Br* on the temperature distribution within the channel. We discover in figure 11 for isothermal heating process that the fluid temperature profiles increase with increasing Brinkman number *Br*. Also in the case of isoflux heating process, in figure 12, the fluid temperature increases as *Br* increases. Alternatively, this also implies that decrease in *Br* decreases the temperature. We can deduce here that, heat produced by viscous dissipation is being transported away by conduction. In figure 13, we show the simultaneous effects of the Brinkman number, *Br* and ratio of viscosities, *M* with isoflux boundary condition, this confirms that the temperature increases with increase in Brinkman number and decreases with increase in ratio of viscosities, *M*.

Comparatively from these figures above, the magnitude of heat generated in the isoflux heating channel is greater than that of isothermal heating channel in the presence of the dimensionless emerging parameters mentioned.

5. Conclusion

We have extensively studied the theory of laminar flow through a saturated porous media with isothermal and isoflux heating walls. The various combinations of emerging parameters revealed much insight into the behaviour of the flow.

From our results, we established that:

(i) Porous medium may be used to control the flow in saturated porous media using less permeable materials which offer greater resistance to the boundary layer momentum development.

(ii) Input of viscous term on the flow behaviour is insignificant for extremely lower values of Darcy number (*Da* = 0.001) whereas for large Darcy number, porosity of the medium increases, hence fluid flows quickly.

(iii) The flow is parabolic in nature and symmetrical about y = 0.5

(iv) Heat produced by viscous dissipation is being transported away by conduction

(v) Fluid temperature is more influenced in both isothermal and isoflux processes, if either the Darcy number *Da*, or Brinkman number *Br*, or both are increased and the ratio of viscosities *M* is decreased. However, the magnitude of heat generated in the isoflux heating channel is greater than that of isothermal heating channel in the presence of the dimensionless emerging parameters mentioned.

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Nomenclature

We give the definition of some parameters that feature prominently in this write-up and except otherwise stated, these parameters will assume the definition given.

a	channel width
Br	Brinkman number
Da	Darcy number
G	applied pressure gradient
k	fluid thermal conductivity
K	permeability
M	ratio of viscosities
q	fluid flux rate
T_o	wall temperature
T	absolute temperature
u	dimensionless fluid velocity
$\overline{u}$	fluid velocity
$\overline{x}$	axial coordinate
y	dimensionless transverse coordinate
$\overline{y}$	transverse coordinate

Greek Symbols

μ	fluid viscosity
μ_e	effective viscosity in the Brinkman term
θ	dimensionless temperature
ρ	fluid density

Dimensionless Group

$y = \frac{\bar{y}}{a}$	dimensionless transverse coordinate
$u = \frac{\mu\bar{u}}{Ga^2}$	dimensionless velocity
$M = \frac{\mu_e}{\mu}$	ratio of viscosities
$Da = \frac{K}{a^2}$	dimensionless Darcy number
$Br = \frac{G^2a^3}{q\mu}$	dimensionless Brinkman number
$\theta = \frac{k}{qa}(T - T_0)$	dimensionless temperature

.

Graphs

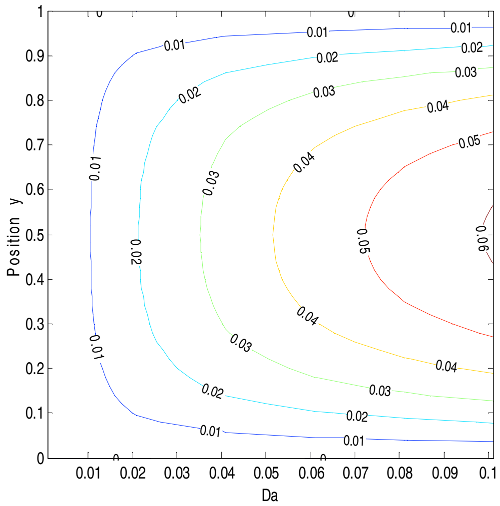

Figure 2: Contour Graph of velocity u(y) at various values of Da with M = 1.0 for equation (3)

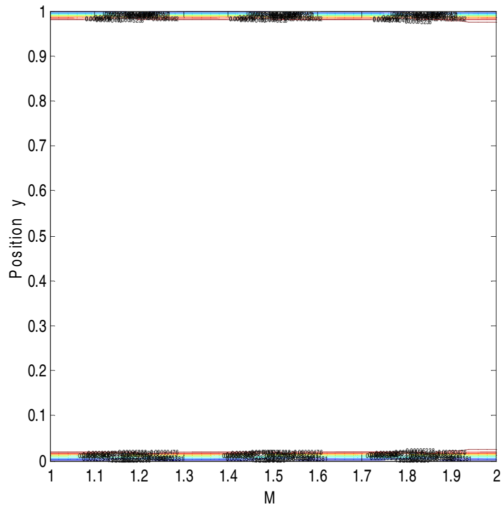

Figure 3: Contour Graph of velocity u(y) at various values of M with Da = 0.001 for equation (3)

