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CONSTRUCTION OF TRANSITIVE SUPERSOLVABLE PERMUTATION GROUPS

M. Bello
Mathematics and computer science department, Federal University Kashere, Gombe State, Nigeria
P.M.B 0182, Gombe State Nigeria.
Tel: +2348060971976, Email: mbatap560@gmail.com

Mustapha Danjuma
Mathematics and Statistics Department, School of science, Abubakar Tatari Ali Polytechnic, Bauchi State Nigeria
Tel: +2348037025131, Email: mustaphdanjuma@gmail.com

Sani Musa
Mathematics and Statistics Department, School of science, Abubakar Tatari Ali Polytechnic, Bauchi State Nigeria
Tel: +2348028909358, Email: aksanimusa@gmail.com

Raliatu Mohammed Kashim
Mathematics and Statistics Department, Abubakar Tatari Ali Polytechnic Bauchi, Bauchi State, Nigeria
Tel: +2347088110109, Email: raliaumax@gmail.com

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ABSTRACT

In this paper, we used wreath products of two permutation groups in constructing transitive supersolvable permutation groups. We verified these groups using some groups theoretical concepts and also validate our work using a standard program; GAP (Groups Algorithm and Programming).

1.0 Introduction

Supersolvable groups are very important in the theory of groups as they play an important role in the theory of groups with finite order. For instance, finite abelian groups can be used to construct a group in which all its subgroups are solvable, because all abelian groups are trivially solvable since a subnormal series being given by just the group itself and the trivial group. But non-abelian groups may or may not be solvable. Thanos Gentimis (2006), showed that any group of order up to 100 and not 60 is solvable. For this research, we shall use Wreath Products of two permutation groups to construct locally solvable groups.

1.1 Definition (Milne, J.S, 2009)
The series of subgroups
$G_0, G_1, G_2, \dots, G_n$ Such that $G = G_0 \trianglelefteq G_1 \trianglelefteq G_2 \trianglelefteq \cdots \trianglelefteq G_n = (1)$ where G_i/G_{i+1} is abelian is called a solvable series.

1.2 Definition (Milne, J.S, 2009)
A group *G* is solvable if there is a finite collection of groups $G_0, G_1, \dots, G_n$ such that $(1) = G_0 \subseteq G_1 \subseteq \cdots \subseteq G_n = G$ where $G_i \trianglelefteq G_{i+1}$ and G_{i+1}/G_i is abelian. If $|G| = 1$, then *G* is a solvable group.

1.3 Definition (Kurosh. A. G, 1956)
The series in 1.1 above where each subgroup is normal in the next one is called a normal series for the group G

1.4 Definition (Kurosh. A. G, 1956)
A group G is called supersolvable if it has normal series with cyclic factors.

1.5 Definition (Audu M.S, 2003)
The Wreath product of C by D denoted by W = C wr D is the semidirect product of P by D, so that $W = \{(f,d) \mid f \in P, d \in D\}$, with multiplication in W defined as $(f_1, d_1)(f_2, d_2) = f_1 f_2^{d_1^{-1}}, (d_1 d_2)$ for all $f_1, f_2 \in P$ and $d_1, d_2 \in D$. Henceforth, we write fd instead of (f, d) for elements of W.

Theorem 1.1 (Audu M.S,2003)
Let C and D be permutation groups on Γand Δ respectively. Let C^{Δ} be the set of all maps of Δ into the permutation group C. That is $C^{\Delta} = \{f : \Delta \longrightarrow C\}\ \forall f_1, f_2 \in C^{\Delta}$. Let $f_1 f_2$ in C^{Δ} be defined $\forall\ \delta$ inΔ by $(f_1 f_2)(\delta) = f_1(\delta) f_2(\delta)$, (M. S. Audu, K.E Osondu, A.R.j. Solarin, 2003)

Remark

i. Thus C^{Δ} is a group with respect to multiplication defined above. (We denote the group by P)
ii. With respect to this operation of multiplication, C^{Δ}acquire a structure of a group.

Lemma 1.1
Assume that *D* acts on *P* as follows: $f^d(\delta) = f(\delta d^{-1})$ for all $\delta \in \Delta, d \in D$. Then D acts on P as a group.

