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The Grand Unified Theory- A Predator Prey Approach, Part Two The Final Solution

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Abstract:

In this final part, we report the consubstantiate model and investigate the Solutional behaviour, stability analysis and asymptotic stability. For details, reader is kindly referred to part one. Philosophy merges with ontology, ontology merges with univocity of being, analogy has always a theological vision, not a philosophical vision, and one becomes adapted to the forms of singular consciousness, self and world. The univocity of being does not mean that there is one and the same being; on the contrary, beings are multiple and different they are always produced by disjunctive synthesis; and they themselves are disintegrated and disjoint and divergent; membra disjuncta.like gravity. Like electromagnetism; the constancy of gravity does not mean there does not exist total gravity, the universal theory depends upon certain parameters and it is disjoint; conservations of energy and momentum is one; but they hold good for each and every disjoint system; so there can be classification of systems based on various parametric representationalitiesof the theory itself. This is very important. Like one consciousness, it is necessary to understand that the individual consciousness exists, so does the collective consciousness and so doth the evolution too. These are the aspects which are to be borne in my mind in unmistakable terms .The univocity of being signifies that being is a voice that is said and it is said in one and the same "consciousness". Everything about which consciousness is spoken about. Being is the same for everything for which it is said like gravity, it occurs therefore as a unique event for everything. For everything for which it happens, eventum tantum, it is the ultimate form for all of the forms; and all these forms are disjointed. It brings about resonance and ramification of its disjunction; the univocity of being merges with the positive use of the disjunctive synthesis, and this is the highest affirmation of its univocity, highest affirmation of a Theory be it GTR or QFT. Like gravity; it is the eternal resurrection or a return itself, the affirmation of all chance in a single moment, the unique cast for all throws; a simple rejoinder for Einstein's god does not play dice; one being, one consciousness, for all forms and all times. A single instance for all that exists, a single phantom for all the living single voice for every hum of voices, or a single silence for all the silences; a single vacuum for all the vacuumes; consciousness should not be said without occuring; if consciousness is one unique event in which all the events communicate with each other. Univocity refers both to what occurs to what it is said, the attributable to all states of bodies and states of affairs and the expressible of every proposition. So univocity of consciousness means the identity of the noematic attribute and that which is expressed linguistically and sensefullly. Univocity means that it does not allow consciousness to be subsisting in a quasi state and but expresses in all pervading reality; Despite philosophical overtones, the point we had to make is clear. There doth exist different systems for which universal laws are applied and they can be classified. And there are situations and conditions under which the law itself breaks; this is the case for dissipations or detritions coefficient in the model.

Introduction:

We incorporate the following forces:

1. Electro Magnetic Force (EMF)
2. Gravity

3. Strong Nuclear Force
4. Weak Nuclear Force

Notation :

Electromagnetism And Gravity:

G_{13} : Category One Of gravity

G_{14} : Category Two Of Gravity

G_{15} : Category Three Of Gravity

T_{13} : Category One Of Electromagnetism

T_{14} : Category Two Of Electromagnetism

T_{15} :Category Three Of Electromagnetism

Strong Nuclear Force And Weak Nuclear Force

G_{16} : Category One Of Weak Nuclear Force

G_{17} : Category Two Of Weak Nuclear Force

G_{18} : Category Three Of Weak Nuclear Force

T_{16} : Category One Of Strong Nuclear Force

T_{17} : Category Two Of Strong Nuclear Force

T_{18} : Category Three Of Strong Nuclear Force

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}$ $(a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$
$(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$: are Accentuation coefficients

$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$,
$(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$ are Dissipation coefficients

Governing Equations: Of The System Electromagnetic Force And Gravitational Force:

The differential system of this model is now

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13}$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14}$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15}$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15}$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Governing Equations: System: Strong Nuclear Force And Weak Nuclear Force:

The differential system of this model is now

$\frac{dG_{16}}{dt} = (a_{16})^{(2)} G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right] G_{16}$

$\frac{dG_{17}}{dt} = (a_{17})^{(2)} G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right] G_{17}$

$\frac{dG_{18}}{dt} = (a_{18})^{(2)} G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right] G_{18}$

$\frac{dT_{16}}{dt} = (b_{16})^{(2)} T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19}), t)\right] T_{16}$

$\frac{dT_{17}}{dt} = (b_{17})^{(2)} T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}((G_{19}), t)\right] T_{17}$

$\frac{dT_{18}}{dt} = (b_{18})^{(2)} T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19}), t)\right] T_{18}$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}((G_{19}), t) =$ First detritions factor

