Journal of Natural Sciences Research www.iiste.org
ISSN 2224-3186 (Paper) ISSN 2225-0921 (Online)
Vol.5, No.5, 2015

An Elzaki Transform Decomposition Algorithm Applied to a Class of Non-Linear Differential Equations

M. Khalid* Mariam Sultana Faheem Zaidi Uroosa Arshad
Department of Mathematical Sciences, Federal Urdu University of Arts, Sciences & Technology, Karachi-75300
*E-mail of corresponding author: khalidsiddiqui@fuuast.edu.pk

Abstract
Another version of the classic Sumudu Transform called the Elzaki Transform, was put forward as closely related to the Laplace Transform. In the following paper, the Elzaki Transform Algorithm, which has been built on the Decomposition Method, is presented to be applied to find approximate solution of a class of non-linear, initial value problems. This method gives an approximate solution in a Convergent-Series form with easily computable components necessitating no linearization or a low perturbation criterion. The most important part of this paper is the error analysis conducted between exact solutions and pade approximate solutions; it proves that our approximate solutions narrow in rapidly to the exact solutions. Moreover, as we will discuss after the results are resented, this algorithm can also be applicable to more general classes of linear and nonlinear differential equations.
Keywords: Elzaki Transform, Adomian Decomposition Method; Nonlinear Differential Equation, Series Solution, Convergence.

1. Introduction

The Elzaki transform of the function's belonging to a class B, the set is given by $B=\{f(t)\,|\,\exists M,k_1,k_2>0 \text{ such that} |f(t)|<Me^{|t|/k_j} \text{ if } t\in(-1)^j\times[0,\infty)\}$ Where $f(t)$ is denoted by $E[f(t)]=T(u)$, and is defined [1, 2, 3] as introduced a new integral transform called Elzaki Transform defined by

$$T(u)=u^2\int_0^\infty f(ut)e^{-t}dt\,;\quad k_1,k_2>0 \tag{1}$$

The introduction of the new transform is evidently a convenient tool in solving various problems related to differential equations as seen in [1, 2, 4, 5] and the references cited therein. Elzaki et al., in [1, 2, 4] and the references therein, already established the basics of this transform.

The Elzaki Transform [1] and Adomain Decomposition Method [7, 8] have been combined in this paper to solve nonlinear differential equations. The order is as follows: the numerical method (Elazki Transform Method) is outlined in Section 2 and used to solve the non-linear differential equation. In Section 3, the method under question is applied to the differential equation. Three examples of the method being easily and conveniently applied on different equations will be presented. A conclusion and discussion follows in Section 4.

2 Elzaki Transform Decomposition Algorithm

The Elzaki Transform Decomposition Algorithm is employed in the solution of the following class of non-linear initial value problem.

$$y''(x)+a(x)y'(x)+b(x)y(x)=f(y) \tag{2}$$

Subject to initial conditions

$$y(0)=\alpha \quad ; \quad y'(0)=\beta \tag{3}$$

Here $f(y)$ is a non-linear operator and $a(x), b(x)$ represent functions in the underlying function space. The technique consists first of applying Elzaki Transformation (denoted through out his paper by E) to both side of Eq. (2), hence

$$E[y''(x)]+E[a(x)y'(x)]+E[b(x)y(x)]=E[f(y)] \tag{4}$$

Applying the formula on Elzaki transform [5], we attain

$$\frac{E[y(x)]}{u^2}-y(0)-uy'(0)+E[a(x)y'(x)]+E[b(x)y(x)]=E[f(y)] \tag{5}$$

$$\frac{E[y(x)]}{u^2}=\alpha+\beta u-E[a(x)y']-E[b(x)y(x)]+E[f(y)] \tag{6}$$

Multiplying both sides of the above expansion by u^2

$$E[y(x)] = \alpha u^2 + \beta u^3 - u^2 E[a(x)y'(x)] - u^2 E[b(x)y(x)] + u^2 E[f(y)] \tag{7}$$