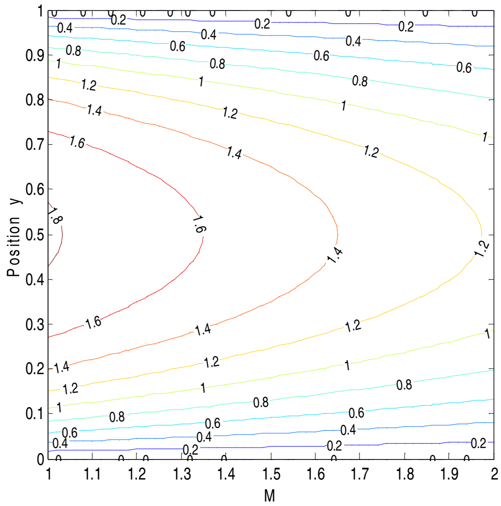

Figure 4: Contour graph of velocity u(y) at various values of M with Da = 0.1 for equation (3)

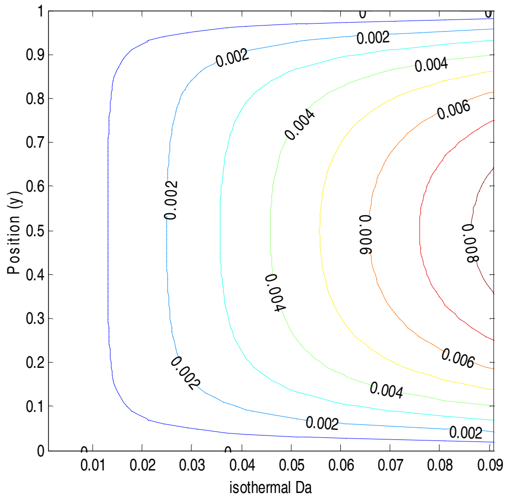

Figure 5: Contour graph of temperature θ(y) at different values of Da with Br = 0.3, M = 1.0 for equation (9)

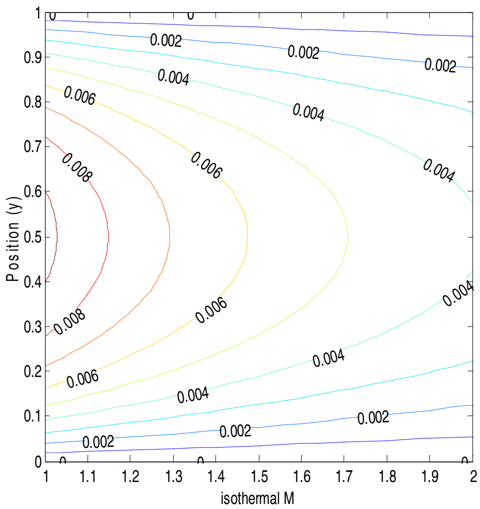

Figure 6: Contour graph of temperature θ(y)at different values of M with Br = 0.3, Da = 0.1 for equation (9)

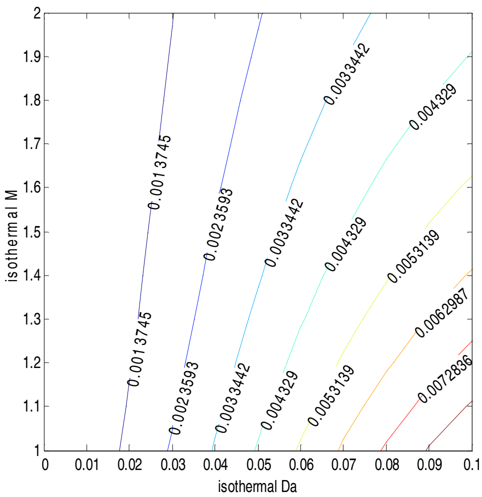

Figure 7: Contour graph of temperature θ(y) at various values of Da and M with Br = 0.3, y = 0.5 for equation (9)

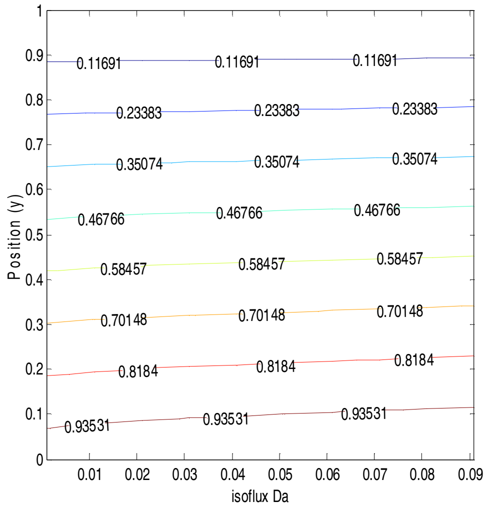

Figure 8: Contour graph of temperature θ(y) at various values of Da with Br = 0.3, M = 1.0 for equation (9)

