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Balancing of Chemical Equations using Matrix Algebra

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Abstract
This study describes a procedure employing Gaussian elimination method in matrix algebra to balance chemical equations from easy to relatively complex chemical reactions. The result shows that 2 atom of sodium (Na), 6 atoms of oxygen (O), 4 atoms of Hydrogen (H), and 1 atom of sulfur (S) each on both the reactants and products makes the chemical equation balance. This result satisfies the law of conservation of matter and confirms that there is no contradiction to the existing way(s) of balancing chemical equations.
Keywords: Balanced equation, Conservation of matters, Reactants, Products, Matrixes.

1. Introduction

The balancing of chemical equations can be made much easier, especially for those who find it difficult, by moving the procedures toward the algorithmic and away from the heuristic. That is, a "step to step" procedure is simpler to master than is the haphazard hopping of inspection, even a highly refined inspection (Hutchings *et al*., 2007).

A popular, casual approach of coping with the challenges of balancing chemical-reaction equations is "by inspection" (Toth and Guo, 1997). The standard balancing-by-inspection approach is to make successive, hopefully intelligent guesses at the coefficients that will balance an equation, continuing until balance is achieved. This can be a straightforward, speedy approach for simple equations. But it rapidly becomes both lengthy and requires more skill for complex reactions that involve many reactants and products that require balancing.

Indeed, more systematic approaches to implementing the balancing-by-inspection method have been published for use with such challenging equations, and these are typically slightly faster and more reliable than simple inspection/guesswork. They usually recommend identifying either the "least adjustable" coefficients or the coefficients for a set of sub-reactions and balancing them by inspection first, then balancing the undetermined coefficients, algebraically, in a final step. But even these improved approaches overlook a critical problem. Balancing by inspection does not produce a systematic evaluation of all of the sets of coefficients that would potentially balance an equation; in other words, the technique encourages the notion that there is one, and only one, correct solution for any skeletal equation. In fact, there may be no possible solution, one unique solution, or an infinite number of solutions.

Basically, the substances taking part in a chemical reaction are represented by their molecular formulae, and their symbolic representation is termed as chemical equation (Roa, 2007). Chemical equation therefore is an expression showing symbolic representation of the reactants and the products usually positioned on the left side and on right side in a particular chemical reaction (Risteski, 2009). Unlike mathematical equations, left side and right side of chemical equations are usually separated using a single arrow pointing to the direction of the products for cases of one way reactions which are most times irreversible whereas a double arrow pointing either direction indicating a reversible reaction (Lay, 2006). Chemical equations play great role in theoretical as well as industrial chemistry. Mass balance of chemical equations as a century old problem is one of the most highly studied topics in chemical education base on the rule called the law of conservation of matter,which states that atoms are not created or destroyed in a chemical reaction. It always has the biggest interest for science students on every level. The qualitative and quantitative understanding of the chemical process estimating reactants, predicting the nature and amount of products and determining reaction conditions is necessary to balance the chemical equation. Every student which has general chemistry as a subject is bound to come across balancing chemical equations. Actually, balancing the chemical equations provide an excellent demonstrative and instructive example of interconnection between stoichiometry principles and linear algebra. The illustration of Chemical equation is shown by example:

The reactants sodium hydroxide (NaOH) and sulfuric acid (H_2SO_4) when react yields the products sodium sulfate (Na_2SO_4) and water and its chemical equation representation is as

$$NaOH + H_2SO_4 \longrightarrow Na_2SO_4 + H_2O \qquad \text{[Not Balanced]}$$

A chemical equation is said to be balanced, provided that the number of atoms of each type on the left is same as the number of atoms of the corresponding type on the right (Lay, 2006). This leads to the concept of stoichiometry which is defined as the quantitative relationship between reactants and products in a chemical equation is (Hill *et al*., 2000). In other words, stoichiometry is the proportional relationship between two or more substances during a chemical reaction (Myers *et al*., 2006). The ratio of moles of reactants and products is given by the coefficients in a balanced chemical equation (Hill *et al*., 2000). It is through this that the amount of reactant needed to produce a given quantity of product, or how much of a product is formed from a given quantity of reactant is determined (Myers *et al*., 2006).

Linear algebra at present is of growing importance in engineering research, science, frameworks, electrical networks, traffic flow, economics, statistics, technologies, and many others (Clugston and Flemming, 2002). It forms a foundation of numeric methods and its main instrument is matrices that can hold enormous amounts of data in a form readily accessible by the computer. This study thus illustrates how to construct a homogenous system of equations whose solution provides appropriate value to balance the atoms in the reactants and with those in the products using matrix algebra.