Theorem 1.2 (Audu M.S, 2003)
Let *D* act on *P* as a group. Then the set of all ordered pairs (f, d) with $f \in P$ and $d \in D$ acquires the structure of a group when we define for all $f_1, f_2 \in P$
and $d_1, \quad d_2 \in D (f_1, d_1)(f_2, d_2) = (f_1 f_2^{d_1^{-1}}, d_1 d_2)$

Theorem 1.3 (Audu M.S, 2003)
Let D act on P as $f^d(\delta) = f(\delta d^{-1})$ where $f \in P, d \in D$ and $\delta \in \Delta$. Let W be the group of all juxtaposed symbols fd, with $f \in P, d \in D$ and multiplication given by $(f_1, d_1)(f_2, d_2) = f_1 f_2^{d_1^{-1}}, (d_1 d_2)$. Then W is a group called the semi-direct product of P by D with the defined action.
Based on the forgoing we note the following:

- If C and D are finite groups, then the wreath product W determined by an action of D on a finite set is a finite group of order $|W| = |C|^{|\Delta|} . |D|$.
- P is a normal subgroup of W and D is a subgroup of W.
- The action of W on Γ × Δ is given by $(\alpha, \beta) fd = (\alpha f(\beta), \beta d)$ where $\alpha \in \Gamma$ and $\beta \in \Delta$.

2.0 Preliminaries
The following Theorems are important in this work.
Theorem 2.1 (Thanos G., 2006)
G is solvable if and only if $G^{(n)} = 1$, for some n.

Theorem 2.3
A group G is solvable if and only if it has a solvable series.

Proof
Suppose G is solvable. Then by the definition of "solvable," in the derived series of commutator subgroups we have $G^{(n)}$ = (1), for some $n \in N$. By Theorem 2.2, in the series
$G > G^{(1)} > G^{(2)} > \cdots > G^{(2)} = (1)$, we have that$G^{(i+1)}$ is normal in $G^{(i)}$ and $G^{(i)}/G^{(i+1)}$ is abelian. So the series is subnormal (because each subgroup is normal in each previous subgroup) and is also solvable (since the quotient groups are abelian).
Now suppose $G = G_0 > G_1 > \cdots > G_n = (1)$ is a solvable series. Then
$G_i / G_{i+1}$is abelian (by definition of solvable series) for $0 \leq i \leq n-1$. By Theorem 2.2, $G_{i+1} > (G_i)'$for $0 \leq i \leq n-1$.
Since in the derived series of commutator subgroups we have $G > G^{(1)} > G^{(2)} > \cdots > G^{(n)}$, then

$$G_1 > G_0{}' = G' = G^{(1)}$$
$$G_2 > G_1{}' = \left(G^{(1)}\right)' = G^{(2)}$$
$$G_3 > G_2{}' = \left(G^{(2)}\right)' = G^{(3)}$$
$$\vdots$$
$$G_{i+1} > G_i{}' = \left(G^{(i)}\right)' = G^{(i+1)}$$
$$\vdots$$
$$G_n > G'_{n-1} = \left(G^{(n-1)}\right)' = G^{(n)}.$$

But $G_n = (1)$ so it must be that $G^{(n)} = (1)$ and G is solvable.

Thus, we give the following illustrations:

(i) $G = \{(1), (48765), (47586), (46857), (45678), (132), (132)(48765), (132)(47586), (132)(46857), (132)(45678), (123), (123)(48765), (123)(47586), (123)(46857), (123)(45678)\}$
has the subgroups as follows;

$H_0 = (1)$
$H_1 = \{(1), (123), (132)\}$
$H_2 = \{(1), (48765), (47586), (46857), (45678)\}$
$H_3 = \{(1), (48765), (47586), (46857), (45678), (132), (132)(48765), (132)(47586), (132)(46857), (132)(45678), (123), (123)(48765), (123)(47586), (123)(46857), (123)(45678)\}$
has a solvable series which is $(1) = H_0 \lhd H_1 \lhd H_3 = G$ hence solvable by Theorem 2.3

(ii) The dihedral group D_n is solvable since $D_n \rhd (p) \rhd \{1\}$
Let D_{16} be the Dihedral group of Degree 8 given by:

$D_{16} = \{(1), (28)(37)(46), (15)(26)(37)(48), (15)(24)(68), (1753)(2864), (17)(26)(35), (1357)(2468), (13)(48)(57), (18765432), (18)(27)(36)(45), (14725836), (14)(23)(58)(67), (16385274), (16)(25)(34)(78), (12345678), (12)(38)(47)(56)\}$

whose subgroups are as follows;

$H_1 = (1)$
$H_2 = \{(1), (15)(26)(37)(48)\} = (p)$
$H_3 = \{(1), (28)(37)(46), (15)(26)(37)(48), (15)(24)(68), (1753)(2864), (17)(26)(35), (1357)(2468), (13)(48)(57), (18765432), (18)(27)(36)(45), (14725836), (14)(23)(58)(67), (16385274), (16)(25)(34)(78), (12345678), (12)(38)(47)(56)\}$

Hence $D_{16} = H_3 \rhd H_2 \rhd H_1 = (1)$

Proposition 2.1

Let *G* be solvable and $H \leq G$. Then

3. *H* is solvable.
4. If $H \lhd G$, then G/H is solvable.

Proof

Start from a series with abelian slices. $G = G_0 \trianglerighteq G_1 \trianglerighteq \cdots \trianglerighteq G_n = \{1\}$. Then $H = H \cap G_0 \trianglerighteq H \cap G_1 \trianglerighteq \cdots \trianglerighteq H \cap G_n = \{1\}$. When *H* is normal, we use the canonical projection $\pi: G \to G/H$ to get $G/H = \pi(G_0) \trianglerighteq \cdots \trianglerighteq \pi(G_n) = \{1\}$; the quotients are abelian as well, so G/H is still solvable.

Proposition 2.2

Let $N \trianglelefteq G$. Then G is solvable if and only if N and G/N are solvable.

Proof

(⇒) This is obvious by Proposition 2.1.

(⇐) Stick together a series for N with abelian slices with the lift to *G* of a series for G/N, using the fourth isomorphism law.

Proposition 2.3

Let *G* be a group with a composition series. Then the following are equivalent:

i. *G* is solvable.
ii. All composition factors are abelian.
iii. All composition factors are cyclic of prime order.

Proof

i. ⇒ii. Apply Proposition 2.2 to a composition series.

ii.⇒ iii. Simple abelian groups are cyclic of prime order.

iii. ⇒i. Take the composition series as the subnormal series with abelian factors.

Thus, $n \geq 4$, S_n is not solvable. A_n is not abelian. Note that solvable groups with a composition series are finite.

Theorem

Supersolvable groups are closed under passage to subgroups, quotients, and direct products

Corollary

Let G be a group with normal subgroups H and K. If G/H and G/K are supersolvable then G/(H ∩ K) is supersolvable

Theorem

Let G be any group and N C G. If N is cyclic and G/N is supersolvable then G is supersolvable

RESULTS

2.1 Consider the permutation groups $C_1 = \{(1),(123),(132)\}$ and $D_1 = \{(1),(12)\}$ acting on $X = \{1,2,3\}$ and $\Delta = \{1,2\}$ respectively. Let $F_1 = C_1^{\Delta} = \{f: \Delta_1 \to C_1\}$. Then $|F_1| = |C_1|^{|\Delta_1|} = 3^2 = 9$. The order of the wreath product is given by $|W_1| = |C_1|^{|\Delta_1|} \times |D_1| = 3^2 \times 2.$

The mappings are as follows

$f_1 : 1 \to (1), 2 \to (1)$
$f_2 : 1 \to (123), 2 \to (123)$
$f_3 : 1 \to (132), 2 \to (132)$
$f_4 : 1 \to (1), 2 \to (123)$
$f_5 : 1 \to (1), 2 \to (132)$
$f_6 : 1 \to (123), 2 \to (1)$
$f_7 : 1 \to (132), 2 \to (1)$
$f_8 : 1 \to (132), 2 \to (123)$
$f_9 : 1 \to (123), 2 \to (132)$