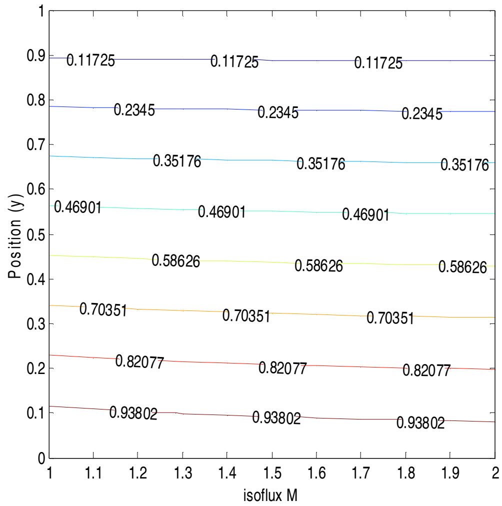

Figure 9: Contour graph of temperature θ(y) at different values of M with Br = 0.3, Da = 0.1 for equation (9)

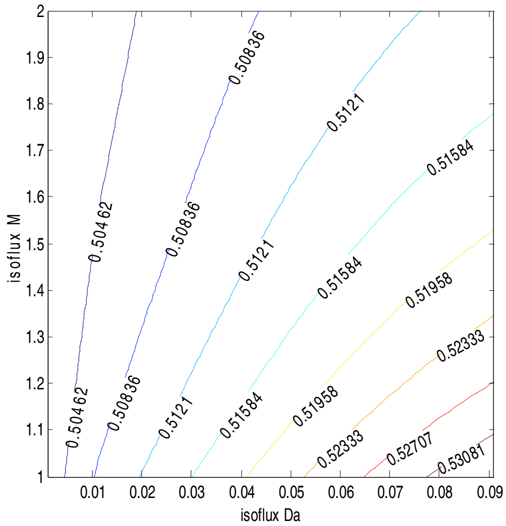

Figure 10: Contour graph of temperature θ(y) at various values of Da and M with Br = 0.3, y = 0.5 for equation (9)

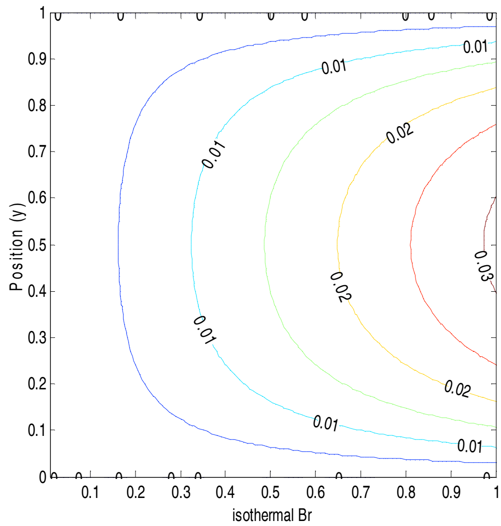

Figure 11: Contour graph of temperature θ(y) at different values of Br with Da = 0.1, M = 1.0 for equation (9)

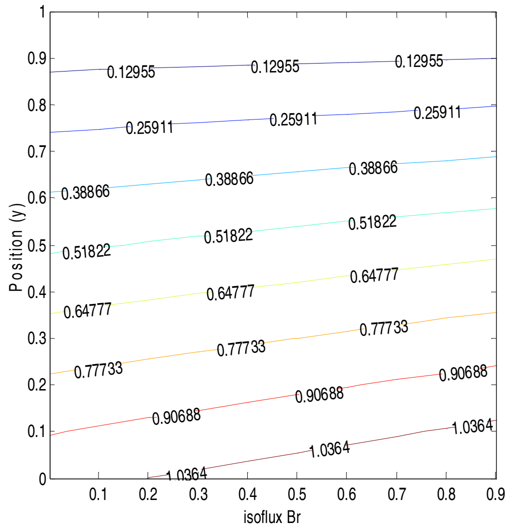

Figure 12: Contour graph of temperature θ(y) at various values of Br with Da = 0.1, M = 1.0 for equation (9)

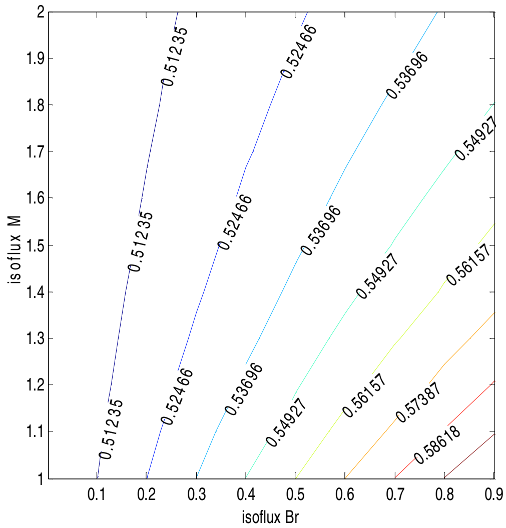

Figure13: Contour graph of temperature θ(y) at different values of Br and M with Da = 0.1, y = 0.5 for equation (9)