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Nonlinear Modulation of Wave Propagation in Spherical Shell Model and Modified Zhang Model Using Free Space Model as a Bench-Mark

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Abstract
This paper presents nonlinear modulation of wave propagation in spherical shell model (SSM) and modified Zhang model (MZM) using free space model (FSM) as a bench-mark. A typical non-linearity is the change in the dielectric constant due to electromagnetic (EM) wave field that propagates through a medium. By modulation, we mean the characteristic departure of EM waves' propagation in both media as opposed to the free space propagation. Maxwell's equations were used to derive the basic equation that govern the propagation of electromagnetic waves in nonlinear media. The equations of the models were found to be nonlinear and their solution were obtained numerically using Runge-Kutta scheme implemented in Matlab software. The spatial EM wave profile graphic displays were supplemented by the symmetric spatial Fast Fourier Transform (FFT) analysis. The symmetric implementation of the FFT meant that the actual number of modes present in any solution was half the number of observed spikes. The free space model (FSM) showed periodic propagation for all frequencies ($\frac{f_o}{3}, \mathrm{f}_0, 10\mathrm{f}_0,$ and $25\mathrm{f}_0$) examined corresponding to a wavenumber per frequency. The result only serves to give some level of confidence that the algorithm performed well. The MZM supports a variety of characteristics. There are amplitude amplifications or wave steeping, lossless or solitary propagation and multiplicity of modes for all frequencies examined. However, at the fundamental frequency $\mathrm{f}_0 = 47.7 \times 10^6\mathrm{Hz}$, the SSM is capable of exhibiting amplitude amplification without attenuation. The EM wave propagation characteristics of the MZM and SSP showed that materials which could be fabricated according to this model would be very useful as EM wave guides as they could support waves without losses as opposed to the present known commercial optical fibers.
Keywords: Nonlinear modulation, Wave propagation, Spatial Electromagnetic wave.

1.0 Introduction

The study of electromagnetic wave propagation in free space has been well known and documented [1]. Many applications such as radio, television, radar and microwave transmissions are examples of free space propagation. Telecommunication is already transformed with the help of nonlinear optics and similar impact is expected very soon on technology that involves computer science. Nonlinear methods cover a wide region of different applications now, such as harmonic generations and frequency sum [2]. In free space the wave amplitude and hence the energy remains constant in space and time [3]. However, the power density decreases in accordance with the inverse square law. This leads to eventual fading of signals from transmission sources. When the waves are guided, it is possible to achieve transmission in which the power density is constant [4]. Apart from resistive losses, signals on wave guides are supposed to be pure and clear over the whole length of the guide. The important property of a medium which alter or modulate wave propagation through it is the dielectric tensor or the refractive index in the case of isotropic assumption [5]. The rapid development in the field of quantum optics and nanotechnology has made it possible to manipulate the dielectric constants of some media [6].

The nonlinear propagation of strong interacting electromagnetic fields which is explored by the standard theoretical way is based on the nonlinear solution of the equation for a specific nonlinearity (quadratic, cubic, etc.) [7]. But in many experiments the laser field's intensity is very high that the conventional expansion of the nonlinear polarization in a Taylor series over the electric field strength stops to be a good approximation, and the corresponding language of susceptibilities is no longer valid [8]. It is also a common view that for future use more sophisticated understanding of the basic mechanisms underlying non-linear phenomena of this kind will be required. This motivates us to explore nonlinear modulation of wave propagation in model media where details of material properties can been manipulated so that it can guide optical signals with much purity. In the light of this anticipation, models of field dependent materials can be assumed and the nature of EM wave's propagation in them can be studied.

It is therefore very important to study computationally how media property will modulate electromagnetic wave propagation in them. If a particular model gives an interesting and useful wave modulation, it will open up experimental challenges on how to realize the materials with such model properties. The problem of how media properties alter or modulate the electromagnetic wave propagation in them remains an open one. The exact solution of the electromagnetic wave equation for model media shall be investigated by numerical

techniques in this study.
For an understanding of the nonlinear modulation of wave propagation in model media, it is necessary to consider the theory of electromagnetic wave propagation normal to the plane of propagation with respect to a material medium [9].

2.0 Theoretical Consideration

2.1 Maxwell's Equations

The propagation of electromagnetic waves is determined by Maxwell's equations [10]. That is:

$$\nabla \times \vec{E} = -\frac{\partial B}{\partial t} \quad (1)$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad (2)$$

$$\nabla . \vec{D} = \rho_f \quad (3)$$

$$\nabla . \vec{B} = 0 \quad (4)$$

where $\vec{E}$ is the electric field vector and $\vec{H}$ is the magnetic field vector. $\vec{D}$ is the electric flux density and $\vec{B}$ is the magnetic flux density. The current density vector is $\vec{J}$. The flux densities $\vec{D}$ and $\vec{B}$ appear in response to the electric and magnetic fields $\vec{E}$ and $\vec{H}$ propagating inside the medium and are related to them through the constitutive relations given [11] by

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \quad (5)$$

$$\vec{B} = \mu_0 \vec{H} + \vec{M} \quad (6)$$

where ε_0 is the vacuum permittivity, μ_0 is the vacuum permeability, $\vec{P}$ is the induced electric polarization and $\vec{M}$ is the induced magnetic polarization.

2.2 Assumptions of the model

The basic equations that govern propagation of electromagnetic waves can be obtained from Maxwell's equations which describe the time and space evolution of magnetic and electric fields [12]. Here, time discretization is completely irrelevant since time must disappeared in the final equation as treated in accordance with Helmholtz decomposition. A number of assumptions are necessary to realize the computation that follow and to arrive at a good results without time.
(a) We shall assume a rectangular symmetry so that Cartesian coordinates x,y, z can be used. (b)The direction of propagation of the EM waves is the x direction.
(c)The electric and magnetic vectors of the EM waves are in the y and z directions respectively, and that they vary only in the x direction, i.e. $E = E_y(x)j, H = H_z(x)k$, where j and k are unit vectors in y and z directions respectively.
(d)The media are perfect dielectrics and non-magnetic.
(e)The electric and magnetic fields are harmonic in time.
(f)The dielectric properties of the media respond to the spatial component of the electric field only and that it is nonlinear only in the x direction.

2.3 Derivation of the working equations

Recall Faraday equation (1)

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

The Faraday equation in rectangular coordinates is given by

$$\nabla \times E = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) i + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) j + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) k$$

$$= -\left(\frac{\partial B_x}{\partial t} i + \frac{\partial B_y}{\partial t} j + \frac{\partial B_z}{\partial t} k\right)$$

This means that

$$\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) i + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) j + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) k = -\left(\frac{\partial B_x}{\partial t} i + \frac{\partial B_y}{\partial t} j + \frac{\partial B_z}{\partial t} k\right) \quad (7)$$

By assumption (c) the surviving terms are:

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \quad (8)$$

Similarly, the Ampere equation (2)

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

Can be expressed in rectangular coordinates as:

$$\nabla \times H = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) i + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right) j + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) k$$

$$= \left(J_x + \frac{\partial D_x}{\partial t}\right) i + \left(J_y + \frac{\partial D_y}{\partial t}\right) j + \left(J_z + \frac{\partial D_z}{\partial t}\right) k$$

Or

$$\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) i + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right) j + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) k = \left(J_x + \frac{\partial D_x}{\partial t}\right) i + \left(J_y + \frac{\partial D_y}{\partial t}\right) j + \left(J_z + \frac{\partial D_z}{\partial t}\right) k$$

(9)

By assumption (c) and (d; J=0), the surviving terms are:

$$\frac{\partial H_z}{\partial x} = -\frac{\partial D_y}{\partial t} \quad (10)$$

For a material medium the constitutive relations enables us to express (9) and (10) as:

$$\frac{\partial E_y}{\partial x} = -\frac{\partial \mu H_z}{\partial t} \quad (11)$$

$$\frac{\partial H_z}{\partial x} = -\frac{\partial \varepsilon E_y}{\partial t} \quad (12)$$

