Journal of Natural Sciences Research www.iiste.org
ISSN 2224-3186 (Paper) ISSN 2225-0921 (Online)
Vol.8, No.6, 2018

On Radical Groups of Permutation Groups

M. Bello[1] A. B. Umar[2] Mustapha Danjuma[3] Bulus Simon[3]
1.Mathematics and Computer Science Department, Federal University Kashere, Gombe State, Nigeria
2.Department of Mathematical Science, Abubakar Tafawa Balewa University Bauchi, Bauchi State, Nigeria
3.Department of Mathematics and Statistics, Abubakar Tatari Ali Polytechnic, Bauchi State, Nigeria

Abstract
In this paper a new theorem has been stated and proved concerning the radical group of permutation groups. Symmetric groups, alternating groups, dihedral groups and groups generated by semidirect products of two permutation groups have been considered in the research being them as permutation groups.

Introduction
The concept of radical group plays great role in the theory of finite group being it the largest solvable normal subgroup of that group.

Definition 1.1
A subgroup N of a group G is normal in G if the left and right cosets are the same, that is if $gH = Hg \ \forall g \in G$ and a subgroup H of G.
Definition 1.2 (Milne, J.S, 2009)
A group G is solvable if there is a finite collection of groups $G_0 G_1 \ldots G_n$ such that$(1) = G_0 \subseteq G_1 \subseteq \cdots \subseteq G_n = G$ where $G_i \trianglelefteq G$ and $G_{i+1}/G_i$is abelian.If $|G| = 1$ then $G$is considered as solvable group.
Definition 1.3
A radical group of a group G is the largest solvable normal subgroup.
WREATH PRODUCT (Audu M.S, 2003)
The Wreath product of C by D denoted by W = C wr D is the semidirect product of P by D so that$W = \{(fd) \mid f \in Pd \in D\}$ with multiplication in W defined as $(f_1 d_1)(f_2 d_2) = f_1 {f_2}^{{d_1}^{-1}}(d_1 d_2)$ for all $f_1 f_2 \in Pand d_1 d_2 \in D$. Henceforth we write f d instead of (fd) for elements of W.
Theorem 1.1 (Audu M.S, 2003)
Let D act on P as $f^d(\delta) = f(\delta d^{-1})$ where $f \in Pd \in Dand\delta \in \Delta$. Let W be the group of all juxtaposed symbols f d with $f \in Pd \in D$ and multiplication given by $(f_1 d_1)(f_2 d_2) = f_1 {f_2}^{{d_1}^{-1}}(d_1 d_2)$. Then W is a group called the semi-direct product of P by D with the defined action.
Based on the forgoing we note the following:

- If C and D are finite groups then the wreath product W determined by an action of D on a finite set is a finite group of order $|W| = |C|^{|\Delta|}.|D|$.
- P is a normal subgroup of W and D is a subgroup of W.
- The action of W on $\Gamma \times \Delta$ is given by $(\alpha\beta)fd = (\alpha f(\beta)\beta d) where \alpha \in \Gamma and \beta \in \Delta$.

We shall at this point identify the conditions under which a sup group will be soluble or nilpotent and study them for further investigation.
Theorem 1.2 (Thanos G., 2006)
G is solvable if and only if $G^{(n)} = 1$, for some n.

RESULT
Theorem 1.3
Let G be a finite group, then the radical group of G is G itself if G is solvable and identity if G is not solvable.
Proof
Since G is a group then it has a composition series by proposition 2.0. Suppose G is solvable and that its composition series is $(1) = G_1 \leq G_2 \leq \cdots \leq G_n = G$ - - - - - $(*)$
The solvability of G imply that $G_i \trianglelefteq G_{i+1}$. Since $(*)$ is a composition series for G then each subgroup G_n is normal in G, but the largest among them is $G_n = G$ which is solvable being the trivial subgroup of G, implying that G is the radical group of itself.
On the other hand, if G is not solvable then it has no solvable normal subgroup except the trivial subgroup (1) meaning that in this situation the radical group of G is (1).

APPLICATION

2.1 Symmetric groups

2.1.1 Consider the symmetric group acting on $\Omega_1 = \{1,2,3\}$

$S_3 = \{(1),(23),(13),(132),(123),(12)\}$

S_3 is solvable with radical group S_3 it self.