Next, the solution is presented as an infinite series, as in:

$$y(x) = \sum_{n=0}^{\infty} y_n(x) \tag{8}$$

Where, in the above equation, the term $y_n(x)$ is to be recursively computed. Moreover, the non-linear operator $f(y)$ is decomposed as follows:

$$f(y) = \sum_{n=0}^{\infty} A_n \tag{9}$$

Where $A_n = A_n(y_o, y_1, y_2, ..., y_n)$ are the so-called Adomian Polynomials [6, 7]. The first few Polynomials are given by

$$A_o = f(y_o),$$
$$A_1 = y_1(x) f^{(1)}(y_o),$$
$$A_2 = y_2(x) f^{(1)}(y_o) + \frac{1}{2!} y_1^2(x) f^{(2)}(y_o),$$
$$A_3 = y_3(x) f^{(1)}(y_o) + y_1(x) y_2(x) f^{(2)}(y_o) + \frac{1}{3!} y_1^3(x) f^{(3)}(y_o), \tag{10}$$

Substituting Eq. (8) and Eq. (9) into Eq. (7)

$$E\left[\sum_{n=0}^{\infty} y_n(x)\right] = \alpha u^2 + \beta u^3 - u^2 E\left[a(x)\sum_{n-0}^{\infty} y_n'(x)\right] - u^2 E\left[b(x)\sum_{n=0}^{\infty} y_n(x)\right] + u^2 E\left[\sum_{n=0}^{\infty} A_n\right] \tag{11}$$

Employing the linearity of the Elzaki Transform, it follows that

$$\sum_{n=0}^{\infty} E[y_n(x)] = \alpha u^2 + \beta u^3 - u^2 \sum_{n=0}^{\infty} E[a(x)y_n'(x)] - u^2 \sum_{n=0}^{\infty} E[b(x)y_n(x)] + u^2 \sum_{n=0}^{\infty} E[A_n] \tag{12}$$

Matching both sides of Eq. (12) yield the following iterative algorithm:

$$E[y_o(x)] = \alpha u^2 + \beta u^3 \tag{13}$$
$$E[y_1(x)] = -u^2 E[a(x)y_o'(x)] - u^2 E[b(x)y_o(x)] + u^2 E[A_o] \tag{14}$$
$$E[y_2(x)] = -u^2 E[a(x)y_1'(x)] - u^2 E[b(x)y_1(x)] + u^2 E[A_1] \tag{15}$$

In general,

$$E[y_{n+1}(x)] = -u^2 E[a(x)y_n'(x)] - u^2 E[b(x)y_n(x)] + u^2 E[A_n] \tag{16}$$

Applying the Inverse Elzaki Transform to Eq. (13), we get

$$y_o(x) = \alpha + \beta x \tag{17}$$

Substituting this value of y_o into Eq. (14) gives

$$E[y_1(x)] = -u^2 E[a(x)\beta] - u^2 E[b(x)\cdot(\alpha + \beta x)] + u^2 E[A_o] \tag{18}$$

The Elzaki Transform is applied on the right hand side of Eq. (18) then again, the inverse Elzaki Transform. Through this, the value of $y_1(x)$ is obtained. The other terms $y_2(x), y_3(x), ..., y_n(x)$ can be obtained recursively in a similar fashion by using Eq. (16).

3. Numerical Examples:

The Elzaki Transform Decomposition Algorithm, described in section 2, is applied to some special cases of class of non-linear initial value problems given in (2) and (3)

Example 1: First, we take into account a non-linear ordinary differential equation:

$$y'' + (1 - x)y' - y = 2y^3 \tag{19}$$

Subject to initial conditions

$$y(0) = 1 \quad ; \quad y'(0) = 1 \tag{20}$$

The closed form solution of which is $y = \frac{1}{1-x}$ (21)

Applying Elzaki Transform on both sides of (19), using initial conditions (20) gives

$$E[y] = u^2 + u^3 - u^2 E[(1-x)y'] + u^2 E[y] + 2u^2 E[y^3] \quad (22)$$

Following the technique, if we assume an infinite series solution of the form (8), we obtain

$$E\left[\sum_{n=0}^{\infty} y(u)\right] = u^2 + u^3 - u^2 E\left[(1-x)\sum_{n=0}^{\infty} y_n'\right] + u^2 E\left[\sum_{n=0}^{\infty} y_n\right] + 2u^2 E\left[\sum_{n=0}^{\infty} A_n\right] \quad (23)$$

Where the non-linear operator $f(y) = y^3$ is decomposed in terms of equation (9), the Adomian polynomials. From (10) the first few Adomian polynomials for $f(y) = y^3$ are given by

$$A_o = y_o^3$$
$$A_1 = 3y_o^2 y_1$$
$$A_2 = 3y_o^2 y_2 + 3y_0 y_1^2$$
$$A_3 = 3y_o^2 y_3 + 6y_o y_1 y_2 + y_1^3 \quad (24)$$