2. Methodology

In this section will be applied the linear algebra method on many chemical equations for their balancing. All chemical equations balanced here appears in many chemistry textbooks and they are chosen with an intention to balance them using matrix algebra by method of Guassian elimination method giving us an upper triangular matrix or Echelon matrix. To this end, we begin with the following examples immediately;

Example 2.1 Sodium hydroxide (NaOH) reacts with sulfuric acid (H_2SO_4) yields sodium sulfate (Na_2SO_4) and water.

The chemical equation is;

$$NaOH + H_2SO_4 \longrightarrow Na_2SO_4 + H_2O \quad \text{[Not Balanced]}$$

To balance this equation, we insert unknowns, multiplying the reactants and the products to get an equation of the form

$$wNaOH + xH_2SO_4 \longrightarrow yNa_2SO_4 + zH_2O$$

Next, we compare the number of sodium (Na), oxygen (O), hydrogen (H), and sulfur (S) atoms of the reactants with the number of the products. We obtain four linear equations

Na: $w = 2y$

O: $w + 4x = 4y + z$

H: $w + 2x = 2z$

S: $x = y$

It is important to note that we made use of the subscripts because they count the number of atoms of a particular element. Rewriting these equations in standard form, we see that we have a homogenous linear system in four unknowns, that is, w, x, y and z

$$\begin{aligned} w + 0x - 2y + 0z &= 0 \\ w + 4x - 4y - z &= 0 \\ w + 2x + 0y - 2z &= 0 \\ 0w + x - y + 0z &= 0 \end{aligned}$$

Alternatively;

$$\begin{aligned} w - 2y \quad &= 0 \\ w + 4x - 4y - z &= 0 \\ w + 2x - 2z &= 0 \\ x - y \quad &= 0 \end{aligned}$$

Writing this equations or system in matrix form, we have the augmented matrix

$$\left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 1 & 4 & -4 & -1 & 0 \\ 1 & 2 & 0 & -2 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{array}\right)$$

Solving using Echelon form:

Keeping the first row constant and subtracting the second row from that,

$$\left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 1 & 4 & -4 & -1 & 0 \\ 1 & 2 & 0 & -2 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{array}\right) R_1 - R_2 \sim \left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & -4 & 2 & 1 & 0 \\ 1 & 2 & 0 & -2 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{array}\right)$$

Now, subtracting the third row from the first row, we obtain

$$\left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & -4 & 2 & 1 & 0 \\ 1 & 2 & 0 & -2 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{array}\right) R_1 - R_3 \sim \left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & -4 & 2 & 1 & 0 \\ 0 & -2 & -2 & 2 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{array}\right)$$

Keeping the first and second row constant and multiplying the third row by (-2) and the fourth row by (4), we obtain

$$\left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & -4 & 2 & 1 & 0 \\ 0 & -2 & -2 & 2 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & -4 & 2 & 1 & 0 \\ 0 & 4 & 4 & -4 & 0 \\ 0 & 4 & -4 & 0 & 0 \end{array}\right)$$

Now, adding row two and three, we obtain

$$\left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & -4 & 2 & 1 & 0 \\ 0 & 0 & 6 & -3 & 0 \\ 0 & 4 & -4 & 0 & 0 \end{array}\right)$$

Adding second and fourth rows, we get

$$\left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & -4 & 2 & 1 & 0 \\ 0 & 0 & 6 & -3 & 0 \\ 0 & 0 & -2 & 1 & 0 \end{array}\right)$$

Multiplying the fourth row by 3, we have

$$\left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & -4 & 2 & 1 & 0 \\ 0 & 0 & 6 & -3 & 0 \\ 0 & 0 & -6 & 3 & 0 \end{array}\right)$$

Adding third and fourth rows, then we obtain

$$\left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & -4 & 2 & 1 & 0 \\ 0 & 0 & 6 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Then the solution is quickly found from the corresponding equations;

$$w + 0x - 2y + 0z = 0 \quad (1)$$
$$0w - 4x + 2y + z = 0 \quad (2)$$
$$0w + 0x + 6y - 3z = 0 \quad (3)$$

From equation (3)

$$6y - 3z = 0$$
$$6y = 3z$$
$$y = \frac{1}{2}z \quad (4)$$

Substituting the value of y into equation (1)

$$w - 2y = 0$$
$$w - 2\left(\frac{1}{2}\right)z = 0$$
$$w = z \quad (5)$$

Putting equation (4) and (5) into (2)

$$-4x + 2y + z = 0$$
$$-4x + 2\left(\frac{1}{2}\right)z + z = 0$$
$$-4x + 2z = 0$$