The elements of W are

$(f_1,d_1),(f_1,d_2),(f_2,d_1),(f_2,d_2),(f_3,d_1),(f_3,d_2),(f_4,d_1),(f_4,d_2),(f_5,d_1),$
$(f_5,d_2),(f_6,d_1),(f_6,d_2),(f_7,d_1),(f_7,d_2),(f_8,d_1),(f_8,d_2),(f_9,d_1),(f_9,d_2),$

$(\alpha,\delta)^{fd} = (\alpha f(\delta), \delta d).$

Further, $\Gamma \times \Delta = \{(1,1),(1,2),(2,1),(2,2),(3,1),(3,2)\}$

We obtain the following permutations by the action of W on $\Gamma \times \Delta$

$(\alpha,\beta) f_1 d_1 = (\alpha f_i(\delta), \beta d_i)$
$(1,1) f_1 d_1 = (1 f_1(1), 1 d_1) = (1,1)$
$(1,2) f_1 d_1 = (1 f_1(2), 2 d_1) = (1,2)$
$(2,1) f_1 d_1 = (2 f_1(1), 1 d_1) = (2,1)$
$(2,2) f_1 d_1 = (2 f_1(2), 2 d_1) = (2,2)$
$(3,1) f_1 d_1 = (3 f_1(1), 1 d_1) = (3,1)$
$(3,2) f_1 d_1 = (3 f_1(2), 2 d_1) = (3,2)$

Rename the symbols as

$(1,1) \to 1$	$(2,1) \to 2$	$(3,1) \to 3$
$(1,2) \to 4$	$(2,2) \to 5$	$(3,2) \to 6$

And in summary,

$$(\Gamma \times \Delta)^{(f_1,d_1)} = \begin{Bmatrix} (1,1)(1,2)(2,1)(2,2)(3,1)(3,2) \\ (1,1)(1,2)(2,1)(2,2)(3,1)(3,2) \end{Bmatrix}$$

$$(\Gamma \times \Delta)^{(f_1,d_2)} = \begin{Bmatrix} (1,1)(1,2)(2,1)(2,2)(3,1)(3,2) \\ (1,2)(1,1)(2,2)(2,1)(3,2)(3,1) \end{Bmatrix}$$

$$(\Gamma \times \Delta)^{(f_2,d_1)} = \begin{Bmatrix} (1,1)(1,2)(2,1)(2,2)(3,1)(3,2) \\ (2,1)(2,2)(3,1)(3,2)(1,1)(1,2) \end{Bmatrix}$$

$$(\Gamma \times \Delta)^{(f_2,d_2)} = \begin{Bmatrix} (1,1)(1,2)(2,1)(2,2)(3,1)(3,2) \\ (2,2)(2,1)(3,2)(3,1)(1,2)(1,1) \end{Bmatrix}$$

$$(\Gamma \times \Delta)^{(f_3,d_1)} = \begin{Bmatrix} (1,1)(1,2)(2,1)(2,2)(3,1)(3,2) \\ (3,1)(3,2)(1,1)(1,2)(2,1)(2,2) \end{Bmatrix}$$

$$(\Gamma \times \Delta)^{(f_3,d_2)} = \begin{Bmatrix} (1,1)(1,2)(2,1)(2,2)(3,1)(3,2) \\ (3,2)(3,1)(1,2)(1,1)(2,2)(2,1) \end{Bmatrix}$$

