

Matrix Growth Models for a Natural Forest in Shasha Forest Reserve Nigeria

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Abstract

Matrix growth models were developed for a natural forest stand in Shasha forest reserve Nigeria. Data were collected from permanent sample plots in the forest in 2006 and 2012 by complete enumeration of trees with a minimum diameter of 5cm at breast height. All observed trees were classified into three species groups; top, middle, and under storey species. Within each species group, trees were classified into diameter classes with a width of 5cm. Linear regressions were used to estimate the ingrowth, mortality and upgrowth transition probabilities for a six year growth period. The models were developed, calibrated and validated with data from the permanent sample plots. Simulations suggested a recovery in the tree population of the forest stand given minimal disturbances. Validation of the models gave reliable predictions of stand population and diameter distributions for the forest. The χ^2 statistics for the species groups revealed no significant differences between the predicted and observed number of trees at 0.05 significance level. The models provided some vital information which can be employed for sustainable management of the natural forest.

Keywords: Matrix models, Transition probabilities, Diameter distribution, Forest dynamics, Forest management

1. Introduction

Forests are continuously changing thus projecting the changes that occur during forest growth over time is necessary in order to obtain relevant information for sustainable management. Forest research has developed rapidly in Nigeria during recent years. However, only a few local models have been developed for projecting forest growth. Meanwhile, population growth and human activities such as settlements, agriculture and construction are threatening forests. The natural forest in Shasha forest reserve has been abused through illegal timber exploitation. This ecosystem is an important ecological resource providing many functions and values such as wildlife habitat, water quality protection, biodiversity conservation, timber production and carbon sequestration. Sustainable management which would restore the forest has become imperative.

Matrix growth models are a type of empirical models that are often preferred by researchers and resource managers because of their ability to make detailed and accurate predictions of tree and stand dynamics, particularly of species and stem size distribution that aggregate accurately to the stand level (Monserud 2003). Matrix models of forest dynamics allow the projection of tree stem density in size classes, thus assessment of the changes that occur in a forest stand and the time required to recover lost or exploited stock. Many studies have been carried out to develop fixed-parameter matrices, and variable-parameter matrices that take into consideration density dependence and even environmental factors for various forest types (Buongiorno & Michie 1980; Osho 1991; Lin and Buongiorno 1997; Boltz and Carter 2006; Zhou & Buongiorno 2006; Tahvonon 2009; Liang 2010; Picard *et al.* 2010; Liang *et al.* 2011; Escalante *et al.* 2011). The models are easy to build and require only the diameter structure of the population(s) under investigation as input variables. Matrix models are particularly advantageous in that they can be used to simulate a range of conditions based upon minimal data requirements (Gourlet-Fleury *et al.* 2005).

This study develops matrix models for a natural forest in Shasha forest reserve which will facilitate the sustainable management of the forest.

2. Methodology

2.1 Study Area

The data to estimate the parameters of the matrix model were collected from a 1.35ha Permanent Sample Plot (PSP 133) in Shasha Forest Reserve, Nigeria. The plot which consists of 15 subplots of 30m x 30m in size was measured at two consecutive inventories with a six year interval (2006–2012). Shasha forest reserve is located in Osun State in South Western Nigeria. It is under the management of the State Department of Forestry. The PSP lies on Latitude 9°4'N and Longitude 3°54'E at altitude 122m above sea level with a mean annual rainfall of 1421mm. Soil type is ferruginous tropical soils on crystalline acid rock. The topography is gently undulating to undulating plain. The vegetation is mainly of the high forest type.

The diameter at breast height measurement for all trees larger than 5cm in the PSP was carried out and the tree species identified. All observed trees were grouped according to their ecological and morphological criteria (Lin *et al.* 1996; Namaalwa *et al.* 2005). The criteria include shade tolerance and maximum attainable heights based on (Keay 1989), resulting into three main species groups; top, middle and under storey trees.

2.2 Model description

The growth model was based on a characterization of forest structure and composition using tree sizes and species groups. The matrix model has the following form:

$$Y_{t+1} = G_t(y_t) + I_t \quad (1)$$

$y_t = [y_{ijt}]$ is a column vector representing the number of live trees per unit area. Where i is the tree species group ($i = 1, 2, \dots, m$), j is the diameter class ($j = 1, 2, \dots, n$) and t is the year. I_t is a column vector which contains the ingrowth for each species group at time t .