By employing the linearity of Elzaki Transform, and matching both sides of (23), it results in the following iterative scheme

$$E[y_o] = u^2 + u^3 \quad (25)$$
$$E[y_1] = -u^2 E[(1-x)y_o'] + u^2 E[y_o] + 2u^2 E[A_o] \quad (26)$$
$$E[y_2] = -u^2 E[(1-x)y_1'] + u^2 E[y_1] + 2u^2 E[A_1] \quad (27)$$
$$E[y_3] = -u^2 E[(1-x)y_2'] + u^2 E[y_2] + 2u^2 E[A_2] \quad (28)$$

In general,

$$E[y_{n+1}] = -u^2 E[(1-x)y_n'] + u^2 E[y_n] + 2u^2 E[A_n] \quad (29)$$

Operating with Elzaki Inverse Transform on both sides of (25) yields

$$y_o = 1 + x \quad (30)$$

Substituting this value of y_o and of $A_o = y_o^3$ given in (24) into (26), we get

$$E[y_1] = -u^2 E[(1-x)y_o'] + u^2 E[y_o] + 2u^2 E[(A_o)^3] \quad (31)$$
$$E[y_1] = 2u^4 + 8u^5 + 12u^6 + 12u^7 \quad (32)$$

The Inverse Elzaki Transform applied to (32) gives

$$y_1 = x^2 + \frac{4}{3}x^3 + \frac{1}{2}x^4 + \frac{1}{10}x^5 \quad (33)$$

Substituting (33) into (27) and using the value of A_1 given in (24) we have:

$$E[y_2] = -u^2 E[(1-x)y_1'] + u^2 E[y_1] + 2u^2 E[A_1]$$
$$E[y_2] = -u^2 E\left[(1-x)\left(2x + 4x^2 + 2x^3 + \frac{1}{2}x^4\right)\right] + u^2 E\left[x^2 + \frac{4}{3}x^3 + \frac{1}{2}x^4 + \frac{1}{10}x^5\right] + 2u^2 E\left[3(1+x)^2\left(x^2 + \frac{4}{3}x^3 + \frac{1}{2}x^4 + \frac{1}{10}x^5\right)\right] \quad (34)$$

By simplifying the right-hand side of eq. (34), we get

$$E[y_2] = -2u^5 + 10u^6 + 140u^7 + 648u^8 + 1824u^9 + 3024u^{10} + 3024u^{11} \quad (35)$$

The Inverse Elzaki Transform applied to (35) yields

$$y_2 = -\frac{1}{3}x^3 + \frac{5}{12}x^4 + \frac{7}{6}x^5 + \frac{9}{10}x^6 + \frac{38}{105}x^7 + \frac{3}{40}x^8 + \frac{1}{120}x^9 \quad (36)$$

Substituting (36) into (28) and using the value of A_2 given in (24)

$$E[y_3] = -u^2 E[(1-x)y_2'] + u^2 E[y_2] + 2u^2 E[A_2] \quad (37)$$

Simplifying the right-hand side of eq. (37), we get

$$E[y_3] = 2u^6 - 30u^7 + 18u^8 + 4032u^9 + 42000u^{10} + 242592u^{11} + 931104u^{12} + 2505600u^{13} + 4390848u^{14} + 4390848u^{15} \tag{38}$$

The Inverse Elzaki Transform applied to (38) yields

$$y_3 = \frac{1}{12}x^4 - \frac{1}{4}x^5 + \frac{1}{40}x^6 + \frac{4}{5}x^7 + \frac{25}{24}x^8 + \frac{361}{540}x^9 + \frac{3233}{12600}x^{10} + \frac{29}{462}x^{11} + \frac{11}{1200}x^{12} + \frac{11}{15600}x^{13} \tag{39}$$

Substituting this value of y_3 and that of A_3 given in (39) and (24) into (29)

$$E[y_4] = -u^2 E[(1-x)y_3'] + u^2 E[y_3] + 2u^2 E[A_3] \tag{40}$$

$$E[y_4] = -2u^7 + 52u^8 - 738u^9 - 7278u^{10} + 141120u^{11} + 3356976u^{12} + 35047200u^{13} + 239901408u^{14} + 1190263104u^{15} + 4376550528u^{16} + 11758763520u^{17} + 21224560896u^{18} + 21224560896u^{19} \tag{41}$$