Multiply through by (-1)

$$4x - 2z = 0$$
$$4x = 2z$$
$$z = 2x \quad (6)$$

Hence equation (5) becomes

$$w = 2x$$

Now, putting equation (6) into (4)

$$y = \frac{1}{2}(2x)$$
$$y = x$$

But $y = \frac{1}{2}z$

Using reversible equations we obtain,

$$w = z$$
$$x = \frac{1}{2}z$$

$$y = \frac{1}{2}z$$

$\Rightarrow$

$$w + 0x + 0y - z = 0$$
$$0w + x + 0y - \frac{1}{2}z = 0$$
$$0w + 0x + y - \frac{1}{2}z = 0$$

become the reduced linear system or, the reduced row echelon form,

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Hence, since z can be chosen arbitrary and we are dealing with atoms, it is convenient to choose values so that all the unknowns are positive integers. One of such choice is z = 2 which yields

$w = 2,\ x = 1$, and $y = 1$

Corresponding solution using Matlab

>> $[wxyz]$ = solve ('w - 2*y = 0', 'w + 4*x - 4*y - z = 0', 'w + 2*x - 2*z = 0', '$x - y = 0$')

$$w = z$$
$$x = {}^{z}/_{2}$$
$$y = {}^{z}/_{2}$$
$$z = z$$

Since$z$ is an arbitrary constant, let $z = 2$; then we have

$$w = 2, x = 1, y = 1, z = 2$$

In this case our balance equation is

$$2NaOH + H_2SO_4 \longrightarrow Na_2SO_4 + 2H_2O$$

Example 2.2 Rust is formed when there is a chemical reaction between iron and oxygen. The compound that is formed is redish-brown scales that cover the iron object. Rust is an iron oxide whose chemical formula is Fe_2O_3, so the chemical equation for rust is

$$Fe + O_2 \longrightarrow Fe_2O_3$$

Balance the equation.

Solution: To balance this equation, we insert unknowns, multiplying the reactants and the products to get an equation of the form

$$xFe + yO_2 \longrightarrow zFe_2O_3$$

Next, we compare the number of iron (Fe), and oxygen (O) atoms of the reactants with the number of the products. We obtain two linear equations

Fe: $x = 2z$

O: $2y = 3z$

Rewriting these equations in standard form, we see that we have a homogenous linear system in three unknowns, that is, $x, yandz$.

$$x + 0y - 2z = 0$$
$$0x + 2y - 3z = 0$$

Alternatively;

$$x + 0y - 2z = 0$$
$$0x + 2y - 3z = 0$$
$$0x + 0y + 0z = 0$$

Writing this equations or system in matrix form, we have the augmented matrix

$$\left(\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 2 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Then the solution is quickly found from the corresponding equations;

$$x + 0y - 2z = 0 \quad (1)$$
$$0x + 2y - 3z = 0 \quad (2)$$

From equation (1), we have

$$z = {}^{x}/_{2} \quad (3)$$

substituting equation (3) to (2), we obtain

$$2y - 3\left(\frac{x}{2}\right) = 0$$
$$x = \frac{4}{3}y \quad (4)$$

substituting equation (4) to (3), we obtain

$$z = \frac{4}{3}y \Big/ 2$$

$$z = \frac{2}{3}y$$

multiplying throughout by 3

$$3z = 2y$$

Using reversible equations we obtain,

$$y = 3, z = 2$$

substituting the values of $z$to equation (1), we have

$$x = 4$$

$$\therefore x = 4, y = 3 \text{ and } z = 2$$

Corresponding solution using Matlab

>> $[xyz]$ = solve($'x - 2 * z = 0', \ '2 * y - 3 * z = 0', '0x + 0y + 0z = 0'$)

$$x = 2 * z$$

$$y = (3 * z)/2$$

$$z = z$$

Since z is an arbitrary constant, let $z = 2$ so we could have a whole number, we have

$$x = 4, y = 3, z = 2$$

Hence our balance equation is

$$4Fe + 3O_2 \longrightarrow 2Fe_2O_3$$

Example 2.3 Ethane is a gas similar to methane that burns in oxygen to give carborndioxide gas and steam. The steam condenses to form water droplets. The chemical equation for this reaction is

$$C_2H_6 + O_2 \longrightarrow CO_2 + H_2O$$

Balance the equation.