The growth matrix G_t is defined as:

$$G_t = \begin{bmatrix} G_{1t} & & & & \\ & G_{2t} & & & \\ & & G_{3t} & & \\ & & & \ddots & \\ & & & & G_{mt} \end{bmatrix} \quad (2)$$

Each matrix G_{it} contains the transition probabilities for each species group;

$$G_{it} = \begin{bmatrix} a_{i1t} & \cdot & \cdot & \cdot & \cdot & \cdot \\ b_{i1t} & a_{i2t} & \cdot & \cdot & \cdot & \cdot \\ \cdot & b_{i2t} & a_{i3t} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \ddots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & b_{i,m-1,t} & a_{i,m,t} \end{bmatrix} \quad (3)$$

Where: b_{ijt} is the probability that a tree of species i and in diameter class j at time t is alive and in diameter class $j+1$ at $t+1$, a_{ijt} is the probability that the same tree is alive and still in diameter class j at $t+1$, and n is the number of diameter classes.

Trees in the largest size n remain there until they are either harvested or lost to mortality. These probabilities which are functions of the tree and stand level variables (Buongiorno *et al.* 1995) were estimated using linear regression models:

$$b_{ijt} = \beta_{i0} + \beta_{i1}D + \beta_{i2}D^2 + \beta_{i3} \sum_{i=1}^m \sum_{j=1}^n B_{ij}(y_{ijt}) \quad (4)$$

$$m_{ijt} = \delta_{i0} + \delta_{i1}D + \delta_{i2}D^2 + \delta_{i3} \sum_{i=1}^m \sum_{j=1}^n B_{ij}(y_{ijt}) \quad (5)$$

Where the β s and δ s are parameters, D is the diameter (in cm, measured at breast height) and B_{ij} is the basal area of the average tree of species class i in diameter class j . m_{ijt} is the probability that a tree of species i in diameter class j dies between t and $t+1$. Hence a_{ijt} can be given as:

$$a_{ijt} = 1 - b_{ijt} - m_{ijt} \quad (6)$$

Ingrowth, I_{it} , is the recruitment of new trees of species i that enter the smallest diameter class between year t and $t+1$, was estimated using the model:

$$I_{it} = \alpha_{i0} + \alpha_{i1} \sum_{j=1}^m \sum_{j=1}^n B_{ij}(y_{ijt}) + \alpha_{i2} \sum_{j=1}^n (y_{ijt}) \quad (7)$$

where B_{ij} is the basal area of the average tree of species class i in diameter class j , and α s are the parameters to be estimated.

2.3 Model Estimation

The plot data were divided into an estimation data set (10 plots) and a validation data set (5 plots). The estimation data set was used to develop the growth models while the validation data set was used to test the efficiency of the growth model.

Within each species group, trees larger than 5cm were classified into ten 5cm diameter classes ranging from 5cm to ≥ 50 cm and each class was represented by its midpoint diameter. The ingrowth was the number of trees that became larger than 5cm between the two forest inventories. A tree would be expected to be in one of the three states after a six year interval: remaining in the same diameter class, moving up one class, or dead.

The plots at the two growth periods were pooled, and the probabilities of a , b , and m were calculated by dividing the number of trees that stayed in a diameter class, moved up one class or died by the number of trees in that class at the beginning of the six year period. For each species group, b_{ijt} and m_{ijt} values were estimated as given in Equations (4) and (5). The a_{ijt} values were estimated as given in Equation (6).

To obtain estimates of ingrowth I_{ijt} , the ingrowth was firstly modeled according to (Osho 1991) as being a uniform contribution by the mature trees in the higher diameter classes but weighted proportionally to the number of trees present in each class. Then the average recruitment per individual tree in a class was treated as the dependent variables in Equation (7). Substitution of B_t and the number of live trees in the ingrowth Equation (7) gave the I_{ijt} values thus the ingrowth matrix. Then the $G_t + I$ matrix is gotten. Multiplying the diameter distribution vector (y_t) by $G_t + I$ gave the diameter distribution at time $t + 1$. Future diameter distributions can then be calculated by successive application of Equation (1).

For model verification and validation of the growth model, the model was used to predict the state of the validation plots at the time of their current measurement, given their state at the previous inventory. The average observed growth data (change in number of trees in each species-size class) was compared with the average predicted growth data estimated from the validation data sets. The accuracy of the matrix model in estimating the stand growth was tested using the chi – square goodness of fit test. A valid model would show no significant difference between the observed and the predicted values at 5% level of significance.