2.1.2 Consider the symmetric group acting on $\Omega_2 = \{1,2,3,4\}$

$S_4 = \{(1),(34),(24),(243),(234),(23),(14),(143),(142),(1432),(1423),(14)(23),(124),(1243),(12),(12)(34),(123),(1234),(134),(13),(1342),(132),(13)(24),(1324)\}$

S_4 is solvable with radical group S_4 it self.

2.1.3 Consider the symmetric group acting on $\Omega_3 = \{1,2,3,4,5\}$

$S_5 = \{(1),(45),(35),(354),(345),(34),(25),(254),(253),(2543),(2534),(25),(34),(235),(2354),(23),(23)(45),(234),(2345),(245),(24),(2453),(243),(24)(35),(2435),(15),(154),(153),(1543),(1534),(15)(34),(152),(1542),(1532),(15432),(15342),(152)(34),(1523),(15423),(15)(23),(154)(23),(15)(234),(15234),(1524),(15)(24),(15324),(15)(243),(153)(24),(15243),(125),(1254),(1253),(12543),(12534),(125)(34),(12),(12)(45),(12)(35),(12)(354),(12)(345),(12)(34),(123),(123)(45),(1235),(12354),(12345),(1234),(124),(1245),(124)(35),(12435),(12453),(1243)(135),(1354),(13),(13)(45),(134),(1345),(1352),(13542),(132),(132)(45),(1342),(13452),(13)(25),(13)(254),(1325),(13254),(13425),(134)(25),(13524),(135)(24),(1324),(13245),(13)(24),(13)(245),(145),(14),(1453),(143),(14)(35),(1435),(1452),(142),(14532),(1432),(142)(35),(14352),(14523),(1423),(145)(23),(14)(23),(14235),(14)(235),(14)(25),(1425),(14)(253),(14325),(14253),(143)(25)\}$

S_5 is not solvable and its radical group (1) .

2.2 Alternating group

2.2.1 Consider the alternating group acting on $\Omega_4 = \{1,2,3\}$

$A_3 = \{(1),(123),(132)\}$

A_3 is solvable with radical group A_3 it self.

2.2.2 Consider the alternating group acting on $\Omega_4 = \{1,2,3,4\}$

$A_4 = \{(1),(243),(234),(143),(14)(23),(142),(134),(132),(13)(24),(124),(12)(34),(123)\}$

A_4 is solvable with radical group A_4 it self.

2.2.3 Consider the alternating group acting on $\Omega_4 = \{1,2,3,4,5\}$

$A_5 = \{(1),(354),(345),(254),(25)(34),(253),(245),(243),(24)(35),(235),(23)(45),(234),(154),(15)(34),(153),(15)(24),(15243),(15324),(152),(15432),(15342),(15234),(15)(23),(15423),(145),(143),(14)(35),(142),(14352),(14532),(14)(25),(14325),(14253),(14523),(14)(23),(14235),(125),(12543),(12534),(12)(45),(12)(34),(12)(35),(124),(12435),(12453),(123),(12354),(12345),(135),(13)(45),(134),(13542),(13452),(132),(13524),(13245),(13)(24),(13)(25),(13254),(13425)\}$

A_5 is not solvable and its radical group (1).

2.3 Dihedral group

2.3.1 Consider the dihedral group acting on $\Omega_4 = \{1,2,3\}$

$D_6 = \{(1),(23),(132),(13),(123),(12)\}$

D_3 is solvable with radical group D_3 it self.

2.3.2 Consider the dihedral group acting on $\Omega_4 = \{1,2,3,4\}$

$D_8 = \{(1),(24),(13)(24),(13),(1432),(14)(23),(1234),(12)(34)\}$

D_8 is solvable with radical group D_8 it self.

2.3.3 Consider the dihedral group acting on $\Omega_4 = \{1,2,3,4,5,6,7\}$

$D_{14} = \{(1),(27)(36)(45),(1765432),(17)(26)(35),(1642753),(16)(25)(34),(1526374),(15)(24)(67),(1473625),(14)(23)(57),(1357246),(13)(47)(56),(1234567),(12)(37)(46)\}$

D_{14} is solvable with radical group D_{14} it self.

2.4 Wreath product

2.4.1 Consider the permutation groups M_1 and L_1

$\boldsymbol{M_1} = \{(1),(123),(132)\}$, $\boldsymbol{L_1} = \{(1),(45)\}$ acting on the sets $S_1 = \{1,2,3\}$ *and* $\Delta_1 = \{4,5\}$ respectively.