$$(\Gamma\times\Delta)^{(f_4,d_1)}=\begin{Bmatrix}(1,1)(1,2)(2,1)(2,2)(3,1)(3,2)\\(1,1)(2,2)(2,1)(3,2)(3,1)(1,2)\end{Bmatrix}$$
$$(\Gamma\times\Delta)^{(f_4,d_2)}=\begin{Bmatrix}(1,1)(1,2)(2,1)(2,2)(3,1)(3,2)\\(1,2)(2,1)(2,2)(3,1)(3,2)(1,1)\end{Bmatrix}$$
$$(\Gamma\times\Delta)^{(f_5,d_1)}=\begin{Bmatrix}(1,1)(1,2)(2,1)(2,2)(3,1)(3,2)\\(1,1)(3,2)(2,1)(1,2)(3,1)(2,2)\end{Bmatrix}$$
$$(\Gamma\times\Delta)^{(f_5,d_2)}=\begin{Bmatrix}(1,1)(1,2)(2,1)(2,2)(3,1)(3,2)\\(1,2)(3,1)(2,2)(1,1)(3,2)(2,1)\end{Bmatrix}$$
$$(\Gamma\times\Delta)^{(f_6,d_1)}=\begin{Bmatrix}(1,1)(1,2)(2,1)(2,2)(3,1)(3,2)\\(2,1)(1,2)(3,1)(2,2)(1,1)(3,2)\end{Bmatrix}$$
$$(\Gamma\times\Delta)^{(f_6,d_2)}=\begin{Bmatrix}(1,1)(1,2)(2,1)(2,2)(3,1)(3,2)\\(2,2)(1,1)(3,2)(2,1)(1,2)(3,1)\end{Bmatrix}$$
$$(\Gamma\times\Delta)^{(f_7,d_1)}=\begin{Bmatrix}(1,1)(1,2)(2,1)(2,2)(3,1)(3,2)\\(3,1)(1,2)(1,1)(2,2)(2,1)(3,2)\end{Bmatrix}$$
$$(\Gamma\times\Delta)^{(f_7,d_2)}=\begin{Bmatrix}(1,1)(1,2)(2,1)(2,2)(3,1)(3,2)\\(3,2)(1,1)(1,2)(2,1)(2,2)(3,1)\end{Bmatrix}$$
$$(\Gamma\times\Delta)^{(f_8,d_1)}=\begin{Bmatrix}(1,1)(1,2)(2,1)(2,2)(3,1)(3,2)\\(3,1)(2,2)(1,1)(3,2)(2,1)(1,2)\end{Bmatrix}$$
$$(\Gamma\times\Delta)^{(f_8,d_2)}=\begin{Bmatrix}(1,1)(1,2)(2,1)(2,2)(3,1)(3,2)\\(3,2)(2,1)(1,2)(3,1)(2,2)(1,1)\end{Bmatrix}$$
$$(\Gamma\times\Delta)^{(f_9,d_1)}=\begin{Bmatrix}(1,1)(1,2)(2,1)(2,2)(3,1)(3,2)\\(2,1)(3,2)(3,1)(1,2)(1,1)(2,2)\end{Bmatrix}$$
$$(\Gamma\times\Delta)(f_9,d_2)=\begin{Bmatrix}(1,1)(1,2)(2,1)(2,2)(3,1)(3,2)\\(2,2)(3,1)(3,2)(1,1)(1,2)(2,1)\end{Bmatrix}$$

Then the permutations in cyclic form is

$$W_1=\begin{Bmatrix}(1),(14)(25)(36),(123)(456),(153426),(132)(465),\\(162435),(456),(142536),(465),(143625),(123),\\(152634),(132),(163524),(134)(456),(35),\\(123)(465),(15)(26)(34).\end{Bmatrix}$$

Some of the normal subgroups of W_1 are;

$H_0=(1)$

H_1 ={(1),(123)(465),(132)(456)}

H_2 ={(1),(132),(123),(465),(132)(465),(123)(465),(456),(132)(456),(123)(456)}

$$H_3=\begin{Bmatrix}(1),(14)(25)(36),(123)(456),(153426),(132)(465),\\(162435),(456),(142536),(465),(143625),(123),\\(152634),(132),(163524),(134)(456),(35),\\(123)(465),(15)(26)(34).\end{Bmatrix}$$

W_1 is transitive with chief series as $W_1=H_3\rhd H_2\rhd H_1\rhd H_0=(1)$ with cyclic factors C_2,C_3 and C_3

2.2 Consider the permutation groups C_1 and D_1

$C_2=\{(1),(12)\},\ D_2=\{(1),(35)(46),(3654),(3456)\}$

acting on the sets $S_2=\{1,2\}\ and\ \Delta_2=\{3,4,5,6\}$ respectively.

Let $P=C_2^{\Delta_2}=\{f:\Delta_2\longrightarrow C_2\} then |P|=|C_2|^{\Delta_2}=2^4=16$

We can easily verify that G_3 is a group with respect to the operations

$(f_1f_2)\delta_1=f_1(\delta_1)f_2(\delta_1) where\ \delta_1\in\Delta_1$.