3. Results and Discussion

Parameter estimates for the upgrowth equation for each species group are given in Table 1. The coefficient of determination (R^2) of the upgrowth equation for the under, middle and top storey species groups are 0.85, 0.69, and 0.40 respectively. A positive β_1 sign and a negative β_2 implies that growth rate would increase with tree diameter reach a maximum and then decline, a negative β_3 imply that tree diameter growth is faster in less dense stands, other things being equal (Ralston *et al.* 2003). All parameters of the variables for the upgrowth equation had the expected signs and were significant.

Parameter estimates for the mortality equation for each species group is given in Table 2. The coefficient of determination (R^2) of the mortality equation for the under, middle and top storey species groups are 0.57, 0.35 and 0.31 respectively. Higher stand density was related to a higher mortality probability for the under and middle storey species groups. However, the reverse was the case for the top storey species; basal area did not necessarily contribute to the mortality of the top storey species groups.

Parameter estimates for the ingrowth equation for each species group is given in Table 3. A negative α_1 or α_2 implies that ingrowth decreases as the stand becomes more dense.

Table 1. Equations for the transition probabilities upgrowth

Statistics	Constant	Independent variables		
		Diameter	Diameter Square	Basal area
Under storey species				
Coefficients	0.0903= β_0	0.0106= β_1	-0.00009= β_2	-0.00018= β_3
S.E	0.0276	0.0102	0.00018	0.00024
R^2	0.85			
Middle storey species				
Coefficients	-0.1839= β_0	0.0435= β_1	-0.00072= β_2	-0.00029= β_3
S.E	0.1808	0.0134	0.00022	0.00019
R^2	0.69			
Top storey species				
Coefficients	0.0906= β_0	0.0141= β_1	-0.00028= β_2	-0.00009= β_3
S.E	0.0659	0.0105	0.00024	0.00113
R^2	0.40			

S.E = standard error, R^2 = coefficient of determination, $\beta_0, \beta_1, \beta_2$ and β_3 = regression coefficients.

Table 2. Equations for the probability of mortality

Statistics	Constant	Independent variables		
		Diameter	Diameter Square	Basal area
Under storey species				
Coefficients	0.1336= δ_0	-0.00577= δ_1	0.00014= δ_2	0.00035= δ_3
S.E	0.0492	0.00739	0.00018	0.00014
R^2	0.57			
Middle storey species				
Coefficients	0.1682= δ_0	-0.00378= δ_1	0.00004= δ_2	0.00013= δ_3
S.E	0.0489	0.00376	0.00007	0.00016
R^2	0.35			
Top storey species				
Coefficients	0.1417= δ_0	0.00651= δ_1	-0.00026= δ_2	-0.00001= δ_3
S.E	0.0475	0.00778	0.00021	0.00003
R^2	0.31			

S.E = standard error, R^2 = coefficient of determination, $\delta_0, \delta_1, \delta_2$ and δ_3 = regression coefficients

Table 3. Equations for the probability of Ingrowth.

Statistics	Constant	Independent variables	
		Basal area	stem density
Under storey species			
Coefficients	6.2005 = α_0	0.0034 = α_1	-0.0279 = α_2
S.E	2.7713	0.0045	0.0324
R^2	0.27		

Middle storey species

Coefficients	2.3559 = α_0	0.00063 = α_1	-0.0401 = α_2
S.E	0.4813	0.00045	0.0113
R^2	0.65		

Top storey species

Coefficients	0.9680 = α_0	0.00074 = α_1	-0.0546 = α_2
S.E	0.5469	0.0023	0.0588
R^2	0.12		

S.E = standard error, R^2 = coefficient of determination, $\delta_0, \delta_1, \delta_2$ and δ_3 = regression coefficients

The coefficient of determination (R^2) of the ingrowth equation for the under, middle and top storey species groups are 0.27, 0.65 and 0.12 respectively. The low R^2 values of some of the equations do not mean that the equations are inadequate. It only implies that those models explained only a very small part of the variation in the upgrowth, mortality and ingrowth rates.

The developed matrix models were applied to project the proportions of different sizes of trees and the ingrowth at various time periods in the future given no further logging. In applying the matrix growth models, the data was grouped into the same diameter classes as was done with the estimation data. To predict future stand states given the current diameter distributions by species groups the growth matrix was applied iteratively, each iteration simulating a six year growth.