By assumption (d), the medium being non-magnetic, we have (11) as

$$\frac{\partial E_y}{\partial x} = -\mu_0 \frac{\partial H_z}{\partial t} \quad (13)$$

And by assumption (f), we can write (12) as

$$\frac{\partial H_z}{\partial x} = -\varepsilon \frac{\partial E_y}{\partial t} \quad (14)$$

Elimination of H_z between (13) and (14) leads to wave-like equation:

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon \frac{\partial^2 E_y}{\partial t^2} \quad (15)$$

Assumption (f) is restricted to x-coordinate only, so $\varepsilon = \varepsilon(E(x))$. In this case the time harmonic assumption (e) for electric fields:

$$E(x,t) = W(x)e^{-i\omega t} \quad (16)$$

where, $W(x)$ is y-component of electric field that depends on x-coordinate, gives the equation

$$\frac{d^2W}{dx^2} + \mu_0 \omega^2 \varepsilon(w) W = 0 \quad (17)$$

Mathematically, the solution of (15) consists of finding the solution of (17) and substituting in (16), but spatial modulation of waves is what is of practical importance, so we do not need to go back to $E(x,t)$; $W(x)$ is just what we need.

2.4 Models of Dielectric Function

The behavior of material towards electric field using free space model (FSM), spherical shell model (SSM) and modified Zhang model (MZM) is modelled by the following equations:

FSM $\varepsilon(w(x)) = \varepsilon_0$ (18)

SSM $\varepsilon(W(x)) = \varepsilon_0 \left(1 + \gamma W^{\frac{a}{b}}\right)$ (19)

MZM $\varepsilon(w(x)) = \frac{\varepsilon(0)}{\{1-\alpha W^2\}^{1/3}}$ (20)

where, γ is an attenuation parameter that depends on the thickness of the spherical shell material with outer radius, a in meters and inner radius b in meters. $W = E_1$ in Zhang's model, however since our field is EM, $W = W(x)$, hence $\varepsilon(W)$ is a functional, α is a parameter that plays the role of making the units consistent.

2.5 Numerical Solution of the Models

The solutions of the models were obtained using Runge-Kutta 4th order method. The second order ordinary differential equation was reduced into a system of two first order ordinary differential equation. The ODE45 module in Matlab which is built based on Runge-Kutta 4th and 5th order method was used to implement Runge-Kutta Algorithm in Matlab. The equations were converted as follows:

A substitution of FSM into equation (17) gives

$$\frac{d^2W}{dx^2} + (\mu_0 \omega^2 \varepsilon_0) W = 0 \quad (21)$$

but, $\omega^2 = 4\pi^2 f_0^2$

So that, equation (21) becomes

$$\frac{d^2W}{dx^2} + 4\pi^2 f_0^2 \varepsilon_0 \mu_0 W = 0 \quad (22)$$

$$\frac{d^2W}{dx^2} + k_0 W = 0 \quad (23)$$

where $k_0 = 4\pi^2 f_0^2 \varepsilon_0 \mu_0$ (24)

We let $W = P_1(x)$ and $W' = \frac{dW}{dx} = P_2(x)$ then we obtain the system

$$\frac{dW}{dx} = P_2 \Longrightarrow \frac{dP_1}{dx} = P_2.$$

and

$$\frac{d}{dx}\left[\frac{dW}{dx}\right] + k_0 W = 0$$

$$\frac{dP_2}{dx} + k_0 P_1 = 0$$

$$\frac{dP_2}{dx} = -k_0 P_1$$

Hence our new system of equations is given by

$$\frac{dP_1}{dx} = P_2 \quad (25)$$

$$\frac{dP_2}{dx} = -k_0 P_1 \quad (26)$$

A substitution of SSP into equation (17) gives

$$\frac{d}{dx}\left[\frac{dW}{dx}\right] + \mu_0 \omega^2 \varepsilon_0 \left(1 + \gamma W^{\frac{a}{b}}\right) W = 0 \quad (27)$$

$$\frac{d}{dx}\left[\frac{dW}{dx}\right] = -\mu_0 \omega^2 \varepsilon_0 \left(1 + \gamma W^{\frac{a}{b}}\right) W \quad (28)$$

Or, $$\frac{d}{dx}\left[\frac{dW}{dx}\right] = -k_0 \left(1 + \gamma W^{\frac{a}{b}}\right) W \quad (29)$$

Let $W = k_1$; $W' = k_2$ and

$$\frac{dW}{dx} = k_2 \Longrightarrow \frac{dz_1}{dx} = k_2 \text{ and we have}$$

$$\frac{dk_1}{dx} = k_2 \quad (30)$$

$$\frac{dk_2}{dx} = -k_0 \left(1 + \gamma k_1^{\frac{a}{b}}\right) k_1 \quad (31)$$

Also a substitution of MZM into equation (17) gives

$$\frac{d}{dx}\left[\frac{dW}{dx}\right] + \mu_0 \omega^2 \frac{\varepsilon(0)}{\{1-\alpha W^2\}^{1/3}} W = 0 \quad (32)$$

$$\frac{d}{dx}\left[\frac{dW}{dx}\right] = -\frac{\mu_0 \omega^2 \varepsilon(0)}{\{1-\alpha W^2\}^{1/3}} W$$

since, $\varepsilon(0) = \varepsilon_0$

we have, $$\frac{d}{dx}\left[\frac{dW}{dx}\right] = -\frac{k_0}{\{1-\alpha W^2\}^{1/3}} W \quad (33)$$

Let $W = z_1(x)$; $W' = z_2(x)$ and

$$\frac{dW}{dx} = z_2 \Longrightarrow \frac{dz_1}{dx} = z_2 \text{ and we have}$$

$$\frac{dz_1}{dx} = z_2 \quad (34)$$

$$\frac{dz_2}{dx} = -\frac{k_0}{\{1-\alpha W^2\}^{1/3}} z_1 \quad (35)$$

Table 1: Parameters used in the simulation.

Parameter	Value
Fundamental EM wave, f_0	$47.7 \times 10^6 Hz$
Permittivity of free space, ε_0	$8.854 \times 10^{-12} Fm^{-1}$
Permeability of free space, μ_0	$12.566 \times 10^{-7} NA^{-1}$
Wave frequency, ω	$2\pi f_0$
Alpha, α	0.005
Gamma, γ	0.005
Outer radius of the shell, a	$0.5m$
Inner radius of the shell, b	$0.003m$

The solution of equations (25) and (26) are found in the interval $0 \leq x \leq 100$, with step size of 0.001(FSM). The solutions of equations (30), (31), (34) and (35) are considered in the interval $0 \leq x \leq 200$ (SSM and MZM) with step size of 0.001. The EM wave propagation is varied by $f = f_0, 10f_0, 25f_0, 30f_0, and\ 50f_0$ for our entire models.

Propagation of EM waves from free- space medium to other media models is considered with a boundary region from $(0 \leq x \leq 100)$ to $(100 \leq x \leq 300)$, with the step size of 0.001.

In general, the electric field of the EM wave propagation is

$$W(x) = P_1(x) = k_1(x) = Z_1(x) \quad (36)$$

The output of the ODE45 solver (Electric field W(x)) is introduced into FFT-codes as we transform from wave position domain to wavenumber domain.