Electro Magnetic Force-Gravity-Strong Nuclear Force-Weak Nuclear Force-

The Final Governing Equations

$\frac{dG_{13}}{dt} = (a_{13})^{(1)} G_{14} - \left[(a'_{13})^{(1)} \boxed{+(a''_{13})^{(1)}(T_{14}, t)} \boxed{+(a''_{16})^{(2,2)}(T_{17}, t)}\right] G_{13}$

$\frac{dG_{14}}{dt} = (a_{14})^{(1)} G_{13} - \left[(a'_{14})^{(1)} \boxed{+(a''_{14})^{(1)}(T_{14}, t)} \boxed{+(a''_{17})^{(2,2)}(T_{17}, t)}\right] G_{14}$

$\frac{dG_{15}}{dt} = (a_{15})^{(1)} G_{14} - \left[(a'_{15})^{(1)} \boxed{+(a''_{15})^{(1)}(T_{14}, t)} \boxed{+(a''_{18})^{(2,2)}(T_{17}, t)}\right] G_{15}$

Where $\boxed{(a''_{13})^{(1)}(T_{14}, t)}$, $\boxed{(a''_{14})^{(1)}(T_{14}, t)}$, $\boxed{(a''_{15})^{(1)}(T_{14}, t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2)}(T_{17}, t)}$, $\boxed{+(a''_{17})^{(2,2)}(T_{17}, t)}$, $\boxed{+(a''_{18})^{(2,2)}(T_{17}, t)}$ are second augmentation coefficients for category 1, 2 and 3

$\frac{dT_{13}}{dt} = (b_{13})^{(1)} T_{14} - \left[(b'_{13})^{(1)} \boxed{-(b''_{13})^{(1)}(G, t)} \boxed{+(b''_{16})^{(2,2)}(G_{19}, t)}\right] T_{13}$

$\frac{dT_{14}}{dt} = (b_{14})^{(1)} T_{13} - \left[(b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)}(G, t)} \boxed{+(b''_{17})^{(2,2)}(G_{19}, t)}\right] T_{14}$

$\frac{dT_{15}}{dt} = (b_{15})^{(1)} T_{14} - \left[(b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G, t)} \boxed{+(b''_{18})^{(2,2)}(G_{19}, t)}\right] T_{15}$

Where $\boxed{-(b''_{13})^{(1)}(G, t)}$, $\boxed{-(b''_{14})^{(1)}(G, t)}$, $\boxed{-(b''_{15})^{(1)}(G, t)}$ are first detrition coefficients for category 1, 2 and 3
$\boxed{+(b''_{16})^{(2,2)}(G_{19}, t)}$, $\boxed{+(b''_{17})^{(2,2)}(G_{19}, t)}$, $\boxed{+(b''_{18})^{(2,2)}(G_{19}, t)}$ are second augmentation coefficients for category 1, 2 and 3

$\frac{dG_{16}}{dt} = (a_{16})^{(2)} G_{17} - \left[(a'_{16})^{(2)} \boxed{+(a''_{16})^{(2)}(T_{17}, t)} \boxed{+(a''_{13})^{(1,1)}(T_{14}, t)}\right] G_{16}$

$\frac{dG_{17}}{dt} = (a_{17})^{(2)} G_{16} - \left[(a'_{17})^{(2)} \boxed{+(a''_{17})^{(2)}(T_{17},t)} \boxed{+(a''_{14})^{(1,1)}(T_{14},t)} \right] G_{17}$

$\frac{dG_{18}}{dt} = (a_{18})^{(2)} G_{17} - \left[(a'_{18})^{(2)} \boxed{+(a''_{18})^{(2)}(T_{17},t)} \boxed{+(a''_{15})^{(1,1)}(T_{14},t)} \right] G_{18}$

$Where \boxed{+(a''_{16})^{(2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1)}(T_{14},t)}$ are second detrition coefficients for category 1, 2 and 3

$\frac{dT_{16}}{dt} = (b_{16})^{(2)} T_{17} - \left[(b'_{16})^{(2)} \boxed{-(b''_{16})^{(2)}(G_{19},t)} \boxed{-(b''_{13})^{(1,1)}(G,t)} \right] T_{16}$

$\frac{dT_{17}}{dt} = (b_{17})^{(2)} T_{16} - \left[(b'_{17})^{(2)} \boxed{-(b''_{17})^{(2)}(G_{19},t)} \boxed{-(b''_{14})^{(1,1)}(G,t)} \right] T_{17}$