The matrix models were first applied for short-term (60 years) predictions of stand growth. Figure 1 shows the summary of projected population for the pooled species.

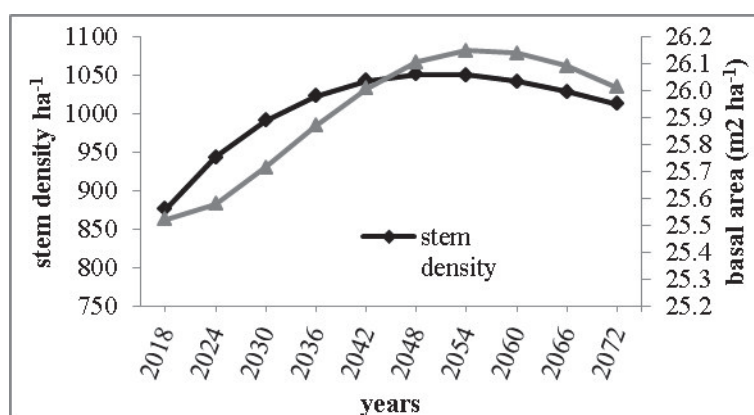


Figure 1. Projected total population and basal area for the stand (60yrs).

The projected population for the whole stand increased for a period of 36 years and declined thereafter. The simulation shows an increase in the overall number of trees in the stand without any logging operations. The predicted basal area was calculated from the predicted diameter distributions of each species group; that is multiplying the number of trees of a diameter class with the basal area of the average tree with the midpoint of that diameter class (Zhao et al. 2005). Figure 1 shows an increased basal area from the initial basal area, which suggests recovery of the natural forest. The summary of the projection for diameter distribution for the pooled species is shown in Figure 2.

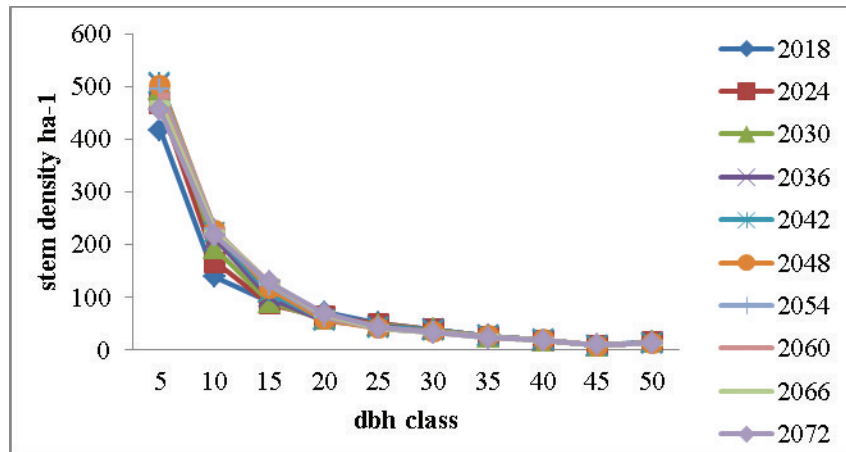


Figure 2. Projected diameter distribution for the pooled species

The diameter distributions have the classical inverse J shape of uneven aged stands. The projections show the stem density per diameter class, and this answers the question about the structure of the resource. In addition to the short-term prediction, the model was applied to predict how a minimally disturbed forest stand would evolve over a long time period. The results are summarized in Figure 3. The purpose of the long term simulation was to predict the condition of the stand at any future point in time, given initial conditions and how the stand grows over various time intervals. Figure 3 shows the expected stand state up to 360 years, the initial stand state being the average predicted state of the stand after six years (year 2018).

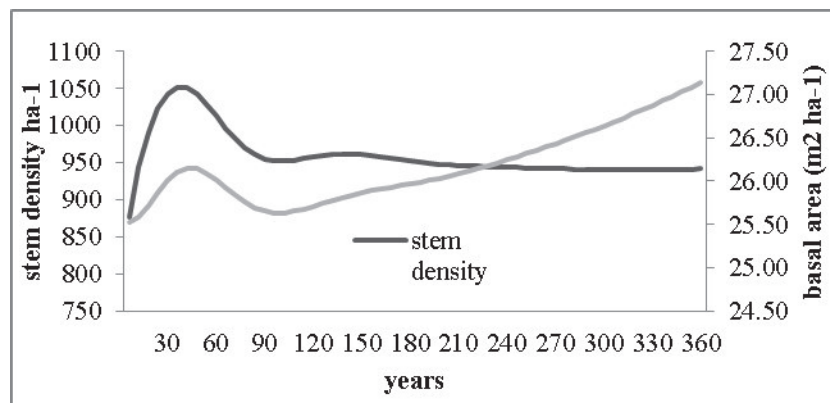


Figure 3. Projected long term population and basal for the stand (360yrs)