The wreath product of C_2 and D_2 is given by W_2, where

W_2
$=\{(1),(78),(56)(78),(56),(34)(78),(34),(34)(56),(34)(56)(78),(12)(34)(56)(78),(12)(34)(56),(12)(34),(12)(34)(78),(12)(56),(12)(56)(78),(12)(78),(12),(1753)(2864),(17642853),(1763)(2854),(17542863),(1764)(2853),(17532864),(1754)(2863),(17632854),(1854)(2763),(18632754),(1864)(2753),(18532764),(1863)(2754),(18542763),(1853)(2764),$

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(18642753), (15)(26)(37)(48), (15)(26)(3748), (1526)(3748), (1526)(37)(48), (15)(26)
(38)(47), (15)(26)(3847), (1526)(3847), (1526)(38)(47), (16)(25)(38)(47), (16)(25)(3847), (1625)
(3847), (1625)(38)(47), (16)(25)(37)(48), (16)(25)(3748), (1625)(3748), (1625)(37)(48),
(1357)(2468), (13572468), (1358)(2467), (13582467), (1368)(2457), (13682457), (1367)
(2458), (13672458), (1458)(2367), (14582367), (1457)(2368), (14572368), (1467)
(2358), (14672358), (1468)(2357), (14682357)}

Some of the normal subgroups of W_2 are;
$H_0 = (1)$
$H_1 = \{(1), (12)(34)(56)(78)\}$
$H_2 = \{(1), (12)(56), (34)(78), (12)(34)(56)(78)\}$
$H_3 = \{(1), (34)(78), (56)(78), (34)(56), (12)(78), (12)(34), (12)(56), (12)(34)(56)(78)\}$
$H_4 = \{(1), (12)(34), (12)(34)(56), (12)(56), (34)(56), (12), (34), (56), (78), (12)(78), (34)(78),$
$(12)(34)(78), (56)(78), (12)(56)(78), (34)(56)(78), (12)(34)(56)(78)\}$

$H_4 =$
{(1), (56), (78), (56)(78), (34)(78), (34)(56)(78), (34), (34)(56), (15)(26)(37)(48), (1526)(37)(48),

(15)(26)(3748), (1526)(3748), (15)(26)(38)(47), (1526)(38)(47), (15)(26)(3847), (1526)(3847), (16)
(25)(37)(48), (1625)(37)(48), (16)(25)(3748), (1625)(3748), (16)(25)(38)(47), (1625)(38)
(47), (16)(25)(3847), (1625)(3847), (12)(56), (12), (12)(56)(78), (12)(78), (12)(34)(56)(78), (12)(34)(78),
(12)(34)(56), (12)(34)}
H_5
= {(1), (78), (56)(78), (56), (34)(78), (34), (34)(56), (34)(56)(78), (12)(34)(56)(78), (12)(34)(56), (12)
(34), (12)(34)(78), (12)(56), (12)(56)(78), (12)(78), (12), (1753)(2864), (17642853), (1763)(2854),
(17542863), (1764)(2853), (17532864), (1754)(2863), (17632854), (1854)(2763),
(18632754), (1864)(2753), (18532764), (1863)(2754), (18542763), (1853)(2764),
(18642753), (15)(26)(37)(48), (15)(26)(3748), (1526)(3748), (1526)(37)(48), (15)(26)
(38)(47), (15)(26)(3847), (1526)(3847), (1526)(38)(47), (16)(25)(38)(47), (16)(25)(3847), (1625)
(3847), (1625)(38)(47), (16)(25)(37)(48), (16)(25)(3748), (1625)(3748), (1625)(37)(48),
(1357)(2468), (13572468), (1358)(2467), (13582467), (1368)(2457), (13682457), (1367)
(2458), (13672458), (1458)(2367), (14582367), (1457)(2368), (14572368), (1467)
(2358), (14672358), (1468)(2357), (14682357)}
W_2 is transitive with chief series as $W_2 = H_5 \trianglerighteq H_4 \trianglerighteq H_3 \trianglerighteq H_2 \trianglerighteq H_1 \trianglerighteq H_0 = (1)$ with cyclic factors C_2, C_2 C_2 C_2 and C_2