Solution: To balance this equation, we insert unknowns, multiplying the reactants and the products to get an equation of the form

$$wC_2H_6 + xO_2 \longrightarrow yCO_2 + zH_2O$$

Next, we compare the number of carbon (C), oxygen (O) and hydrogen (H) atoms of the reactants with the number of the products. We obtain three linear equations;

C: $2w = y$
H: $6w = 2z$
O: $2x = 2y + z$

Rewriting these equations in standard form, we see that we have a homogenous linear system in four unknowns, that is, w x, y and z.

$$2w + 0x - y + 0z = 0$$
$$6w + 0x + 0y - 2z = 0$$
$$0w + 2x - 2y - z = 0$$
$$0w + 0x + 0y + 0z = 0$$

Alternative

$$\begin{aligned} 2w \quad\quad - y \quad\quad &= 0 \\ 6w \quad\quad\quad\quad - 2z &= 0 \\ 2x - 2y - z &= 0 \end{aligned}$$

Writing this equations or system in matrix form, we have the augmented matrix

$$\left(\begin{array}{cccc|c} 2 & 0 & -1 & 0 & 0 \\ 6 & 0 & 0 & -2 & 0 \\ 0 & 2 & -2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Solving using Echelon form:

Keeping the first row constant and interchange second row for third row and third for the second,

$$\left(\begin{array}{cccc|c} 2 & 0 & -1 & 0 & 0 \\ 0 & 2 & -2 & -1 & 0 \\ 6 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Keeping the first and second row constant and multiplying the first row by 3 and subtract third row from the first row, we obtain the third row as;

$$\left(\begin{array}{cccc|c} 2 & 0 & -1 & 0 & 0 \\ 0 & 2 & -2 & -1 & 0 \\ 0 & 0 & -3 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Then the solution is quickly found from the corresponding equations;

$$2w \quad - y \quad = 0 \qquad (1)$$
$$2x - 2y - z = 0 \qquad (2)$$
$$-3y + 2z = 0 \qquad (3)$$

Using elimination method, to eliminate y we have

$$2w \quad - y \quad = 0 \qquad (1) \times 3$$
$$-3y + 2z = 0 \qquad (3) \times 1$$

Subtracting equation (3) from equation (1)

$$\therefore z = 3w$$

Using reversible equations we obtain,

$$w = 1, z = 3$$

substituting the value of $w$into equation (1)

$$y = 2$$

substituting the value of y and z into equation (2)

$$x = {}^{7}/_{2}$$

that is; $w = 1, x = {}^{7}/_{2},\ y = 2,\ z = 3$

Corresponding solution using Matlab

>> $[w\ x\ y\ z]$ = solve ('$2*w - y = 0$', '$6*w - 2*z = 0$', '$2*x - 2*y - z = 0$', '$0*w + 0*x + 0*y = 0$')

$$w = {}^{z}/_{3}$$
$$x = \frac{7*z}{6}$$
$$y = \frac{2*z}{3}$$
$$z = z$$

Since z is an arbitrary constant, we let $z = 3$ then

$$w = 1, x = \frac{7}{2}, y = 2$$

To this end, our balanced equation is

$$C_2H_6 + \frac{7}{2}O_2 \longrightarrow 2CO_2 + 3H_2O$$

Example 2.4 Consider the chemical equation below;

$$KHC_8H_4O_4 + KOH \longrightarrow K_2C_8H_4O_4 + H_2O$$

Balance the equation.

Solution: To balance this equation, we insert unknowns, multiplying the reactants and the products to get an equation of the form

$$wKHC_8H_4O_4 + xKOH \longrightarrow yK_2C_8H_4O_4 + zH_2O$$

Next, we compare the number of potassium (K), carbon (C), oxygen (O) and hydrogen (H) atoms of the reactants with the number of the products. We obtain four linear equations;

K: $w + x = 2y$
H: $5w + x = 4y + 2z$
C: $8w = 8y$
O: $4w + x = 4y + z$

Rewriting these equations in standard form, we see that we have a homogenous linear system in four unknowns, that is, $w, x, y, and\ z$.

$$w + x - 2y \quad = 0$$
$$5w + x - 4y - 2z = 0$$
$$8w \quad - 8y \quad = 0$$

Writing this equations or system in matrix form, we have the augmented matrix

$$\left(\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ 5 & 1 & -4 & -2 & 0 \\ 8 & 0 & -8 & 0 & 0 \\ 4 & 1 & -4 & -1 & 0 \end{array}\right)$$