Let Let $P = {L_1}^{\Delta_2} = \{f: \Delta_1 \longrightarrow L_1\} then |P| = |M_1|^{\Delta_1} = 3^2 = 9$

We can easily verify that G_1 is a group with respect to the operations

$(f_1 f_2)\delta_1 = f_1(\delta_1) f_2(\delta_1) where\ \delta_1 \in \Delta_1$.

The wreath product of P_1 and Q_1 is given by W_1, where

$W_1 = \{(1), (465), (456), (132), (132)(465), (132)(456), (123), (123)(465), (123)(456), (14)(25)(36), (143625), (142536), (163524), (162435), (16)(24)(35), (152634), (15)(26)(34), (153426)\}$

W_1 is solvable with radical group W_1 it self.

2.4.2 Consider the permutation groups M_2 and L_2

$\boldsymbol{M_2} = \{(1), (15432), (14253), (13524), (12345)\}$

, $\boldsymbol{L_2} = \{(1), (678), (687)\}$ acting on the sets $S_2 = \{1,2,3,4,5\}$ *and* $\Delta_2 = \{6,7,8\}$ respectively.

Let Let $P = {L_2}^{\Delta_2} = \{f{:}\,\Delta_2 \longrightarrow L_2\} then |P| = |M_2|^{\Delta_2} = 5^3 = 125$

We can easily verify that G_1 is a group with respect to the operations

$(f_1 f_2)\delta_1 = f_1(\delta_1) f_2(\delta_1) where\ \delta_1 \in \Delta_1$.