2.3 Consider the permutation groups C_3 and D_3
$C_3 = \{(1), (15432), (14253), (13524), (12345)\}$, $D_3 = \{(1), (67)\}$
acting on the sets $S_3 = \{1,2,3,4,5\}$ and $\Delta_3 = \{6,7\}$ respectively.
Let $P = {C_3}^{\Delta_3} = \{f: \Delta_3 \longrightarrow C_3\} then |P| = |C_3|^{\Delta_3} = 5^2 = 25$
We can easily verify that G_3 is a group with respect to the operations
$(f_1 f_2)\delta_1 = f_1(\delta_1) f_2(\delta_1) where\, \delta_1 \in \Delta_1$.
The wreath product of C_3 and D_3 is given by W_3, where

$W_3 =$ {(1), (610 9 8 7), (6 9 710 8), (6 810 7 9), (6 7 8 910), (15432), (1 5 4 3 2)(610 9 8 7),
(1 5 4 3 2)(6 9 710 8), (1 5 4 3 2)(6 810 7 9), (1 5 4 3 2)(6 7 8 910), (14253),
(1 4 2 5 3)(610 9 8 7), (1 4 2 5 3)(6 9 710 8), (1 4 2 5 3)(6 810 7 9), (1 4 2 5 3)
(6 7 8 910), (13524), (1 3 5 2 4)(610 9 8 7), (1 3 5 2 4)(6 9 710 8), (1 3 5 2 4)
(6 810 7 9), (1 3 5 2 4)(6 7 8 910), (12345), (1 2 3 4 5)(610 9 8 7), (1 2 3 4 5)
(6 9 710 8), (1 2 3 4 5)(6 810 7 9), (1 2 3 4 5)(6 7 8 910), (1 6)(2 7)(3 8)(4 9)(510),
(1 6 510 4 9 3 8 2 7), (1 6 4 9 2 7 510 3 8), (1 6 3 8 510 2 7 4 9), (1 6 2 7 3 8 4 9
510), (110 5 9 4 8 3 7 2 6), (110 4 8 2 6 5 9 3 7), (110 3 7 5 9 2 6 4 8), (110 2 6 3 7
4 8 5 9)(110), (2 6)(3 7)(4 8)(5 9), (1 9 4 7 210 5 8 3 6), (1 9 3 6 5 8 210 4 7),
(1 9 210 3 6 4 7 5 8), (1 9)(210)(3 6)(4 7)(5 8), (1 9 5 8 4 7 3 6 210), (1 8 310 5 7
2 9 4 6), (1 8 2 9 310 4 6 5 7), (1 8)(2 9)(310)(4 6)(5 7), (1 8 5 7 4 6 310 2 9),
(1 8 4 6 2 9 5 7 310), (1 7 2 8 3 9 410 5 6), (1 7)(2 8)(3 9)(410)(5 6), (1 7 5 6 410

$3\,9\,2\,8),(1\,7\,10\,2\,8\,5\,6\,3\,9),(1\,7\,3\,9\,5\,6\,2\,8\,4\,10)\}$

Some of the normal subgroups of W_3 are;

$H_0 = (1)$

$H_1 = \{(1),(1\,5\,4\,3\,2)(6\,10\,9\,8\,7),(1\,4\,2\,5\,3)(6\,9\,7\,10\,8),(1\,3\,5\,2\,4)(6\,8\,10\,7\,9),(1\,2\,3\,4\,5)(6\,7\,8\,9\,10)\}$

$H_2 = \{(1),(15432),(14253),(13524),(12345)(6\,10\,9\,8\,7),(1\,5\,4\,3\,2)(6\,10\,9\,8\,7),(1\,4\,2\,5\,3)(6\,10\,9\,8\,7),(1\,3\,5\,2\,4)(6\,10\,9\,8\,7),(1\,2\,3\,4\,5)(6\,10\,9\,8\,7),(6\,9\,7\,10\,8),(1\,5\,4\,3\,2)(6\,9\,7\,10\,8),(1\,4\,2\,5\,3)(6\,9\,7\,10\,8),(1\,3\,5\,2\,4)(6\,9\,7\,10\,8),(1\,2\,3\,4\,5)(6\,9\,7\,10\,8),(6\,8\,10\,7\,9),(1\,5\,4\,3\,2)(6\,8\,10\,7\,9),(1\,4\,2\,5\,3)(6\,8\,10\,7\,9),(1\,3\,5\,2\,4)(6\,8\,10\,7\,9),(1\,2\,3\,4\,5)(6\,8\,10\,7\,9),(6\,7\,8\,9\,10),(1\,5\,4\,3\,2)(6\,7\,8\,9\,10),(1\,4\,2\,5\,3)(6\,7\,8\,9\,10),(1\,3\,5\,2\,4)(6\,7\,8\,9\,10),(1\,2\,3\,4\,5)(6\,7\,8\,9\,10)\}$