$\frac{dT_{18}}{dt} = (b_{18})^{(2)} T_{17} - \left[(b'_{18})^{(2)} \boxed{-(b''_{18})^{(2)}(G_{19},t)} \boxed{-(b''_{15})^{(1,1)}(G,t)} \right] T_{18}$

$Where \boxed{-(b''_{16})^{(2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1)}(G,t)}$ are second detrition coefficients for category 1, 2 and 3

Where we suppose

(A) $(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0,$

$i, j = 13,14,15$

(B) The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}, \ (r_i)^{(1)}$:

$(a''_i)^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$

$(b''_i)^{(1)}(G,t) \leq \ (r_i)^{(1)} \leq (b'_i)^{(1)} \leq (\hat{B}_{13})^{(1)}$

(C) $lim_{T_2 \to \infty} (a''_i)^{(1)} (T_{14},t) = (p_i)^{(1)}$

$\lim_{G \to \infty} (b''_i)^{(1)} (G,t) = \ (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants

and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition:

$|(a''_i)^{(1)}(T'_{14},t) - (a''_i)^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)} |T_{14} - T'_{14}| e^{-(\hat{M}_{13})^{(1)} t}$

$|(b''_i)^{(1)}(G',t) - (b''_i)^{(1)}(G,t)| < (\hat{k}_{13})^{(1)} ||G - G'|| e^{-(\hat{M}_{13})^{(1)} t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a''_i)^{(1)}(T'_{14},t)$ and $(a''_i)^{(1)}(T_{14},t)$. (T'_{14},t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a''_i)^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a''_i)^{(1)}(T_{14},t)$, the first augmentation coefficient would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$:

(D) $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} , \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$:

(E) There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together with $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$

$, (\hat{A}_{13})^{(1)} and \ (\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} \ (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

(F) $(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$

(G) The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}, \ (r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)}$$

$$(b_i'')^{(2)}(G, t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)}$$

(H) $\lim_{T_2 \to \infty} (a_i'')^{(2)} (T_{17}, t) = (p_i)^{(2)}$

$\lim_{G \to \infty} (b_i'')^{(2)} \left((G_{19}), t \right) = \ (r_i)^{(2)}$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$:

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t}$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}\left((G_{19}), t\right)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and $(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)} \right]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the SECOND augmentation coefficient would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

(I) $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}} , \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}} [(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1$$

$$\frac{1}{(\hat{M}_{16})^{(2)}} [(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1$$

Theorem 1: if the conditions IN THE FOREGOING above are fulfilled, there exists a solution satisfying the conditions

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

if the conditions IN THE FOREGOING above are fulfilled, there exists a solution satisfying the conditions

Definition of $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t} \ , \quad T_i(0) = T_i^0 > 0$$

PROOF:

Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i, T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \ , \ T_i(0) = T_i^0 \ , \ G_i^0 \leq (\hat{P}_{13})^{(1)} \ , T_i^0 \leq (\hat{Q}_{13})^{(1)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$$

By

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a_{13}')^{(1)} + a_{13}'')^{(1)} \left(T_{14}(s_{(13)}), s_{(13)} \right) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a_{14}')^{(1)} + (a_{14}'')^{(1)} \left(T_{14}(s_{(13)}), s_{(13)} \right) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a_{15}')^{(1)} + (a_{15}'')^{(1)} \left(T_{14}(s_{(13)}), s_{(13)} \right) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b_{13}')^{(1)} - (b_{13}'')^{(1)} \left(G(s_{(13)}), s_{(13)} \right) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b_{14}')^{(1)} - (b_{14}'')^{(1)} \left(G(s_{(13)}), s_{(13)} \right) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b_{15}')^{(1)} - (b_{15}'')^{(1)} \left(G(s_{(13)}), s_{(13)} \right) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{16})^{(2)}$, $T_i^0 \leq (\hat{Q}_{16})^{(2)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)} (G(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)} (G(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)} (G(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

(a) The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying CONCATENATED EQUATIONS into itself .Indeed it is obvious that

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[((\hat{P}_{13})^{(1)} + G_{14}^0) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for G_{14} , G_{15}, T_{13}, T_{14}, T_{15}

(b) The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} =$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1 \right)$$

From which it follows that

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \leq \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[((\hat{P}_{16})^{(2)} + G_{17}^0) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0} \right)} + (\hat{P}_{16})^{(2)} \right]$$