3.0 Results and Discussion

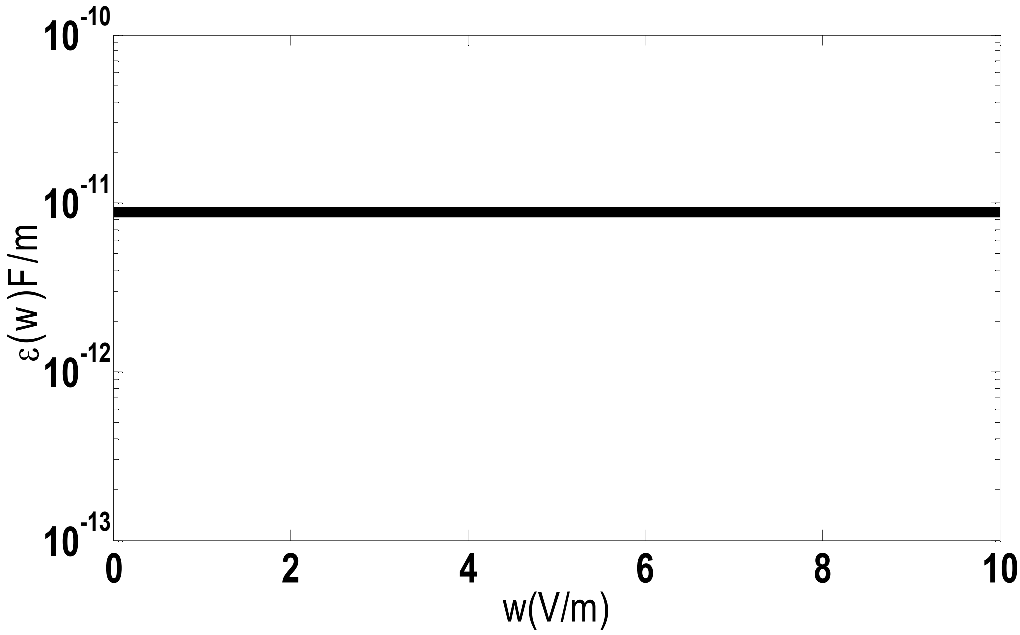

Figure 1. Sketch of free space model.

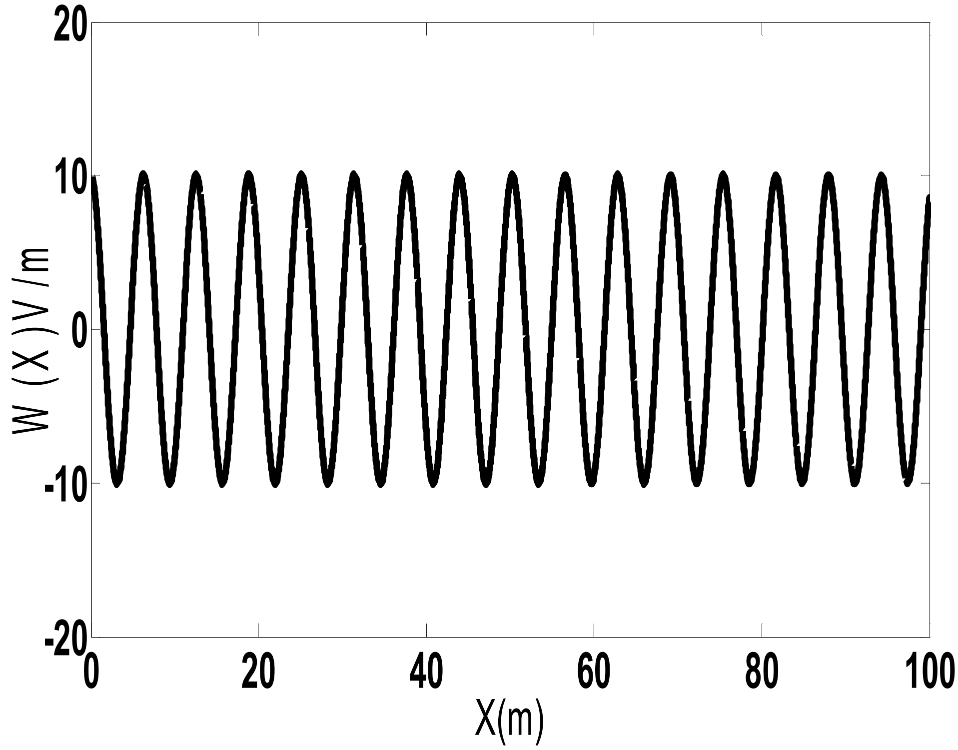

Figure 2(a): Electric field of EM waves $W(x)$ along x-direction in free-space model for $f = f_0$, h=0.001, L=100.

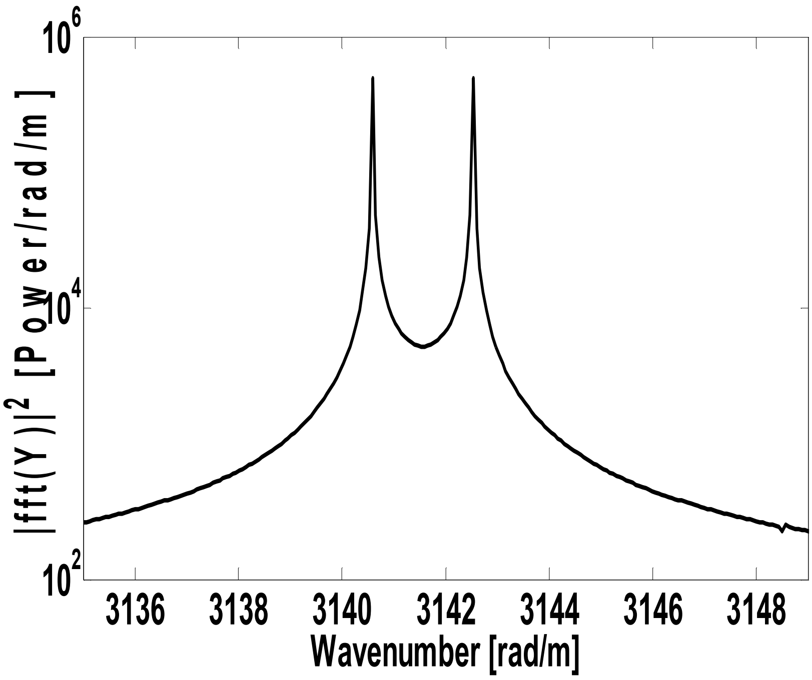

Figure 2(b): Transformation from wave position (using FFT) to wave number for free space model for $f = f_0$.

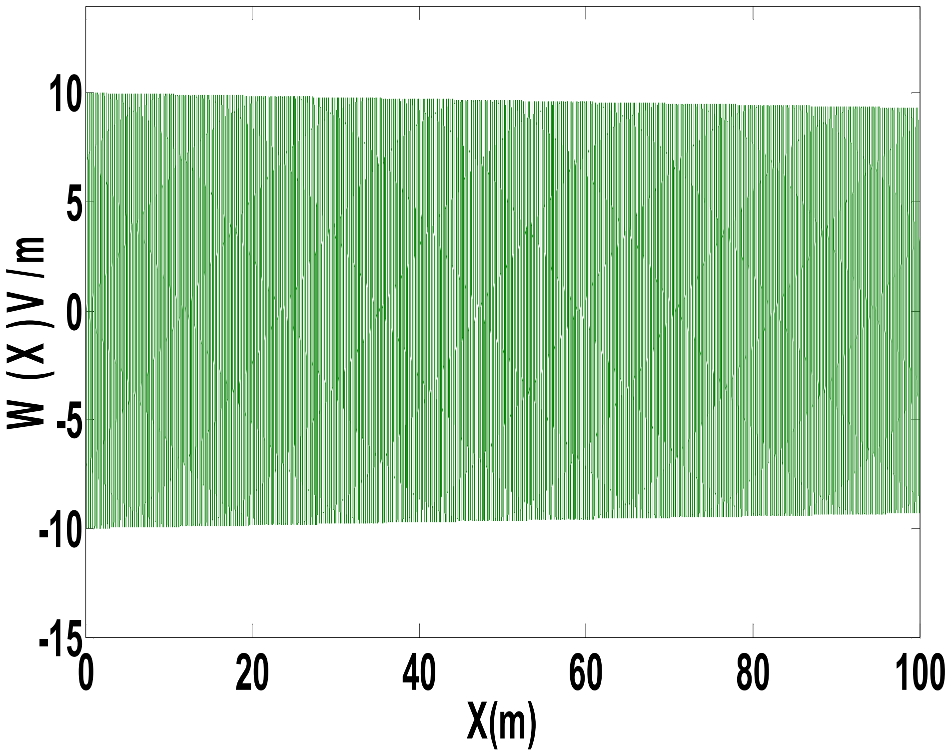

Figure 3(a): Electric field of EM waves $W(x)$ along x-direction for free-space model for $f = 25f_0$, h=0.001, L=100.