The wreath product of M_2 and L_2 is given by W_2, where

W_2 = {(1), (1115141312), (1114121513)(1113151214)(1112131415)(610 9 8 7)(610 9 8 7)
(1115141312)(610 9 8 7)(1114121513)(610 9 8 7)(1113151214)(610 9 8 7)(1112131415)(6 9 710 8)
(6 9 710 8)(1115141312)(6 9 710 8)(1114121513)(6 9 710 8)(1113151214)(6 9 710 8)
(1112131415)(6 810 7 9)(6 810 7 9)(1115141312)(6 810 7 9)(1114121513)(6 810 7 9)
(1113151214)(6 810 7 9)(1112131415)(6 7 8 910)(6 7 8 910)(1115141312)(6 7 8 910)
(1114121513)(6 7 8 910)(1113151214)(6 7 8 910)(1112131415)(15432)(1 5 4 3 2)
(1115141312)(1 5 4 3 2)(1114121513)(1 5 4 3 2)(1113151214)(1 5 4 3 2)(1112131415)
(1 5 4 3 2)(610 9 8 7)(1 5 4 3 2)(610 9 8 7)(1115141312)(1 5 4 3 2)(610 9 8 7)
(1114121513)(1 5 4 3 2)(610 9 8 7)(1113151214)(1 5 4 3 2)(610 9 8 7)(1112131415)
(1 5 4 3 2)(6 9 710 8)(1 5 4 3 2)(6 9 710 8)(1115141312)(1 5 4 3 2)(6 9 710 8)
(1114121513)(1 5 4 3 2)(6 9 710 8)(1113151214)(1 5 4 3 2)(6 9 710 8)(1112131415)
(1 5 4 3 2)(6 810 7 9)(1 5 4 3 2)(6 810 7 9)(1115141312)(1 5 4 3 2)(6 810 7 9)
(1114121513)(1 5 4 3 2)(6 810 7 9)(1113151214)(1 5 4 3 2)(6 810 7 9)(1112131415)
(1 5 4 3 2)(6 7 8 910)(1 5 4 3 2)(6 7 8 910)(1115141312)(1 5 4 3 2)(6 7 8 910)
(1114121513)(1 5 4 3 2)(6 7 8 910)(1113151214)(1 5 4 3 2)(6 7 8 910)(1112131415)
(14253)(1 4 2 5 3)(1115141312)(1 4 2 5 3)(1114121513)(1 4 2 5 3)(1113151214)
(1 4 2 5 3)(1112131415)(1 4 2 5 3)(610 9 8 7)(1 4 2 5 3)(610 9 8 7)(1115141312)
(1 4 2 5 3)(610 9 8 7)(1114121513)(1 4 2 5 3)(610 9 8 7)(1113151214)(1 4 2 5 3)
(610 9 8 7)(1112131415)(1 4 2 5 3)(6 9 710 8)(1 4 2 5 3)(6 9 710 8)(1115141312)
(1 4 2 5 3)(6 9 710 8)(1114121513)(1 4 2 5 3)(6 9 710 8)(1113151214)(1 4 2 5 3)
(6 9 710 8)(1112131415)(1 4 2 5 3)(6 810 7 9)(1 4 2 5 3)(6 810 7 9)(1115141312)
(1 4 2 5 3)(6 810 7 9)(1114121513)(1 4 2 5 3)(6 810 7 9)(1113151214)(1 4 2 5 3)
(6 810 7 9)(1112131415)(1 4 2 5 3)(6 7 8 910)(1 4 2 5 3)(6 7 8 910)(1115141312)
(1 4 2 5 3)(6 7 8 910)(1114121513)(1 4 2 5 3)(6 7 8 910)(1113151214)(1 4 2 5 3)
(6 7 8 910)(1112131415)(13524)(1 3 5 2 4)(1115141312)(1 3 5 2 4)(1114121513)
(1 3 5 2 4)(1113151214)(1 3 5 2 4)(1112131415)(1 3 5 2 4)(610 9 8 7)(1 3 5 2 4)
(610 9 8 7)(1115141312)(1 3 5 2 4)(610 9 8 7)(1114121513)(1 3 5 2 4)(610 9 8 7)
(1113151214)(1 3 5 2 4)(610 9 8 7)(1112131415)(1 3 5 2 4)(6 9 710 8)(1 3 5 2 4)
(6 9 710 8)(1115141312)(1 3 5 2 4)(6 9 710 8)(1114121513)(1 3 5 2 4)(6 9 710 8)
(1113151214)(1 3 5 2 4)(6 9 710 8)(1112131415)(1 3 5 2 4)(6 810 7 9)(1 3 5 2 4)
(6 810 7 9)(1115141312)(1 3 5 2 4)(6 810 7 9)(1114121513)(1 3 5 2 4)(6 810 7 9)
(1113151214)(1 3 5 2 4)(6 810 7 9)(1112131415)(1 3 5 2 4)(6 7 8 910)(1 3 5 2 4)
(6 7 8 910)(1115141312)(1 3 5 2 4)(6 7 8 910)(1114121513)(1 3 5 2 4)(6 7 8 910)
(1113151214)(1 3 5 2 4)(6 7 8 910)(1112131415)(12345)(1 2 3 4 5)(1115141312)
(1 2 3 4 5)(1114121513)(1 2 3 4 5)(1113151214)(1 2 3 4 5)(1112131415)(1 2 3 4 5)
(610 9 8 7)(1 2 3 4 5)(610 9 8 7)(1115141312)(1 2 3 4 5)(610 9 8 7)(1114121513)
(1 2 3 4 5)(610 9 8 7)(1113151214)(1 2 3 4 5)(610 9 8 7)(1112131415)(1 2 3 4 5)
(6 9 710 8)(1 2 3 4 5)(6 9 710 8)(1115141312)(1 2 3 4 5)(6 9 710 8)(1114121513)
(1 2 3 4 5)(6 9 710 8)(1113151214)(1 2 3 4 5)(6 9 710 8)(1112131415)(1 2 3 4 5)
(6 810 7 9)(1 2 3 4 5)(6 810 7 9)(1115141312)(1 2 3 4 5)(6 810 7 9)(1114121513)
(1 2 3 4 5)(6 810 7 9)(1113151214)(1 2 3 4 5)(6 810 7 9)(1112131415)(1 2 3 4 5)
(6 7 8 910)(1 2 3 4 5)(6 7 8 910)(1115141312)(1 2 3 4 5)(6 7 8 910)(1114121513)
(1 2 3 4 5)(6 7 8 910)(1113151214)(1 2 3 4 5)(6 7 8 910)(1112131415)(111 6)(212 7)
(313 8)(414 9)(51510)(11110 515 9 414 8 313 7 212 6)(111 9 414 7 21210 515 8 3
13 6)(111 8 31310 515 7 212 9 414 6)(111 7 212 8 313 9 41410 515 6)(111 6 51510
414 9 313 8 212 7)(11110 414 8 212 6 515 9 313 7)(111 9 313 6 515 8 21210 414 7
)(111 8 212 9 31310 414 6 515 7)(111 7)(212 8)(313 9)(41410)(515 6)(111 6 414 9
212 7 51510 313 8)(11110 313 7 515 9 212 6 414 8)(111 9 21210 313 6 414 7 515