Analogous inequalities hold also for G_{17} , G_{18}, T_{16}, T_{17}, T_{18}

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)}$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)}$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i satisfying GLOBAL EQUATIONS into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric

$$d\left((G^{(1)},T^{(1)}),(G^{(2)},T^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}$: $(\tilde{G},\tilde{T})=\mathcal{A}^{(1)}(G,T)$

It results

$$|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{13})^{(1)}|G_{14}^{(1)}-G_{14}^{(2)}|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}|G_{13}^{(1)}-G_{13}^{(2)}|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{-(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}(T_{14}^{(1)},s_{(13)})|G_{13}^{(1)}-G_{13}^{(2)}|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}|(a''_{13})^{(1)}(T_{14}^{(1)},s_{(13)})-(a''_{13})^{(1)}(T_{14}^{(2)},s_{(13)})|\ e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$|G^{(1)}-G^{(2)}|e^{-(\hat{M}_{13})^{(1)}t}\leq$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\hat{A}_{13})^{(1)}+(\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left((G^{(1)},T^{(1)};\ G^{(2)},T^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (34,35,36) the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ *and* $(\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13,14,15$ depend only on T_{14} and respectively on $G(\text{and not on } t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$

From 19 to 24 it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$:

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below.

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$.

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.

The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions OF THE GLOBAL SYSTEM

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)}$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)}$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying GLOBAL EQUATIONS into itself

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric

$d\left(\left((G_{19})^{(1)}, (T_{19})^{(1)}\right), \left((G_{19})^{(2)}, (T_{19})^{(2)}\right)\right) =$

$sup_{i}\{\max_{t\in\mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\hat{M}_{16})^{(2)}t}\}$

Indeed if we denote

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $(\widetilde{G_{19}}, \widetilde{T_{19}}) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results

$|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t (a_{16})^{(2)} |G_{17}^{(1)} - G_{17}^{(2)}| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}} ds_{(16)} +$

$\int_0^t \{(a'_{16})^{(2)} |G_{16}^{(1)} - G_{16}^{(2)}| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} +$

$(a''_{16})^{(2)}(T_{17}^{(1)}, s_{(16)}) |G_{16}^{(1)} - G_{16}^{(2)}| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}} +$

$G_{16}^{(2)} |(a''_{16})^{(2)}(T_{17}^{(1)}, s_{(16)}) - (a''_{16})^{(2)}(T_{17}^{(2)}, s_{(16)})|\ e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}}\} ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$|(G_{19})^{(1)} - (G_{19})^{(2)}| e^{-(\widehat{M}_{16})^{(2)} t} \leq$

$\frac{1}{(\widehat{M}_{16})^{(2)}} ((a_{16})^{(2)} + (a'_{16})^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)} (\widehat{k}_{16})^{(2)}) d\left(((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)})\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)} t}$ and $(\widehat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(2)}$ and $(b''_i)^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$

From 19 to 24 it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(2)} - (a''_i)^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b'_i)^{(2)} t)} > 0$ for $t > 0$

Definition of $((\widehat{M}_{16})^{(2)})_1, ((\widehat{M}_{16})^{(2)})_2$ and $((\widehat{M}_{16})^{(2)})_3$:

Remark 3: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq ((\widehat{M}_{16})^{(2)})_1 - (a'_{17})^{(2)} G_{17}$ and by integrating

$G_{17} \leq ((\widehat{M}_{16})^{(2)})_2 = G_{17}^0 + 2(a_{17})^{(2)} ((\widehat{M}_{16})^{(2)})_1 / (a'_{17})^{(2)}$

In the same way , one can obtain

$G_{18} \leq ((\widehat{M}_{16})^{(2)})_3 = G_{18}^0 + 2(a_{18})^{(2)} ((\widehat{M}_{16})^{(2)})_2 / (a'_{18})^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 4: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below.

Remark 5: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$.

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded.

The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t),t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

Behavior of the solutions OF THE GLOBAL SYSTEM:

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

(a) $\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$-(\sigma_2)^{(1)} \leq -(a_{13}')^{(1)} + (a_{14}')^{(1)} - (a_{13}'')^{(1)}(T_{14},t) + (a_{14}'')^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b_{13}')^{(1)} + (b_{14}')^{(1)} - (b_{13}'')^{(1)}(G,t) - (b_{14}'')^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$:

(b) By $(\nu_1)^{(1)} > 0, (\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0, (u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$:

By $(\bar{\nu}_1)^{(1)} > 0, (\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0, (\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:-

(c) If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}, \; if \; (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if \; (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}, \; if \; (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \; if \; (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if \; (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined respectively