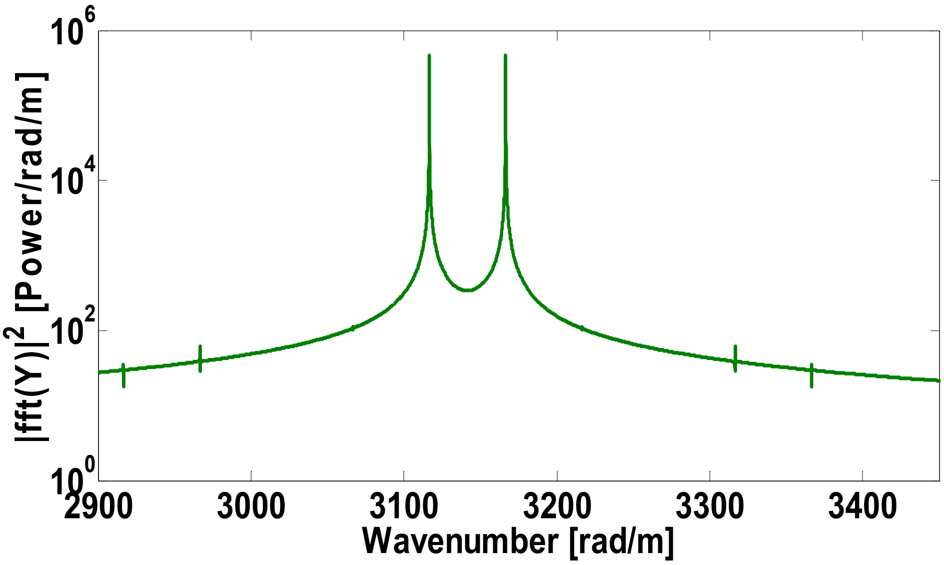

Figure 3(b): Transformation from wave position (using FFT) to wave number for free space model for $f = 25f_0$.

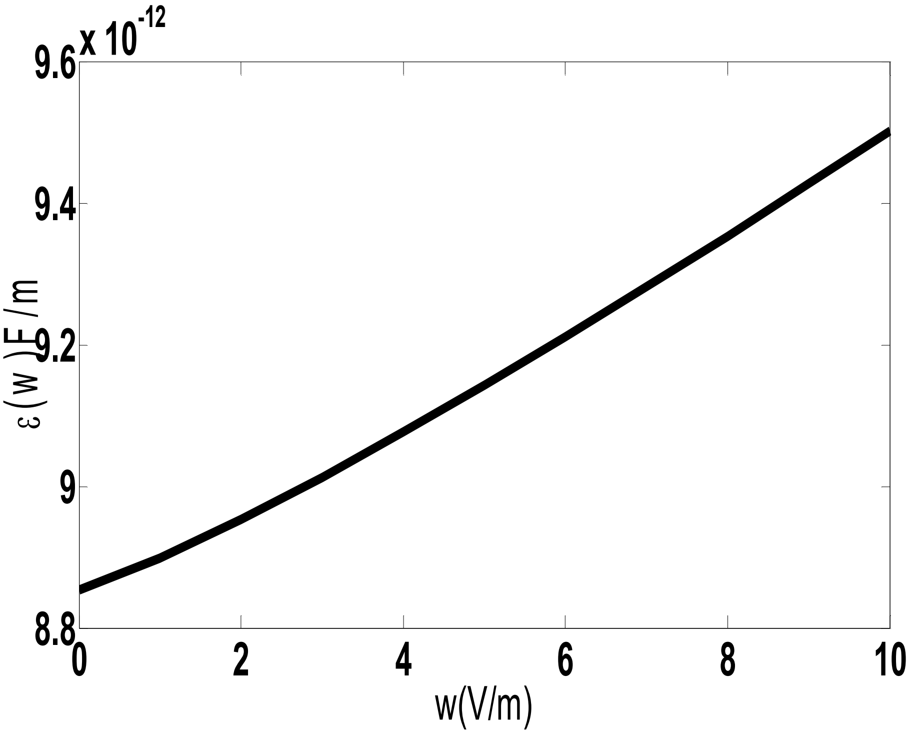

Figure 4: Sketch of spherical shell model.

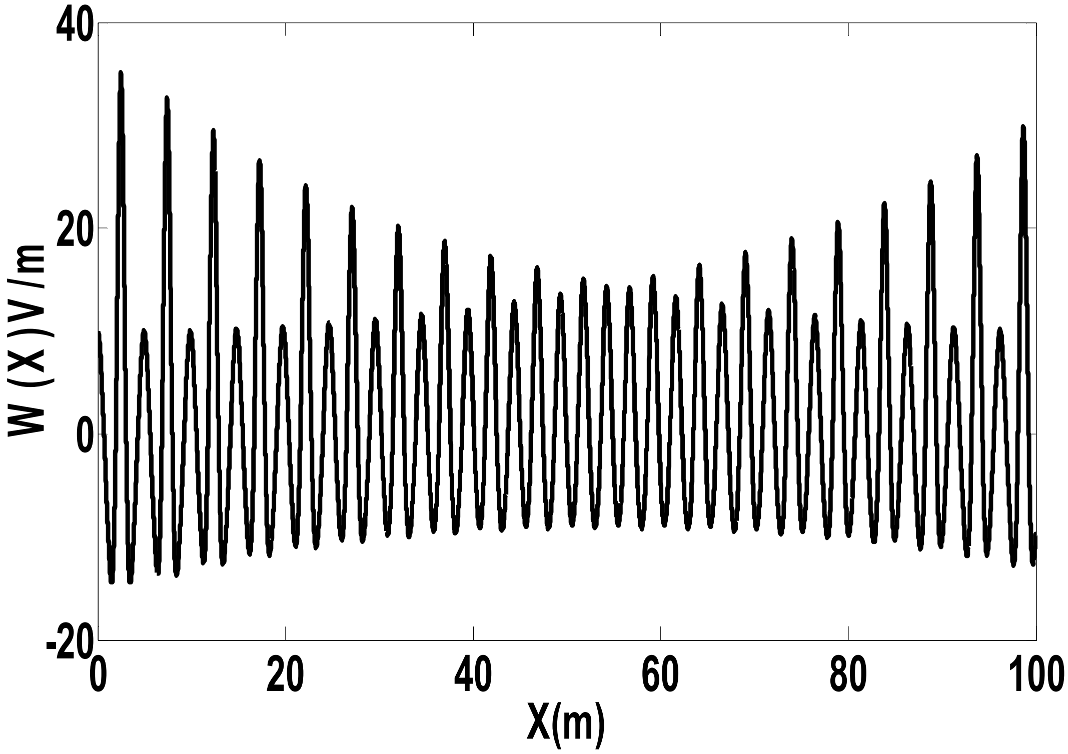

Figure 5(a): Electric field of EM waves $W(x)$ along x-direction for Spherical shell model for f = f_0, h=0.001, L=200.