8)(111 8)(212 9)(31310)(414 6)(515 7)(111 7 515 6 41410 313 9 212 8)(111 6 313 8 51510 212 7 414 9)(11110 212 6 313 7 414 8 515 9)(111 9)(21210)(313 6)(414 7) (515 8)(111 8 515 7 414 6 31310 212 9)(111 7 41410 212 8 515 6 313 9)(111 6 212 7 313 8 414 9 51510)(11110)(212 6)(313 7)(414 8)(515 9)(111 9 515 8 414 7 313 6 21210)(111 8 414 6 212 9 515 7 31310)(111 7 313 9 515 6 212 8 41410)(11510 514 9 413 8 312 7 211 6)(115 9 413 7 21110 514 8 312 6)(115 8 31210 514 7 211 9 4 13 6)(115 7 211 8 312 9 41310 514 6)(115 6)(211 7)(312 8)(413 9)(51410) (11510 413 8 211 6 514 9 312 7)(115 9 312 6 514 8 21110 413 7)(115 8 211 9 312 10 413 6 514 7)(115 7)(211 8)(312 9)(41310)(514 6)(115 6 51410 413 9 312 8 211 7) (11510 312 7 514 9 211 6 413 8)(115 9 21110 312 6 413 7 514 8)(115 8)(211 9) (31210)(413 6)(514 7)(115 7 514 6 41310 312 9 211 8)(115 6 413 9 211 7 51410 3 12 8)(11510 211 6 312 7 413 8 514 9)(115 9)(21110)(312 6)(413 7)(514 8) (115 8 514 7 413 6 31210 211 9)(115 7 41310 211 8 514 6 312 9)(115 6 312 8 514 10 211 7 413 9)(11510)(211 6)(312 7)(413 8)(514 9)(115 9 514 8 413 7 312 6 21110) (115 8 413 6 211 9 514 7 31210)(115 7 312 9 514 6 211 8 41310)(115 6 211 7 312 8 413 9 51410)(114 9 412 7 21510 513 8 311 6)(114 8 31110 513 7 215 9 412 6) (114 7 215 8 311 9 41210 513 6)(114 6)(215 7)(311 8)(412 9)(51310)(11410 513 9 4 12 8 311 7 215 6)(114 9 311 6 513 8 21510 412 7)(114 8 215 9 31110 412 6 513 7) (114 7)(215 8)(311 9)(41210)(513 6)(114 6 51310 412 9 311 8 215 7)(11410 412 8 2 15 6 513 9 311 7)(114 9 21510 311 6 412 7 513 8)(114 8)(215 9)(31110)(412 6) (513 7)(114 7 513 6 41210 311 9 215 8)(114 6 412 9 215 7 51310 311 8)(11410 311 7 513 9 215 6 412 8)(114 9)(21510)(311 6)(412 7)(513 8)(114 8 513 7 412 6 31110 215 9)(114 7 41210 215 8 513 6 311 9)(114 6 311 8 51310 215 7 412 9)(11410 215 6 311 7 412 8 513 9)(114 9 513 8 412 7 311 6 21510)(114 8 412 6 215 9 513 7 3 1110)(114 7 311 9 513 6 215 8 41210)(114 6 215 7 311 8 412 9 51310)(11410) (215 6)(311 7)(412 8)(513 9)(113 8 31510 512 7 214 9 411 6)(113 7 214 8 315 9 4 1110 512 6)(113 6)(214 7)(315 8)(411 9)(51210)(11310 512 9 411 8 315 7 214 6) (113 9 411 7 21410 512 8 315 6)(113 8 214 9 31510 411 6 512 7)(113 7)(214 8) (315 9)(41110)(512 6)(113 6 51210 411 9 315 8 214 7)(11310 411 8 214 6 