The Inverse Elzaki Transform applied to (41) yield

$$y_4 = -\frac{1}{60}x^5 + \frac{13}{180}x^6 - \frac{41}{280}x^7 - \frac{1213}{6720}x^8 + \frac{7}{18}x^9 + \frac{9991}{10800}x^{10} + \frac{14603}{16632}x^{11} + \frac{832991}{1663200}x^{12} + \frac{2066429}{10810800}x^{13} + \frac{20101}{400400}x^{14} + \frac{8101}{900900}x^{15} + \frac{211}{208000}x^{16} + \frac{211}{3536000}x^{17} \tag{42}$$

Higher iteration can be obtained in a similar way

Therefore, the approximate solution is

$$y = y_o + y_1 + y_2 + y_3 + y_4 + \dots\dots\dots$$

$$y = 1 + x + x^2 + x^3 + x^4 + x^5 + \frac{359}{360}x^6 + \frac{853}{840}x^7 + \frac{2097}{2240}x^8 + \frac{1151}{1080}x^9 + \frac{17867}{15120}x^{10} + \frac{15647}{16632}x^{11} + \frac{848237}{1663200}x^{12} + \frac{518513}{2702700}x^{13} + \frac{20101}{400400}x^{14} + \frac{8101}{900900}x^{15} + \frac{211}{208000}x^{16} + \frac{211}{3536000}x^{17} \tag{43}$$

The $[5,5]$ Pade approximation of the solution obtained in (43) is given by

$$y \cong \frac{1 + \frac{53}{7}x + \frac{8703}{392}x^2 - \frac{46607}{2058}x^3 - \frac{178320109}{460992}x^4 + \frac{1}{360}x^5}{1 + \frac{46}{7}x + \frac{5735}{392}x^2 - \frac{369191}{8232}x^3 - \frac{55960047}{153664}x^4 + \frac{2674820843}{6914880}x^5} \tag{44}$$

Table 1 gives the differences between approximate analytical results, obtained by Elzaki Transform Method with the following exact results.

x	Relative Error
0.0	0.000000E+00
0.1	1.609330E-09
0.2	8.539570E-08
0.3	1.897097E-06
0.4	5.028530E-06
0.5	5.207070E-06
0.6	7.809602E-06
0.7	1.305187E-05
0.8	2.447406E-05
0.9	6.038051E-05

Table 1: Calculation of Relative error (example 1) by using Elazaki Transform decomposition Algorithm

Example 2: Consider the initial value problem

$$y' + y^2 = 1 \tag{45}$$

Subject to initial condition

$$y(0) = 3 \tag{46}$$

The closed form of which is

$$y = -1 + \frac{2}{1 - 0.5e^{-2x}} \tag{47}$$

First, we apply Elzaki Transform to both sides of equation (45), using initial condition gives

$$E[y] = u^3 + 3u^2 - uE[y^2] \tag{48}$$

Assuming an infinite series solution of the form (8), we have

$$E\left[\sum_{n=0}^{\infty} y_n\right] = u^3 + 3u^2 - uE\left[\sum_{n=0}^{\infty} A_n\right] \tag{49}$$

Where the non-linear $f(y) = y^2$ is decomposed as in (9) in terms of the Adomian polynomials from (10) the first few Adomian Polynomials are

$$A_o = y_o^2$$
$$A_1 = 2y_o y_1$$
$$A_2 = 2y_o y_2 + y_1^2$$
$$A_3 = 2y_o y_3 + 2y_1 y_2 \tag{50}$$

Following the Elzaki Transform Decomposition Method, if we match both sides of (48) we obtain the iterative scheme

$$E[y_o] = u^3 + 3u^2 \tag{51}$$
$$E[y_1] = -uE[A_o] \tag{52}$$
$$E[y_2] = -uE[A_1] \tag{53}$$