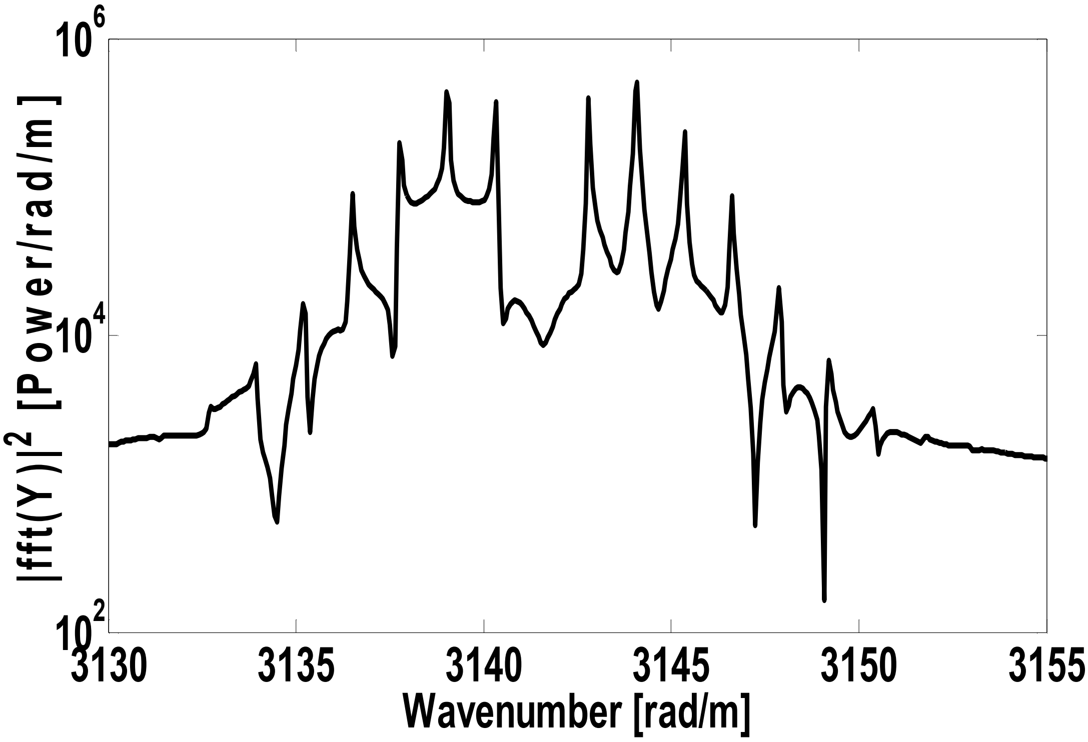

Figure 5(b): Transformation from wave position (using FFT) to wave number for Spherical shell model for $f = f_0$.

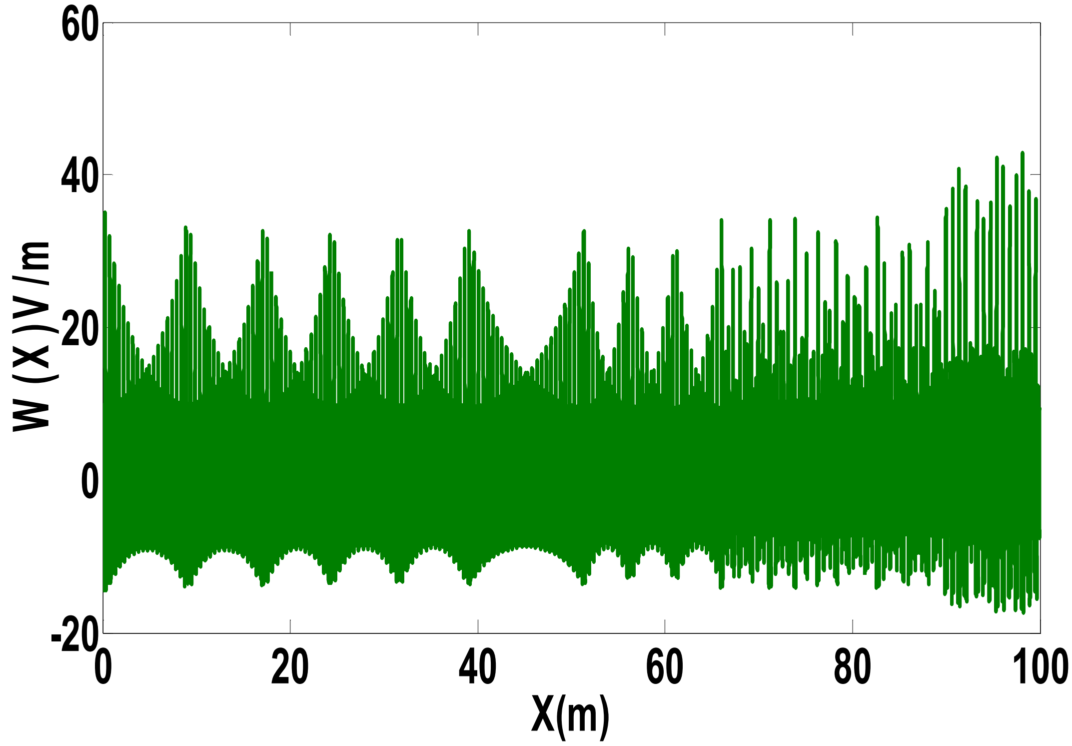

Figure 6(a): Electric field of EM waves $W(x)$ along x-direction for Spherical shell model for f = $10f_0$, h=0.001, L=200.

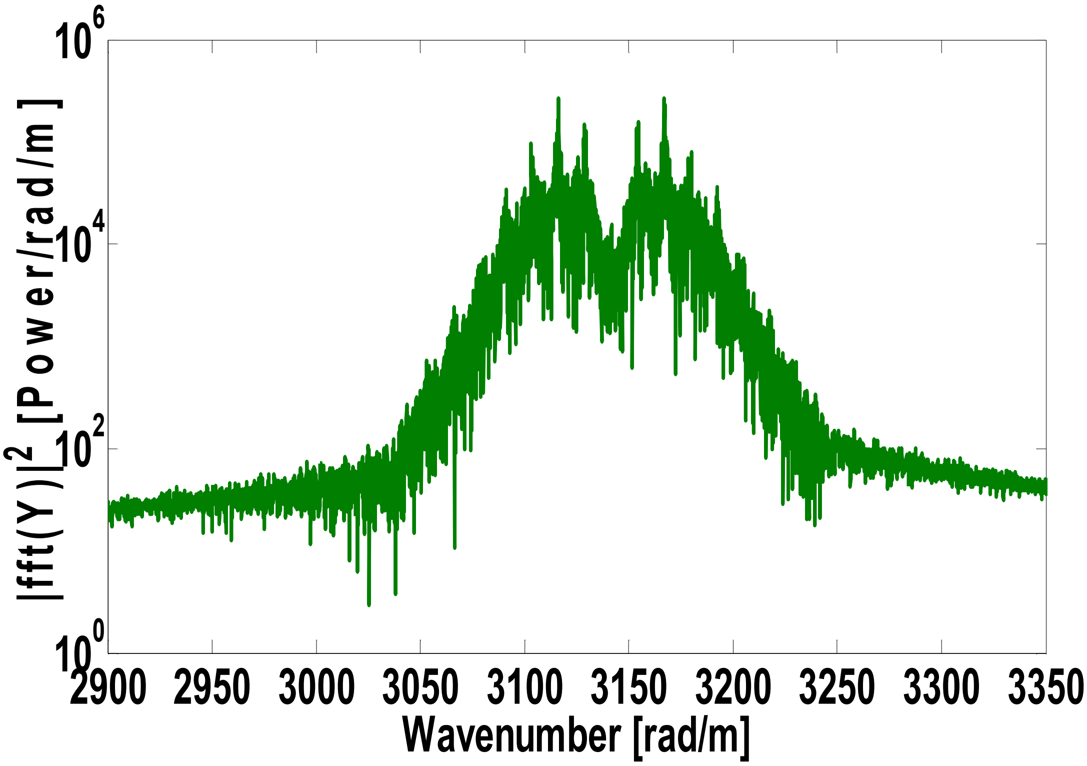

Figure 6(b): Transformation from wave position (using FFT) to wave number for Spherical shell model for $f = 10f_0$.