512 9 3 15 7)(113 9 315 6 512 8 21410 411 7)(113 8)(214 9)(31510)(411 6)(512 7) (113 7 512 6 41110 315 9 214 8)(113 6 411 9 214 7 51210 315 8)(11310 315 7 512 9 214 6 411 8)(113 9 21410 315 6 411 7 512 8)(113 8 512 7 411 6 31510 214 9) (113 7 41110 214 8 512 6 315 9)(113 6 315 8 51210 214 7 411 9)(11310 214 6 315 7 411 8 512 9)(113 9)(21410)(315 6)(411 7)(512 8)(113 8 411 6 214 9 512 7 31510) (113 7 315 9 512 6 214 8 41110)(113 6 214 7 315 8 411 9 51210)(11310)(214 6) (315 7)(411 8)(512 9)(113 9 512 8 411 7 315 6 21410)(112 7 213 8 314 9 41510 5 11 6)(112 6)(213 7)(314 8)(415 9)(51110)(11210 511 9 415 8 314 7 213 6) (112 9 415 7 21310 511 8 314 6)(112 8 31410 511 7 213 9 415 6)(112 7)(213 8) (314 9)(41510)(511 6)(112 6 51110 415 9 314 8 213 7)(11210 415 8 213 6 511 9 3 14 7)(112 9 314 6 511 8 21310 415 7)(112 8 213 9 31410 415 6 511 7)(112 7 511 6 41510 314 9 213 8)(112 6 415 9 213 7 51110 314 8)(11210 314 7 511 9 213 6 415 8)(112 9 21310 314 6 415 7 511 8)(112 8)(213 9)(31410)(415 6)(511 7)(112 7 41510 213 8 511 6 314 9)(112 6 314 8 51110 213 7 415 9)(11210 213 6 314 7 415 8 511 9)(112 9)(21310)(314 6)(415 7)(511 8)(112 8 511 7 415 6 31410 213 9)(112 7 314 9 511 6 213 8 41510)(112 6 213 7 314 8 415 9 51110)(11210)(213 6)(314 7)(415 8) (511 9)(112 9 511 8 415 7 314 6 21310)(112 8 415 6 213 9 511 7 31410)(1 611) (2 712)(3 813)(4 914)(51015)(1 611 51015 4 914 3 813 2 712)(1 611 4 914 2 712 5 1015 3 813)(1 611 3 813 51015 2 712 4 914)(1 611 2 712 3 813 4 914 51015) (1 615 51014 4 913 3 812 2 711)(1 615 4 913 2 711 51014 3 812)(1 615 3 812 510 14 2 711 4 913)(1 615 2 711 3 812 4 913 51014)(1 615)(2 711)(3 812)(4 913)(51014) (1 614 4 912 2 715 51013 3 811)(1 614 3 811 51013 2 715 4 912)(1 614 2 715 3 8 11 4 912 51013)(1 614)(2 715)(3 811)(4 912)(51013)(1 614 51013 4 912 3 811 2 715) (1 613 3 815 51012 2 714 4 911)(1 613 2 714 3 815 4 911 51012)(1 613)(2 714) (3 815)(4 911)(51012)(1 613 51012 4 911 3 815 2 714)(1 613 4 911 2 714 51012 3 815)(1 612 2 713 3 814 4 915 51011)(1 612)(2 713)(3 814)(4 915)(51011) (1 612 51011 4 915 3 814 2 713)(1 612 4 915 2 713 51011 3 814)(1 612 3 814 510 11 2 713 4 915)(11015 5 914 4 813 3 712 2 611)(11015 4 813 2 611 5 914 3 712) (11015 3 712 5 914 2 611 4 813)(11015 2 611 3 712 4 813 5 914)(11015)(2 611)