Then the solution of GLOBAL CONCATENATED EQUATIONS satisfies the inequalities

$$G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \leq G_{13}(t) \leq G_{13}^0 e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \leq G_{14}(t) \leq \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \leq G_{15}(t) \leq \right.$$

$$\left.\frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}}$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \leq T_{15}(t) \leq$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:-

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

Behavior of the solutions of GLOBAL EQUATIONS

If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$:

(d) $\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)} \leq -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \leq -(\sigma_1)^{(2)}$$

$$-(\tau_2)^{(2)} \leq -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \leq -(\tau_1)^{(2)}$$

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$:

By $(\nu_1)^{(2)} > 0, (\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0, (u_2)^{(2)} < 0$ the roots

(e) of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and

Definition of $(\bar{\nu}_1)^{(2)},, (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$:

By $(\bar{\nu}_1)^{(2)} > 0, (\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0, (\bar{u}_2)^{(2)} < 0$ the

roots of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:-

(f) If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}, \ \boldsymbol{if}\ (\nu_0)^{(2)} < (\nu_1)^{(2)}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}, \boldsymbol{if}\ (\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)},$

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$

and analogously

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}, \ \boldsymbol{if}\ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if}\ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}, \ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$

Then the solution of GLOBAL EQUATIONS satisfies the inequalities

$$G_{16}^0 e^{\left((S_1)^{(2)} - (p_{16})^{(2)}\right)t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$$

$(p_i)^{(2)}$ is defined

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{\left((S_1)^{(2)} - (p_{16})^{(2)}\right)t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}\left((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)}\right)}\left[e^{\left((S_1)^{(2)} - (p_{16})^{(2)}\right)t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$$

$$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}\left((S_1)^{(2)} - (a'_{18})^{(2)}\right)}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{\left((R_1)^{(2)} + (r_{16})^{(2)}\right)t}}$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{\left((R_1)^{(2)} + (r_{16})^{(2)}\right)t}$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}\left((R_1)^{(2)} - (b'_{18})^{(2)}\right)}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}\left((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)}\right)}\left[e^{\left((R_1)^{(2)} + (r_{16})^{(2)}\right)t} - e^{-(R_2)^{(2)}t}\right] + T_{18}^0 e^{-(R_2)^{(2)}t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:-

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

PROOF : From GLOBAL EQUATIONS we obtain

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$$\frac{d\nu^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)\nu^{(1)} - (a_{14})^{(1)}\nu^{(1)}$$

Definition of $\nu^{(1)}$:- $\boxed{\nu^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)}\right) \leq \frac{d\nu^{(1)}}{dt} \leq -\left((a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(1)}, (\nu_0)^{(1)}$:-

(a) For $0 < \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (\nu_1)^{(1)} < (\bar{\nu}_1)^{(1)}$

$$\nu^{(1)}(t) \geq \frac{(\nu_1)^{(1)} + (C)^{(1)}(\nu_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(\nu_1)^{(1)} - (\nu_0)^{(1)}}{(\nu_0)^{(1)} - (\nu_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner , we get

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)} - (\nu_0)^{(1)}}{(\nu_0)^{(1)} - (\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

(b) If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case,

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)} + (C)^{(1)}(\nu_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

(c) If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}$, $\boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}$, $\boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$

Now, using this result and replacing it in CONCATENATED SYSTEM OF EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

PROOF : From GLOBAL EQUATIONS we obtain (PLEASE REFER PART ONE OF THE PAPER)

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$

It follows

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

(d) For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$

(e) If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case,

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

(f) If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:-

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(2)}(t)$:-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in GLOBAL SOLUTIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

We can prove the following

Theorem 3: If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with$ $(p_{13})^{(1)}, (r_{14})^{(1)}$ as defined are satisfied , then the system

If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$,

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$

$with$ $(p_{16})^{(2)}, (r_{17})^{(2)}$ as defined are satisfied , then the system

$$(a_{13})^{(1)} G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right] G_{13} = 0$$

$$(a_{14})^{(1)} G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right] G_{14} = 0$$

$$(a_{15})^{(1)} G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right] G_{15} = 0$$

$$(b_{13})^{(1)} T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)] T_{13} = 0$$

$$(b_{14})^{(1)} T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)] T_{14} = 0$$

$$(b_{15})^{(1)} T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)] T_{15} = 0$$

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$

$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0$

$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0$

has a unique positive solution , which is an equilibrium solution for

Proof:

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) +$
$(a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