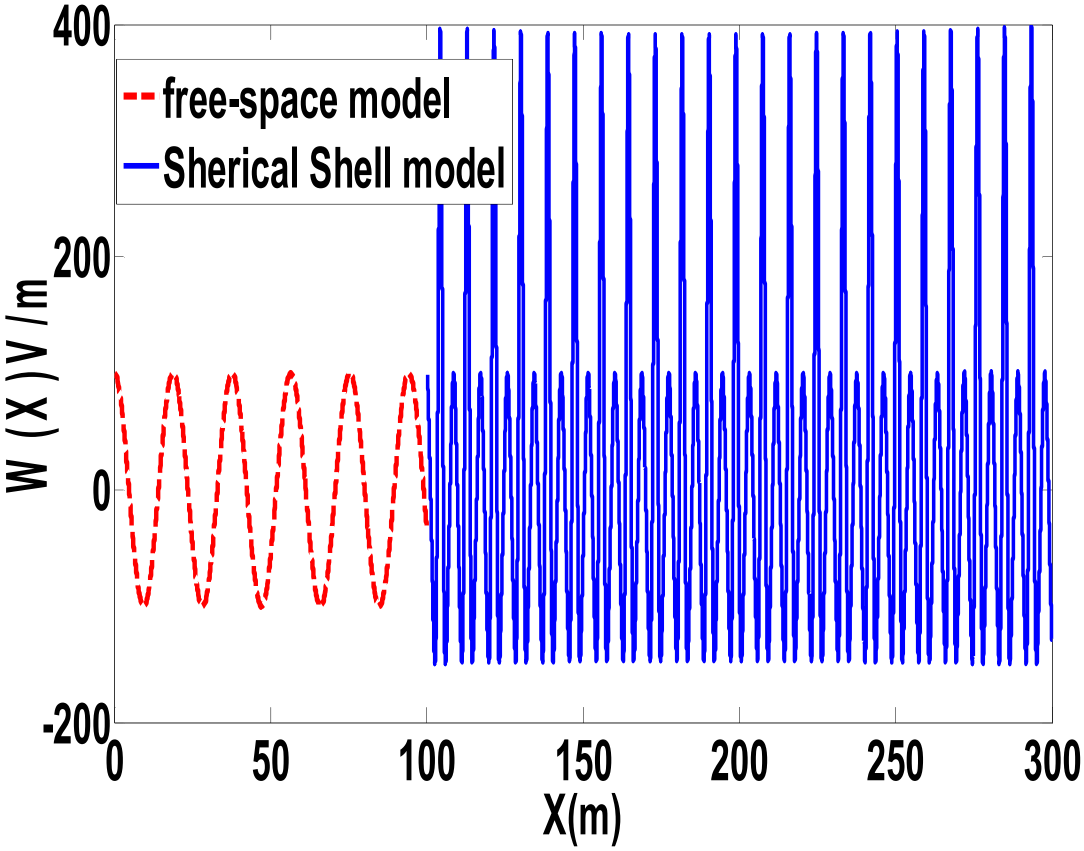

Figure 7: Propagation from free-space medium to Spherical Shell medium at same frequency, $f = f_0/3$. ($from\ 0 \leq x \leq 100\ to\ 100 \leq x \leq 300$.), h=0.001, $\gamma = 0.005$, a=0.5, b=0.003.

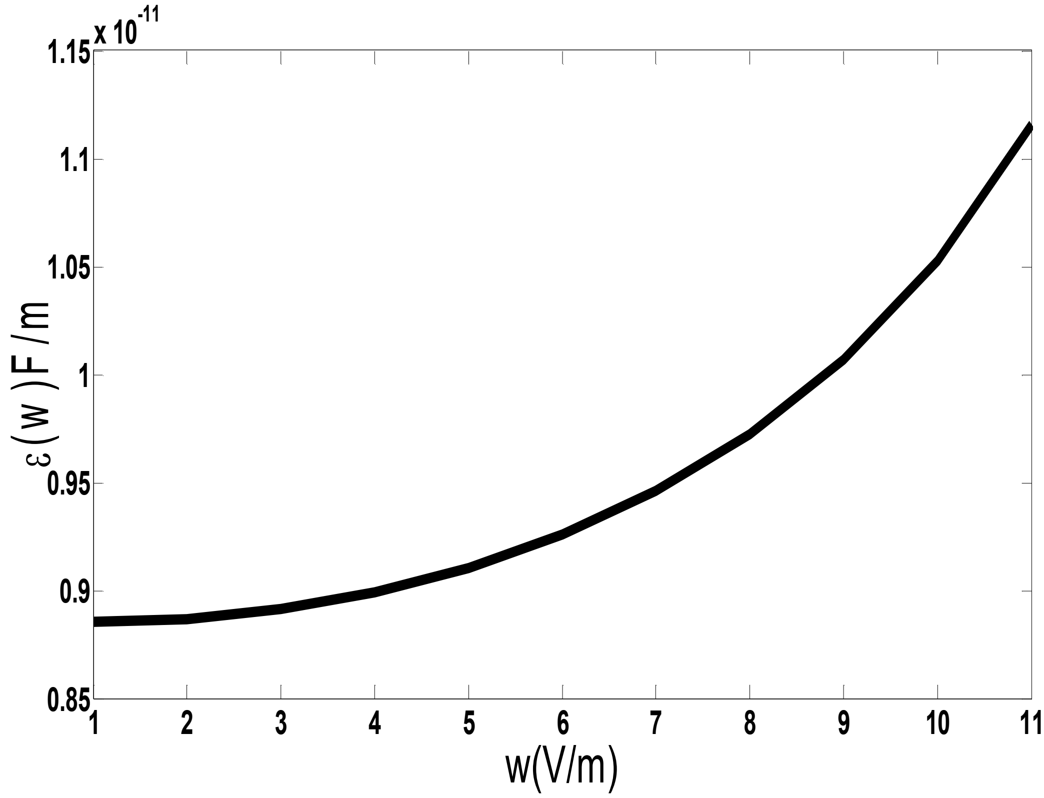

Figure 8. Sketch of modified Zhang model

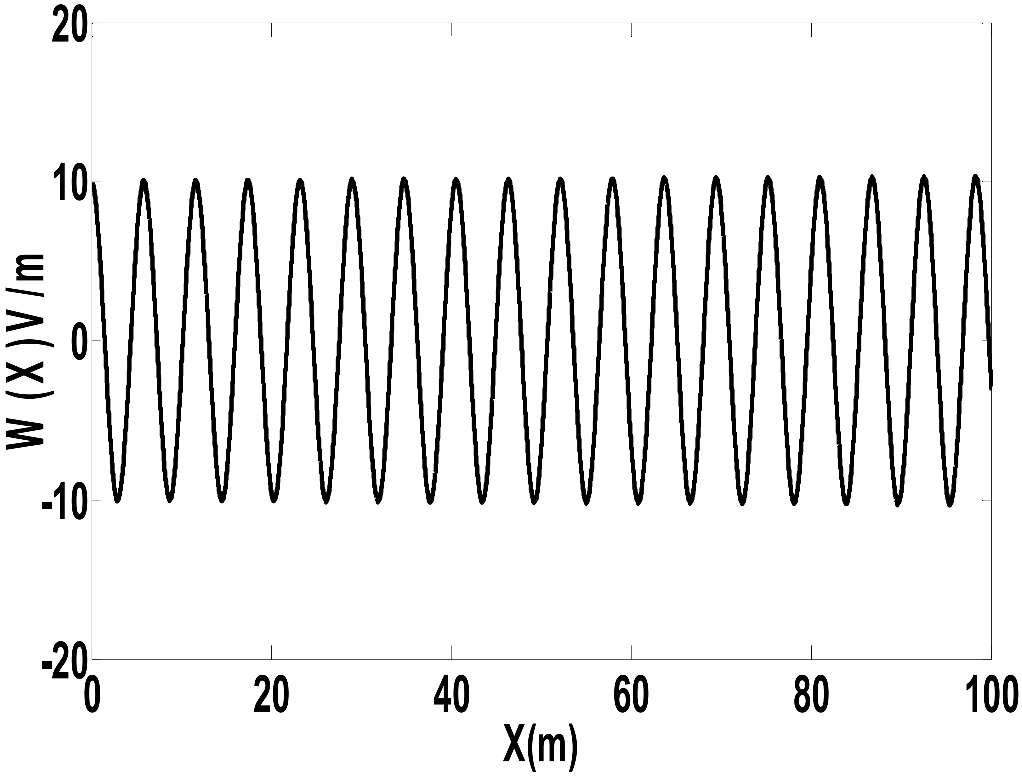

Figure 9(a): Electric field of EM waves $W(x)$ along x-direction for modified Zhang model for $f = f_0$, h=0.001, L=200.