(3 712)(4 813)(5 914)(11014 4 812 2 615 5 913 3 711)(11014 3 711 5 913 2 615 4
812)(11014 2 615 3 711 4 812 5 913)(11014)(2 615)(3 711)(4 812)(5 913)
(11014 5 913 4 812 3 711 2 615)(11013 3 715 5 912 2 614 4 811)(11013 2 614 3 7
15 4 811 5 912)(11013)(2 614)(3 715)(4 811)(5 912)(11013 5 912 4 811 3 715 2 614)
(11013 4 811 2 614 5 912 3 715)(11012 2 613 3 714 4 815 5 911)(11012)(2 613)
(3 714)(4 815)(5 911)(11012 5 911 4 815 3 714 2 613)(11012 4 815 2 613 5 911 3
714)(11012 3 714 5 911 2 613 4 815)(11011)(2 612)(3 713)(4 814)(5 915)
(11011 5 915 4 814 3 713 2 612)(11011 4 814 2 612 5 915 3 713)(11011 3 713 5 9
15 2 612 4 814)(11011 2 612 3 713 4 814 5 915)(1 914 4 712 21015 5 813 3 611)
(1 914 3 611 5 813 21015 4 712)(1 914 21015 3 611 4 712 5 813)(1 914)(21015)
(3 611)(4 712)(5 813)(1 914 5 813 4 712 3 611 21015)(1 913 3 615 5 812 21014 4
711)(1 913 21014 3 615 4 711 5 812)(1 913)(21014)(3 615)(4 711)(5 812)
(1 913 5 812 4 711 3 615 21014)(1 913 4 711 21014 5 812 3 615)(1 912 21013 3 6
14 4 715 5 811)(1 912)(21013)(3 614)(4 715)(5 811)(1 912 5 811 4 715 3 614 21013)
(1 912 4 715 21013 5 811 3 614)(1 912 3 614 5 811 21013 4 715)(1 911)(21012)
(3 613)(4 714)(5 815)(1 911 5 815 4 714 3 613 21012)(1 911 4 714 21012 5 815 3
613)(1 911 3 613 5 815 21012 4 714)(1 911 21012 3 613 4 714 5 815)(1 915 5 814
4 713 3 612 21011)(1 915 4 713 21011 5 814 3 612)(1 915 3 612 5 814 21011 4 713
)(1 915 21011 3 612 4 713 5 814)(1 915)(21011)(3 612)(4 713)(5 814)(1 813 31015
5 712 2 914 4 611)(1 813 2 914 31015 4 611 5 712)(1 813)(2 914)(31015)(4 611)
(5 712)(1 813 5 712 4 611 31015 2 914)(1 813 4 611 2 914 5 712 31015)(1 812 2 9
13 31014 4 615 5 711)(1 812)(2 913)(31014)(4 615)(5 711)(1 812 5 711 4 615 31014
2 913)(1 812 4 615 2 913 5 711 31014)(1 812 31014 5 711 2 913 4 615)(1 811)
(2 912)(31013)(4 614)(5 715)(1 811 5 715 4 614 31013 2 912)(1 811 4 614 2 912 5
715 31013)(1 811 31013 5 715 2 912 4 614)(1 811 2 912 31013 4 614 5 715)
(1 815 5 714 4 613 31012 2 911)(1 815 4 613 2 911 5 714 31012)(1 815 31012 5 7
14 2 911 4 613)(1 815 2 911 31012 4 613 5 714)(1 815)(2 911)(31012)(4 613)(5 714)
(1 814 4 612 2 915 5 713 31011)(1 814 31011 5 713 2 915 4 612)(1 814 2 915 310
11 4 612 5 713)(1 814)(2 915)(31011)(4 612)(5 713)(1 814 5 713 4 612 31011 2 915)
(1 712 2 813 3 914 41015 5 611)(1 712)(2 813)(3 914)(41015)(5 611)(1 712 5 611 4
1015 3 914 2 813)(1 712 41015 2 813 5 611 3 914)(1 712 3 914 5 611 2 813 41015)
(1 711)(2 812)(3 913)(41014)(5 615)(1 711 5 615 41014 3 913 2 812)(1 711 41014 2
812 5 615 3 913)(1 711 3 913 5 615 2 812 41014)(1 711 2 812 3 913 41014 5 615)
(1 715 5 614 41013 3 912 2 811)(1 715 41013 2 811 5 614 3 912)(1 715 3 912 5 6
14 2 811 41013)(1 715 2 811 3 912 41013 5 614)(1 715)(2 811)(3 912)(41013)(5 614)
(1 714 41012 2 815 5 613 3 911)(1 714 3 911 5 613 2 815 41012)(1 714 2 815 3 9
11 41012 5 613)(1 714)(2 815)(3 911)(41012)(5 613)(1 714 5 613 41012 3 911 2 815)
(1 713 3 915 5 612 2 814 41011)(1 713 2 814 3 915 41011 5 612)(1 713)(2 814)
(3 915)(41011)(5 612)(1 713 5 612 41011 3 915 2 814)(1 713 41011 2 814 5 612 3
915)}