Solving to reduce row Echelon form:
Keeping the first row constant, we obtain;

$$\left(\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ 5 & 1 & -4 & -2 & 0 \\ 8 & 0 & -8 & 0 & 0 \\ 4 & 1 & -4 & -1 & 0 \end{array}\right) R_2=5R_1\text{-}R_2 \sim \left(\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ 0 & 4 & -6 & 2 & 0 \\ 8 & 0 & -8 & 0 & 0 \\ 4 & 1 & -4 & -1 & 0 \end{array}\right)$$

Dividing second row by 2 and the third row by 8, we get

$$\left(\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ 0 & 4 & -6 & 2 & 0 \\ 8 & 0 & -8 & 0 & 0 \\ 4 & 1 & -4 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ 0 & 2 & -3 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 4 & 1 & -4 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ 0 & 2 & -3 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 4 & 1 & -4 & -1 & 0 \end{array}\right) R_3= R_1\text{-}R_3 \sim \left(\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ 0 & 2 & -3 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 4 & 1 & -4 & -1 & 0 \end{array}\right)$$

Interchanging second row for third row and third row for first row,

$$\left(\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 2 & -3 & 1 & 0 \\ 4 & 1 & -4 & -1 & 0 \end{array}\right)$$

Hence;

$$\left(\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 2 & -3 & 1 & 0 \\ 4 & 1 & -4 & -1 & 0 \end{array}\right) R_4=4R_1\text{-}R_4 \sim \left(\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 2 & -3 & 1 & 0 \\ 0 & 3 & -4 & 1 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 2 & -3 & 1 & 0 \\ 0 & 3 & -4 & 1 & 0 \end{array}\right) R_3= 2R_2\text{-}R_3 \sim \left(\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 3 & -4 & 1 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 3 & -4 & 1 & 0 \end{array}\right) R_4= 3R_2\text{-}R_4 \sim \left(\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right) R_4= R_3\text{-} R_4 \sim \left(\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Since we have the reduced row echelon matrix, then the solution is quickly found from the corresponding equations;

$$w + x \quad = 0 \qquad (1)$$
$$x - y \quad = 0 \qquad (2)$$
$$y - z = 0 \qquad (3)$$

from equation(3), we have

$$y = z$$

Using reversible equations we obtain,

$$y = 1, z = 1$$

substituting the value of y into equation(2)

$$x = 2$$

this implies that

$$w = 1$$

Hence, $w = 1, x = 1, y = 1, z = 1$

Corresponding solution using Matlab

>> $[w\ x\ y\ z]$ = solve($'w + x = 2 * y', '5 * w + x = 4 * y + 2 * z', '8 * w = 8 * y', '4 * w + x = 4 * y + z'$)

$$w = z$$
$$x = z$$
$$y = z$$
$$z = z$$

Since z is an arbitrary constant, let $z = 1$ we have

$$w = 1, x = 1, y = 1, z = 1$$

To this end, our balanced equation is

$$KHC_8H_4O_4 + KOH \longrightarrow K_2C_8H_4O_4 + H_2O$$

Discussion

In a balanced equation, coefficients specify the number of molecules (or formula units) of each element involved. The coefficients must satisfy Dalton's (Conservation of mass) requirement that atoms are not created or destroyed in a chemical reaction. There is no fixed procedure for balancing an equation. Although a trial-and-error approach is generally used in classrooms, a systematic algebraic approach is a principle of possibility that often works.

This procedure seems to substantially facilitate the balancing of equations that, traditionally, have been considered difficult for many students. It is interesting that the more difficult the equation, the greater this facilitation appears to be. This procedure allows average students and below average students to experience ready success in balancing, thus avoiding a traditional source of frustration and failure which might contribute to their losing interest in chemistry. One interesting serendipity of this procedure is how quickly it turns able pre-matrix students into extremely fast and accurate balancers of chemical equations.

The immediate importance of the procedure lies in the fact that it can remove the heuristic wall of haphazard inspection, replacing it with a near algorithmic procedure that virtually assures balancing success for average students and below average students. Also, it gives able students an unusual facility. A significant, but less immediate, advantage is the preparation the procedure could offer for future matrix techniques.

Conclusion

This allows average, and even low achieving students, a real chance at success. It can remove what is often a source of frustration and failure that turns students away from chemistry. Also, it allows the high achieving to become very fast and very accurate even with relatively difficult equations. A balancing technique based on augmented-matrix protocols was described in this work. Because of its unusual nature, it was best explained through demonstration in the methodology.

The practical superiority of the matrix procedure as the most general tool for balancing chemical equations is demonstrable. In other words, the mathematical method given here is applicable for all possible cases in balancing chemical equations.

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