Next, the general iterative step is

$$E[y_{n+1}] = -uE[A_n] \tag{54}$$

The Inverse Elzaki Transform applied to (51) results

$$y_o = x + 3 \tag{55}$$

Substituting $y_o = x + 3$ and $A_o = y_o^2$ given in (50) into (52) and then the Inverse Elzaki Transform applied, we obtain

$$y_1 = -9x - 3x^2 - \frac{1}{3}x^3 \tag{56}$$

Using this value of y_1 into (53) yields

$$E[y_2] = 54u^4 + 72u^5 + 48u^6 + 16u^7 \tag{57}$$

Taking Inverse Elzaki Transform applied to (57) results in

$$y_2 = 27x^2 + 12x^3 + 2x^4 + \frac{2}{15}x^5 \tag{58}$$

Similarly y_2 and A_2 given in (50) into (54), we obtain

$$y_3 = -81x^3 - 45x^4 - \frac{51}{5}x^5 - \frac{17}{15}x^6 - \frac{17}{315}x^7 \tag{59}$$

In the same way,

$$y_4 = 243x^4 + 162x^5 + \frac{231}{5}x^6 + \frac{248}{35}x^7 + \frac{62}{105}x^8 + \frac{62}{2835}x^9 \tag{60}$$

Higher iteration can be obtained in a like way. Therefore, the approximate solution is:

$$y = y_o + y_1 + y_2 + y_3 + y_4 + ...$$

$$y = 3 - 8x + 24x^2 - \frac{208}{3}x^3 + 200x^4 + \frac{2279}{15}x^5 + \frac{676}{15}x^6 + \frac{443}{15}x^7 + \frac{62}{105}x^8 + \frac{62}{2835}x^9 + \ldots\ldots\ldots \quad (61)$$

The $[3,3]$ Pade approximation of the solution obtained in (61) given by:

$$y \cong \frac{3 - \frac{338993x}{32797} - \frac{1079288529x^2}{1311880} - \frac{1090514809x^3}{3935640}}{1 - \frac{25539x}{32797} - \frac{372982043x^2}{1311880} - \frac{1077295609x^3}{1311880}} \quad (65)$$

To make sure the accuracy of the Elzaki Transform solution, error analysis was made and is displayed in Table 2

x	Relative Error
0.0	0.000000E+00
0.1	1.250900E-10
0.2	1.235068E-08
0.3	1.671428E-07
0.4	1.009246E-06
0.5	3.926337E-06
0.6	1.158095E-05
0.7	2.823738E-05
0.8	5.992332E-05
0.9	1.144124E-04
1.0	2.010472E-04

Table 2: Relative Error obtained in Example 2 using Elzaki transform decomposition algorithm

Example 3: Consider the following non-linear initial value problem

$$y' = 4y - y^3 \quad (63)$$

Subject to initial condition

$$y(0) = 0.5 \quad (64)$$

The closed form of which solution is:

$$y = 2\left(\frac{e^{8x}}{e^{8x} + 15}\right)^{1/2} \quad (65)$$

Operating with Elzaki Transform on both sides of (63)

$$\frac{1}{u}E[y] - uy(0) = 4E[y] - E[y^3] \quad (66)$$

Using the initial condition (64) then simplifying the resulting equation in (66), we obtain

$$E[y] = 0.5u^2 + 4uE[y] - uE[y^3] \quad (67)$$

Assuming an infinite series solution as in (8), we have

$$E\left[\sum_{n=0}^{\infty} y\right] = 0.5u^2 + 4uE\left[\sum_{n=0}^{\infty} y\right] - uE\left[\sum_{n=0}^{\infty} A_n\right] \quad (68)$$

Wherein, the non-linear operator $f(y) = y^3$ is decomposed (as in 9) in terms of the Adomian polynomial. For this case, the first few are given in (24).

$$A_o = y_o^3$$

$$A_1 = 3y_o^2 y_1$$

$$A_2 = 3y_o^2 y_2 + 3y_0 y_1^2$$

$$A_3 = 3y_o^2 y_3 + 6y_o y_1 y_2 + y_1^3$$

Matching both sides of (68), the components of y can be defined as below:

$$E[y_o] = 0.5u^2 \quad (69)$$

$$E[y_1] = 4uE[y_o] - uE[A_o] \quad (70)$$

$$E[y_2] = 4uE[y_1] - uE[A_1] \quad (71)$$

In general,

$$E[y_{n+1}] = 4uE[y_n] - uE[A_n] \quad (72)$$

The term y_n can be obtained in a recursive manner. Taking the Inverse Elzaki Transform of (69) gives

$$y_o = 0.5 \quad (73)$$

Substituting this value of y_o into (70) and using $A_o = y_o^3$ from (24), we obtain