W_2 is solvable with radical group W_2 it self.

3.1 VALIDATION

3.1.1 Algorithm for the result in 2.1.1

```
gap> S3:=SymmetricGroup(3);
Sym( [ 1 .. 3 ] )
gap> RadicalGroup(S3);
Sym( [ 1 .. 3 ] )
gap>quit;
```

3.1.2 Algorithm for the result in 2.1.2

```
gap> S4:=SymmetricGroup(4);
Sym( [ 1 .. 4 ] )
gap> RadicalGroup(S4);
Sym( [ 1 .. 4 ] )
gap>quit;
```

3.1.3 Algorithm for the result in 2.1.3

```
gap> S5:=SymmetricGroup(5);
Sym( [ 1 .. 5 ] )
```

```
gap> RadicalGroup(S5);
Group(())
gap>quit;
```

3.1.4 Algorithm for the result in 2.2.1

```
gap> A3:=AlternatingGroup(3);
Alt( [ 1 .. 3 ] )
gap> RadicalGroup(A3);
Alt( [ 1 .. 3 ] )
gap>quit;
```

3.1.5 Algorithm for the result in 2.2.2

```
gap> A4:=AlternatingGroup(4);
Alt( [ 1 .. 4 ] )
gap> RadicalGroup(A4);
Alt( [ 1 .. 4 ] )
gap>quit;
```

3.1.6 Algorithm for the result in 2.2.3

```
gap> A5:=AlternatingGroup(5);
Alt( [ 1 .. 5 ] )
gap> RadicalGroup(A5);
Group(())
gap>quit;
```

3.1.7 Algorithm for the result in 2.3.1

```
gap> D6:=DihedralGroup(IsGroup,6);
Group([ (1,2,3), (2,3) ])
gap> RadicalGroup(D6);
Group([ (1,2,3), (2,3) ])
gap>quit;
```

3.1.8 Algorithm for the result in 2.3.2

```
gap> D8:=DihedralGroup(IsGroup,8);
Group([ (1,2,3,4), (2,4) ])
gap> RadicalGroup(D8);
Group([ (1,2,3,4), (2,4) ])
gap>quit;
```

3.1.9 Algorithm for the result in 2.3.3

```
gap> D14:=DihedralGroup(IsGroup,14);
Group([ (1,2,3,4,5,6,7), (2,7)(3,6)(4,5) ])
gap> RadicalGroup(D14);
Group([ (1,2,3,4,5,6,7), (2,7)(3,6)(4,5) ])
gap>quit;
```

3.1.10 Algorithm for the result in 2.4.1

```
gap> M1:=Group((1,2,3));
Group([ (1,2,3) ])
gap> L1:=Group((4,5));
Group([ (4,5) ])
gap> W1:=WreathProduct(M1,L1);
Group([ (1,2,3), (4,5,6), (1,4)(2,5)(3,6) ])
gap> RadicalGroup(W1);
Group([ (1,2,3), (4,5,6), (1,4)(2,5)(3,6) ])
gap>quit;
```

3.1.11 Algorithm for the result in 2.4.2

```
gap> M2:=Group((1,2,3,4,5));
Group([ (1,2,3,4,5) ])
gap> L2:=Group((6,7,8));
Group([ (6,7,8) ])
gap> W2:=WreathProduct(M2,L2);
Group([ (1,2,3,4,5), (6,7,8,9,10), (11,12,13,14,15), (1,6,11)(2,7,12)(3,8,13)(4,9,14)(5,10,15) ])
gap> RadicalGroup(W2);
Group([ (1,2,3,4,5), (6,7,8,9,10), (11,12,13,14,15), (1,6,11)(2,7,12)(3,8,13)(4,9,14)(5,10,15) ])
gap>quit;
```

Reference

1. Audu M.S., K.E Osondu, A.R.j. Solarin (2003),"**Research Seminar on Groups, Semi Groups and Loops**," National Mathematical Centre, Abuja, Nigeria (October).
2. Hall M. Jr. (1959)," **The Theory of Groups**" Macmillan Company New York.
3. Hielandt, H. (1964)," **Finite Permutation Groups**", Academic Press, New York, London.
4. Thanos G. (2006), " **Solvable Groups-A numerical Approach**"
5. Joachim Neubüser et al(2016), **"Groups Algorithm and Programming"**, 4.8.5, 25-Sep-2016, build of 2016-09-25 14:51:12 (GMTDT)
6. Kurosh A.G. (1956), "**The theory of groups**" , 1–2 , Chelsea (1955–1956) (Translated from Russian) M. 9.
7. M. Bello et al. (2017), "a numerical search for polycyclic and locally nilpotent permutation groups", International Journal of scientific and engineering research, Vol. 8, issue 8, 2017.
8. M. Bello et al. (2017), "**construction of locally solvable groups using two permutation groups by wreath products**", ABACUS Journal of Mathematical Association of Nigeria.
9. Milne J.S. (2009)," **Theory of groups**" September 21, 2009
10. William Burnside (2012)''**Theory of Groups of Finite order''**Ebook$40395.