The natural processes of growth and mortality would continue as in the past 6 years upon which the model was built, assuming the same pattern of disturbances and no further logging. The simulation shows a fluctuation in number of trees which converges after periods of time but with increasing basal area. The comparisons of the observed and predicted diameter distributions obtained from the matrix model are shown in Figures 4 – 6.

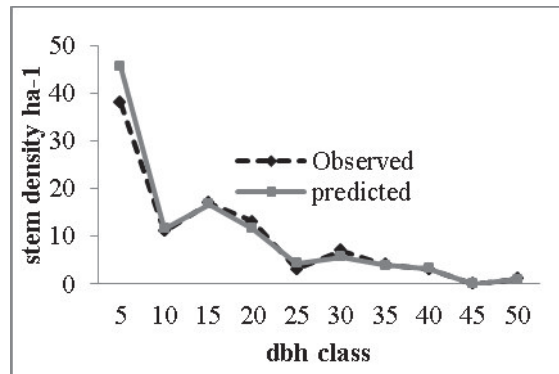


Figure 4. Observed and predicted diameter distribution for under storey species group

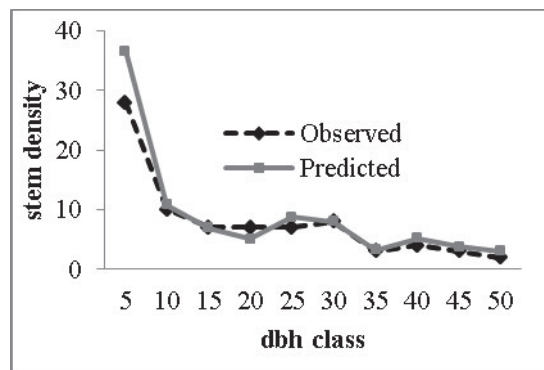


Figure 5. Observed and predicted diameter distribution for middle storey species

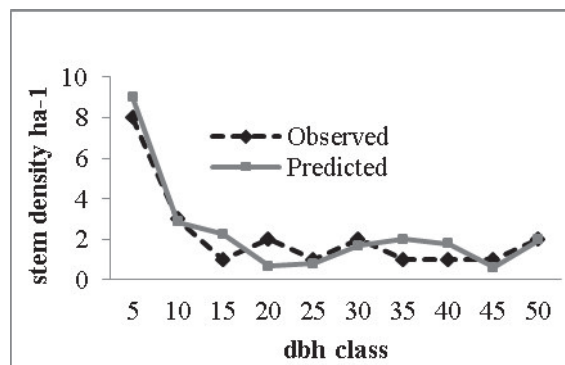


Figure 6. Observed and predicted diameter distribution for top storey species group

It can be seen that the projected distributions agree fairly closely with the actual distributions. The results for the species groups were similar except for an over estimation in the number of ingrowth for the under and middle storey trees. The χ^2 statistic revealed no significant deviations between the predicted and observed number of trees at 0.05 significance level. The comparisons indicated that the predicted species-size distributions over six years were unbiased. The performance of these models demonstrated that the matrix modeling approach was adequate and can be used for the natural forest.

5. Conclusion

Applications of the models demonstrated its usefulness to project stand diameter structure. Therefore the matrix models could serve as a useful tool for predicting stand development. They could also be used in a stochastic framework with many definitions of stand states to compute the probability of transition from one state to another.

The long-term simulations by the model were subject to the bias caused by changes in climate as the model did not take into consideration environmental variables that can represent climate change effects (e.g. temperature, rainfall) on the forest growth. Nevertheless, the model provides a useful tool for the prediction of future stand states of the forest stand.

Given the current state of decline of the natural forest in Shasha forest reserve and based on the projections by the matrix models of possible recovery, it is recommended that a strict ban be placed on timber exploitation and land clearing for a period of time. Assisted regeneration may also be considered as an option to boost tree population recovery rate. The matrix models developed in this study are recommended for sustainable management of the forest.

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