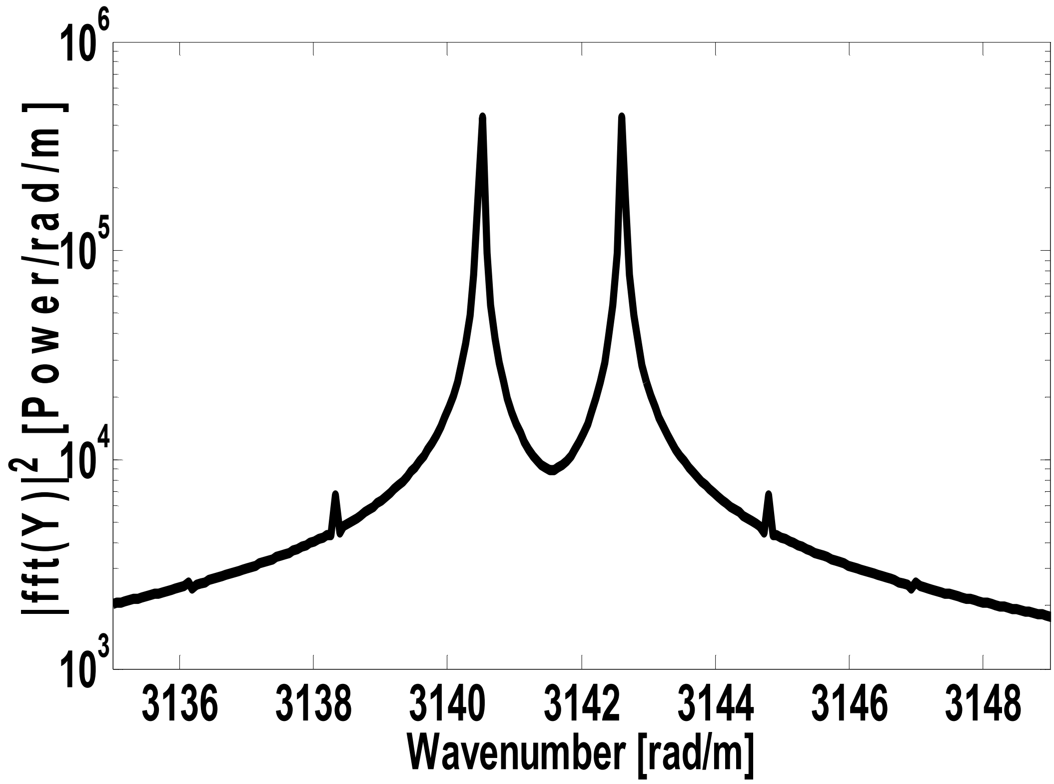

Figure 9(b): Transformation from wave position (using FFT) to wave number for modified Zhang model for $f = f_0$ & $\alpha = 0.005$.

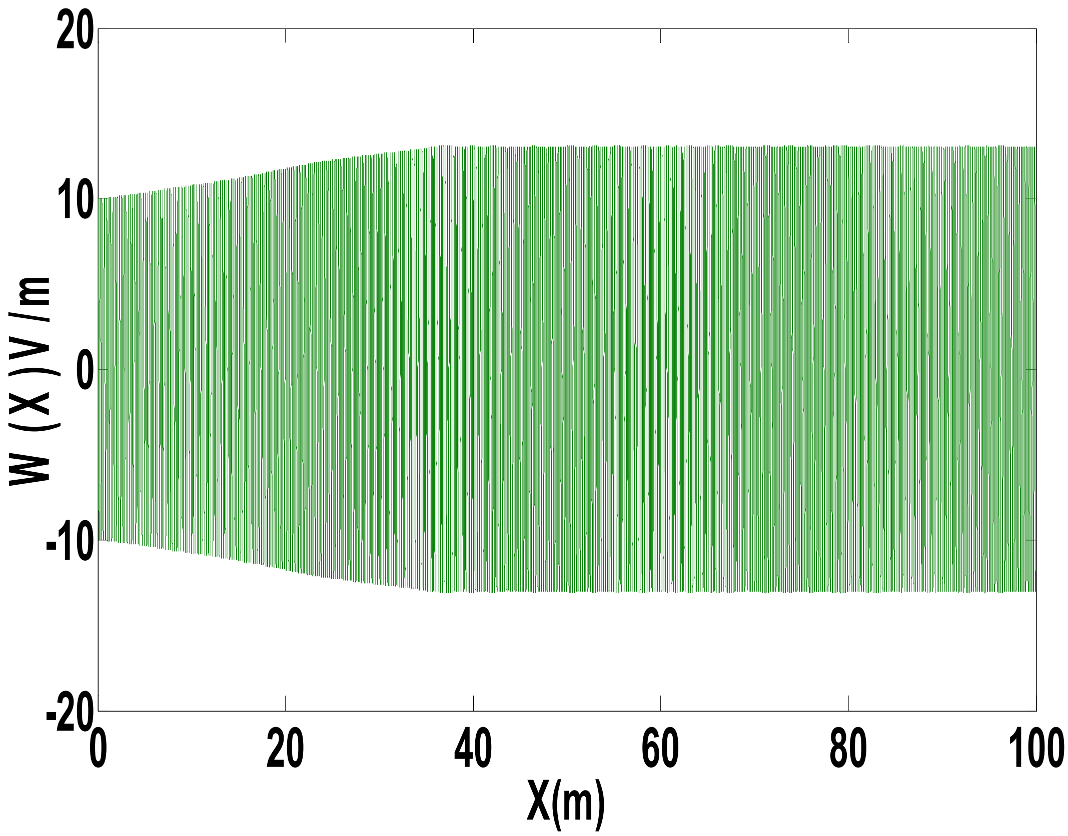

Figure 10(a): Electric field of EM waves $W(x)$ along x-direction for modified Zhang model for $f = 25f_0$, h=0.001, L=200.