$H_3 = \{(1),(6\,10\,9\,8\,7),(6\,9\,7\,10\,8),(6\,8\,10\,7\,9),(6\,7\,8\,9\,10),(15432),(1\,5\,4\,3\,2)(6\,10\,9\,8\,7),(1\,5\,4\,3\,2)(6\,9\,7\,10\,8),(1\,5\,4\,3\,2)(6\,8\,10\,7\,9),(1\,5\,4\,3\,2)(6\,7\,8\,9\,10),(14253),(1\,4\,2\,5\,3)(6\,10\,9\,8\,7),(1\,4\,2\,5\,3)(6\,9\,7\,10\,8),(1\,4\,2\,5\,3)(6\,8\,10\,7\,9),(1\,4\,2\,5\,3)(6\,7\,8\,9\,10),(13524),(1\,3\,5\,2\,4)(6\,10\,9\,8\,7),(1\,3\,5\,2\,4)(6\,9\,7\,10\,8),(1\,3\,5\,2\,4)(6\,8\,10\,7\,9),(1\,3\,5\,2\,4)(6\,7\,8\,9\,10),(12345),(1\,2\,3\,4\,5)(6\,10\,9\,8\,7),(1\,2\,3\,4\,5)(6\,9\,7\,10\,8),(1\,2\,3\,4\,5)(6\,8\,10\,7\,9),(1\,2\,3\,4\,5)(6\,7\,8\,9\,10),(1\,6)(2\,7)(3\,8)(4\,9)(5\,10),(1\,6\,5\,10\,4\,9\,3\,8\,2\,7),(1\,6\,4\,9\,2\,7\,5\,10\,3\,8),(1\,6\,3\,8\,5\,10\,2\,7\,4\,9),(1\,6\,2\,7\,3\,8\,4\,9\,5\,10),(1\,10\,5\,9\,4\,8\,3\,7\,2\,6),(1\,10\,4\,8\,2\,6\,5\,9\,3\,7),(1\,10\,3\,7\,5\,9\,2\,6\,4\,8),(1\,10\,2\,6\,3\,7\,4\,8\,5\,9)(1\,10),(2\,6)(3\,7)(4\,8)(5\,9),(1\,9\,4\,7\,2\,10\,5\,8\,3\,6),(1\,9\,3\,6\,5\,8\,2\,10\,4\,7),(1\,9\,2\,10\,3\,6\,4\,7\,5\,8),(1\,9)(2\,10)(3\,6)(4\,7)(5\,8),(1\,9\,5\,8\,4\,7\,3\,6\,2\,10),(1\,8\,3\,10\,5\,7\,2\,9\,4\,6),(1\,8\,2\,9\,3\,10\,4\,6\,5\,7),(1\,8)(2\,9)(3\,10)(4\,6)(5\,7),(1\,8\,5\,7\,4\,6\,3\,10\,2\,9),(1\,8\,4\,6\,2\,9\,5\,7\,3\,10),(1\,7\,2\,8\,3\,9\,4\,10\,5\,6),(1\,7)(2\,8)(3\,9)(4\,10)(5\,6),(1\,7\,5\,6\,4\,10\,3\,9\,2\,8),(1\,7\,4\,10\,2\,8\,5\,6\,3\,9),(1\,7\,3\,9\,5\,6\,2\,8\,4\,10)\}$

W_3 is transitive with chief series as $W_3 = H_3 \rhd H_2 \rhd H_1 \rhd H_0 = (1)$ with cyclic factors C_2, C_5 C_2 and C_5

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