(a) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) +$
$(a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Definition and uniqueness of T^*_{14} :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]}$, $G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$

Definition and uniqueness of T^*_{17} :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that

there exists a unique T^*_{17} for which $f(\mathrm{T}^*_{17}) = 0$. With this value , we obtain from the three first equations

$G_{16} = \frac{(a_{16})^{(2)}\mathrm{G}_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(\mathrm{T}^*_{17})]}$, $G_{18} = \frac{(a_{18})^{(2)}\mathrm{G}_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(\mathrm{T}^*_{17})]}$

(c) By the same argument, the equations 92,93 admit solutions G_{13}, G_{14} if

$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$
$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi(G^*) = 0$

(d) By the same argument, the equations (SOLUTIONAL EQUATIONS OF THE GLOBAL EQUATIONS) admit solutions G_{16}, G_{17} if

$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$

$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi((G_{19})^*) = 0$

Finally we obtain the unique solution

G^*_{14} given by $\varphi(G^*) = 0$, T^*_{14} given by $f(T^*_{14}) = 0$ and

$$G^*_{13} = \frac{(a_{13})^{(1)} G^*_{14}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G^*_{15} = \frac{(a_{15})^{(1)} G^*_{14}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T^*_{14})]}$$

$$T^*_{13} = \frac{(b_{13})^{(1)} T^*_{14}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T^*_{15} = \frac{(b_{15})^{(1)} T^*_{14}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

G^*_{17} given by $\varphi((G_{19})^*) = 0$, T^*_{17} given by $f(T^*_{17}) = 0$ and

$$G^*_{16} = \frac{(a_{16})^{(2)} G^*_{17}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G^*_{18} = \frac{(a_{18})^{(2)} G^*_{17}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T^*_{17})]}$$

$$T^*_{16} = \frac{(b_{16})^{(2)} T^*_{17}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T^*_{18} = \frac{(b_{18})^{(2)} T^*_{17}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]}$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(1)}$ *and* $(b''_i)^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof:_Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$G_i = G^*_i + \mathbb{G}_i \qquad , T_i = T^*_i + \mathbb{T}_i$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T^*_{14}) = (q_{14})^{(1)} \quad , \quad \frac{\partial (b''_i)^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations GLOBAL EQUATIONS and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G^*_{13} \mathbb{T}_{14}$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G^*_{14} \mathbb{T}_{14}$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G^*_{15} \mathbb{T}_{14}$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T^*_{13} \mathbb{G}_j\right)$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T^*_{14} \mathbb{G}_j\right)$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)} T^*_{15} \mathbb{G}_j\right)$$

If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$

$\frac{\partial (a''_{17})^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial (b''_i)^{(2)}}{\partial G_j}(\,(G_{19})^*\,) = s_{ij}$

taking into account equations (SOLUTIONAL EQUATIONS TO THE GLOBAL EQUATIONS) and neglecting the terms of power 2, we obtain

$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17}$

$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17}$

$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17}$

$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right)$

$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T_{17}^*\mathbb{G}_j\right)$

$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T_{18}^*\mathbb{G}_j\right)$

The characteristic equation of this system is

$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$

$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)}G_{13}^*\right)\right]$

$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T_{14}^* + (b_{14})^{(1)}s_{(13),(14)}T_{14}^*\right)$

$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)}G_{14}^*\right)$

$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T_{14}^* + (b_{14})^{(1)}s_{(13),(13)}T_{13}^*\right)$

$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$

$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$

$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$

$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G_{13}^*\right)$

$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T_{14}^* + (b_{14})^{(1)}s_{(13),(15)}T_{13}^*\right)\} = 0$

$+$

$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$

$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}G_{17}^* + (a_{17})^{(2)}(q_{16})^{(2)}G_{16}^*\right)\right]$

$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}T_{17}^* + (b_{17})^{(2)}s_{(16),(17)}T_{17}^*\right)$

$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}G_{16}^* + (a_{16})^{(2)}(q_{17})^{(2)}G_{17}^*\right)$

$$\left(\left((\lambda)^{(2)}+(b_{16}')^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}_{17}^{*}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}_{16}^{*}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a_{16}')^{(2)}+(a_{17}')^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b_{16}')^{(2)}+(b_{17}')^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a_{16}')^{(2)}+(a_{17}')^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a_{16}')^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}_{17}^{*}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}_{16}^{*}\right)$$

$$\left(\left((\lambda)^{(2)}+(b_{16}')^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}_{17}^{*}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}_{16}^{*}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

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