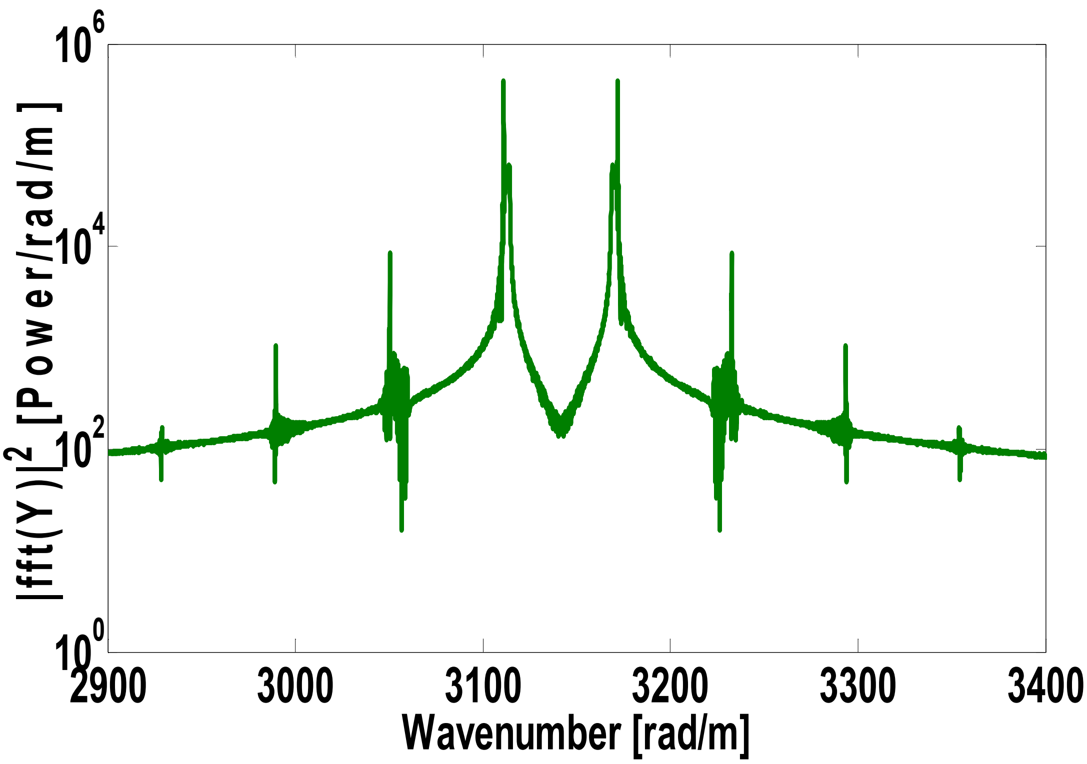

Figure 10(b): Transformation from wave position (using FFT) to wave number for modified Zhang model for $f = 25f_0$ &

$$\alpha = 0.005.$$

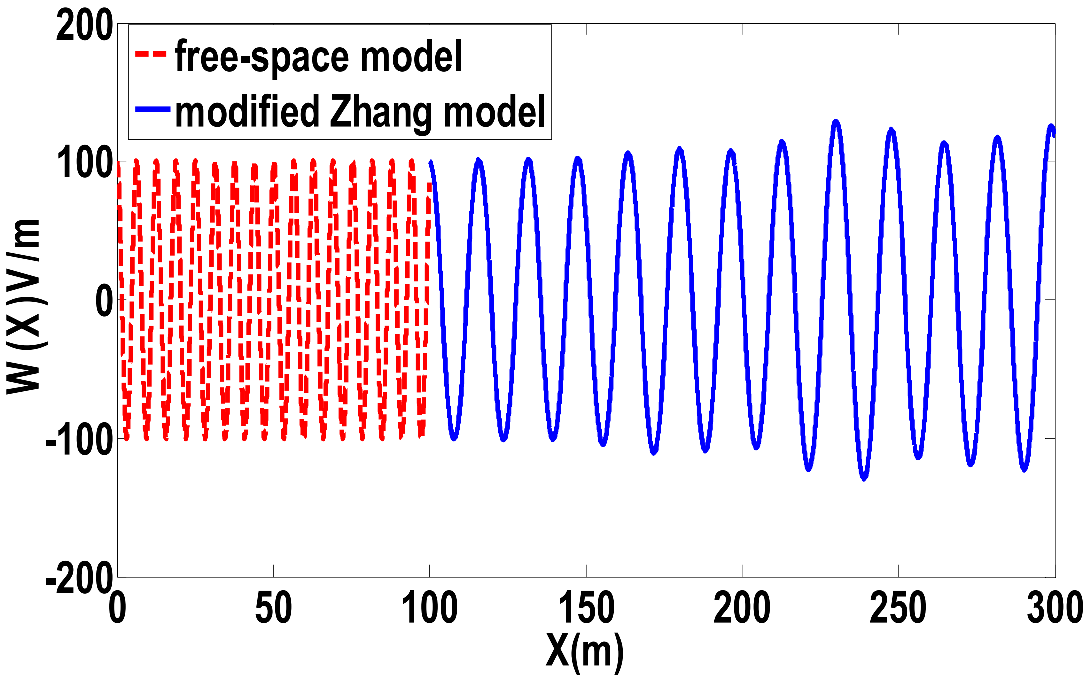

Figure 11: Propagation from free-space medium to modified Zhang model at same frequency $f = f_0(from\ 0 \leq x \leq 100\ to\ 100 \leq x \leq 300)$ with step of h=0.001 and $\alpha = 0.005$.

Figure 1 shows the behavior of material modelled by equation (18) towards electric field. The sketch tells us that, no matter the value of electric field, the permittivity of the medium remains unchanged. This means the permittivity is independent of electric field. The amplitude variation of the EM wave in such a medium is expected to be periodic in space. This corresponds to the well-known straight line propagation in the geometric optic approximation. The symmetric spatial Fourier transform of such a wave ought to show two peaks which are just mirror image of each other. Our simulation results reproduce these well-known facts. Figure 2(a) is a sinusoidal EM wave of one wavelength, λ. The periodic nature is of the same amplitude all through along the direction of propagation. In Figure 2(b), the two peaks represent two half-cycle that are projected as mirror image, when viewing from one side. The single peak represents a single mode. Figures 3(a) shows a sinusoidal wave that over crowds at the middle of the medium with the same amplitude as a result of periodic distribution at the value of $25f_0$. Figures 3(b) show single peak reflected as mirror images without obstruction along the line of propagation of waves. In a nutshell, the free space propagation is periodic characterized by a single wavelength and the periodicity maintained respective of the frequency of incident EM. The results serve as a bench-mark to interpret the results of other models that are wave amplitude dependent.

Figure 4 gives the behavior of model dielectric material governed by equation (19) towards electric field. The sketch tells us that at constant γ, the permittivity of the medium is proportional to the electric field. Figure 5(a) indicates that the amplitude of the periodic propagation decreases from both boundaries towards the middle of the medium by simulation result for the value of f_0. Figure 5(b) shows six distinct peaks projected as mirror image between 3130.9Rad/m and 3140Rad/m represent six spatial wave distributions in the original sample which are of course, odd harmonics of the fundamental wave number k_0 at the value f_0. In Figure 6(a) the number of periodic distributions at this incident frequency cannot be visualize. However, the spatial spectrum in Figure 6(b) reveals the near infinite number of modes that can be propagated in this material. In Figure 7, at the frequency of $f_0/3$, the amplitude of the propagating wave is sinusoidal and uniform while on crossing into the spherical shell it broke into two modes. The amplitude of one of the modes in nearly the same as that of the incident wave, while the amplitude of the second mode is strongly amplified.

A single dielectric component type modelled by equation (20) is part of parabola as shown in Figure 8 and it tells us that at constant α, the permittivity of the medium/material depends on the electric field. Sinusoidal wave propagation with equal amplitude along the direction of propagation is observed in Figure 9(a). In Figure 9(b), there exist two sharp peaks projected as mirror image between 3137Rad/m and 3141Rad/m that correspond to two spatial wave distributions in the original sample when the incident wave is f_0. Figure 10(a) shows an amplitude amplification from $(0-25)m$ and remain solitary along direction of propagation. Figure 10(b) shows the transformation from wave position to wave number with four clear harmonic peaks projected as a mirror image within the range $(2950 - 3130)rad/m$. In Figure 11, the propagation of wave from free space medium has constant amplitude; however, the behavior is completely different on crossing to medium of modified Zhang, as the periodic nature increases in amplitude with respect to the direction of propagation of EM waves. This shows that the model material has the ability to amplify the intensity of the wave at the given incident wave number. This behavior is very important for high quality transmission of data over long distances with wave guides having the model property.

4.0 Conclusion

The propagation of a wave in nonlinear media governed by the wave equation was derived from Maxwell's equations for an inhomogeneous, dielectric media properties with some assumptions. A typical non-linearity is the change in the dielectric constant due to electromagnetic (EM) wave field that propagates through a medium. An investigation of the three models studied using non-linear wave equation reveals that dielectric properties of the media respond to spatial component of EM wave's propagation in them and they are inhomogeneous only in x-direction. Apart from FSM, the SSM and MZM supports a variety of characteristics. There are amplitude amplifications or wave steeping, lossless or solitary propagation and multiplicity of modes for all frequencies examined. The EM wave propagation characteristics of the SSM and MZM, showed that materials which could be fabricated according to this model would be very useful as EM wave guides as they could support waves without losses as opposed to the present known commercial optical fibers.

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