$$E[y_1] = \frac{15}{8}u^3 \quad (74)$$

Taking the Inverse Elzaki Transform of (74) :

$$y_1 = \frac{15}{8}x \quad (75)$$

Using this value of y_1 into (71) yields

$$E[y_2] = \frac{195}{32}u^4 \quad (76)$$

Taking the Inverse Elzaki Transform of (76) gives

$$y_2 = \frac{195}{64}x^2 \quad (77)$$

Using this value of y_2 into (72) gives

$$E[y_3] = \frac{1185}{128}u^5 \quad (78)$$

Taking the Inverse Elzaki Transform of (78) gives

$$y_3 = \frac{395}{256}x^3 \quad (79)$$

Similarly

$$E[y_4] = -\frac{57495}{512}u^6 \quad (80)$$

Taking the Inverse Elzaki Transform of (80) gives

$$y_4 = -\frac{19165}{4096}x^4 \quad (81)$$

The higher iteration is obtained in a similar way. The series solution is, therefore:

$$y = y_o + y_1 + y_2 + y_3 + y_4 + ...$$

$$y = 0.5 + \frac{15}{8}x + \frac{195}{64}x^2 + \frac{395}{256}x^3 - \frac{19165}{4096}x^4 + ... \quad (82)$$

The $[3,3]$ Pade approximation of this approximate solution is

$$y \cong \frac{\frac{1}{2} + \frac{3526406813}{1524125936}x + \frac{29766004321}{6096503744}x^2 + \frac{537242566947}{97544059904}x^3}{1 + \frac{668670683}{762062968}x + \frac{1160659231}{3048251872}x^2 + \frac{56314010537}{48772029952}x^3} \quad (83)$$

Table 3 shows the accuracy of the results of example 3, as estimated by error functions

x	Relative Error
0.0	0.000000E+00
0.1	2.214595E-05
0.2	8.864124E-04
0.3	5.786542E-03
0.4	1.611725E-02
0.5	2.440686E-02
0.6	1.853382E-02
0.7	7.030542E-03
0.8	4.871083E-02
0.9	9.914872E-02
1.0	1.518632E-01

Table 3: Relative Error obtained using Elzaki transform decomposition algorithm with Four Iteration

4. Concluding Remarks

This paper put forward a fresh technique for the determination of the approximate solution of non-linear, differential equations. Three examples in the form of literature are presented so that the efficacy and competence of the method can be made sure of. The method is simple in application, and yet, relatively more accurate than previously used methods. The results attained in this paper suggest that this algorithm can also be readily applied to systems that are more complex than these differential equations.

Acknowledgments

The author would like to thank the academic editor for his invaluable insight during editing, and the referees for their corrections and suggestions; which helped in increasing the clarity of this paper. Thanks to Ms.Wishaal Khalid for proofreading.

References

[1] Tarig M. Elzaki, Salih M. Elzaki, The new transform “Elzaki Transform”, Global Journal of Pure and Applied mathematics, 1, (2011), 57-64.
[2] Tarig M. Elzaki, Salih M. Elzaki, On the connections between Laplace and Elzaki transforms, Advances in Theoretical and Applied mathematics, 6(1), (2011), 1-11.
[3] Khalid, M., Mariam, S., Faheem, Z. and Uroosa, A. “Application of Elzaki Transform Method on Some Fractional Differential Equations”, Mathematical Theory and Modeling, 5 (1), 2015, 89-96
[4] Tarig M. Elzaki, Salih M. Elzaki, “On the Elzaki transforms and system of partial diffrential equations”, Advances in Theoretical and Applid Mathematics, 6(1), (2011), 115-123.
[5] Tarig M. Elzaki, Salih M. Elzaki, “On the Elzaki transform and higher order ordinary diffrential equations”, Advances in Theoretical and Applied mathematics, 6(1), (2011), 107-113.

[6] Tarig M. Elzaki, Salih M. Elzaki, “Application of new transform “Elzaki Transform”, Global Journal of Pure and Applied mathematics, 7(1), (2011), 65-70.
[7] Adomian, G. and Adomian, G.E. “A global method for solution of complex systems”, Math. Model. 5, (1984), 521-568.
[8] Adomian, G. “Solving Frontier Problems of Physics: The Decomposition Method”, Kluwer Academic Publishers, Dordecht, 1994.