Simulation and Modelling of Maximum and Minimum Temperature of Karachi

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Abstract
In the recent past weather derivatives have developed significant interest in surface air temperature and its modeling. In this work, we modelled and simulated daily average temperature. We used three different methods to model and simulate the recorded data at Karachi airport from 2010-2014. The methods used in this work are moving average method, polynomial fitting and maximum entropy method. Both polynomial fitting and maximum entropy methods can be used to find missing data. The simulated graphs and graphs of recorded data are in good agreement.
Keywords: Karachi airport, surface air temperature, moving average, polynomial curve fitting. Maximum Entropy method.

1.1. Introduction:

Weather events and prediction are interesting topics it studied by meteorologists since a long time even employed a variety of model. People want to know what the weather prediction is for now, forthcoming days, and for other interim of time. Awareness of weather condition helps to plan some common activities such as every day traveling to work or school and make financial arrangements energy resources. Since weather is such an integral part of people's lives, one has to know whether there is a pattern that decides when these events happen and how serious they might be, for this, Climate model serve as fundamental tool of climate research, their simulation are able to remake the recent climate, the variability of present climate conditions, and in addition, to give climate scenarios for the future. This presumption of normality is critical, the supposition of normality of the data is required. [1-7].

To study the behavior of climate in different events, researchers have proposed different models such as, Johnson et al. supported the idea of normaility of the temperature distribution[8]. Hansen and Dariscoll developed a condition, which, if explained numerically, changes an ordinary appropriation into one with indicated skewness[9]. Smith et al. suggested the adjustment in neighborhood atmosphere is additionally exhibited by significant environmental changes[10]. Turner et al. have fitted linear tend models to average monthly surface temperatures at different chose Antarctic stations from the datasets accessible from each station. Most of these had warming patterns and some had cooling patterns (one station had next to no information and consequently was overlooked for their examination), however the most measurably critical increment was found at the Faraday/Vernadsky station. The Faraday/Vernadsky station is important to us for two reasons; firstly on the grounds that it has the longest ceaseless record contrasted with some other station, and furthermore in lights of the fact that it has both average monthly temperatures and furthermore least/most extreme month to month temperatures[11]. G¨onc¨u. proposed a seasonal unpredictability model that evaluate daily mean temperatures of Beijing, Shanghai and Shenzhen using the mean-reverting Ornstein-Uhlenbeck process and then derive systematic approximation formulas for the sensitivities of these agreements[12]. Wimmer tested the forecasting temperature[13]. Stone, D. A., & Weaver, A. J. found the effect of climate and suggested the importance of climate for simulation[14]. Zhang, X., Alexander, L., Hegerl, G. C., Jones, P., Tank, A. K., Peterson, T. C., & Zwiers, F. W. reviewed that the accessibility of new variables such as moist enthalpy or apparent temperature which is be required both temperature and humidity data[15]. El Kenawy et all used 47 years daily data to study the spatial and temporal variability of temperature in norther spain [16]. Deryugina, T., & Hsiang, S. M. found that the single environmental parameter is playing an important role overall economic performance: productivity of each days drops and increase in daily average temperature. Weekday estimation costs an average country $20 per person. Hot weekends have slightly effected[17]. Liu, B., Xu, M., Henderson, M., Qi, Y., & Li, Y. concluded that the cloud cover and precipitation is not the primary issues of the decreases in the daily mean temperature in china over the study period, Atmospheric aerosols, particularly sulfur aerosols, represent a possible cause for the decrease in solar irradiance[18]. Chu, J. T., Xia, J., Xu, C. Y., & Singh, V. P. analyzed that the selection of model under the strong impact of monsoon and complex climate condition is a big challenge[19].

1.2. Weather Characteristics of Karachi:

Karachi is located on the shores of the Arabian Sea its tends to have moderate climate due to marine affects, Normally weather of Karachi is very pleasant and beautiful, usually we have four(4) types of season but normally in Karachi we observed six(6) types of season which are describing below:

Season	Time	Minimum temperature	Maximum temperature	Remarks
Winter	Decmeber to Feburary	10	24	Cold
Spring	March to Mid April	22	35	Humid
First Summer	April and May	30	40	Hottest weather
Moon soon	June to Septemebr	26	33	Drizzle, Rainy and Humid
Second Summer	September to October	23.65	34.4	Clear cloud, sudden hot, some heat waves
Autumn	October to Novemeber	12.8	31.8	Cool nights

2. Theory and Methodology:

Various models including simulation through moving average, polynomial curve fitting and Entropy principal are tested for the said data and some suitable model suggested for surface air temperature for Karachi city. Five years (Jan 1, 2010 to Dec 31, 2014) of surface temperature data has been obtained from Meteorological department Karachi. The temperature data are composed of daily observations that are taken from location Karachi Airport. Using MATLAB (matrix laboratory) is a multi-paradigm numerical computing environment and "Pfourth-generation programming language and EXCEL, the daily mean temperatures was analyzed. These daily mean temperatures was then be plotted. We fitted linear trends model by using least square method on the raw daily mean temperature:

$$T_i = \rho + \sigma t_i + \epsilon_i$$

In the above equation "T_i" is temperature of specific year, t_i is the specific year "ρ" is the intercept of the linear trend and calculating by

$$\rho = \bar{T} - b\bar{t}$$

Whereas $\bar{x}$ and $\bar{y}$ is average of the variable.
and "σ" is giving the change and can be determined form

$$\sigma = \frac{n\sum t_i T_i - (\sum t_i)(\sum T_i)}{n\sum t_i^2 - (\sum t_i)^2}$$

Whereas "n" reperesent the size "T_i" is temperature of specific year, "t_i" is the specific year

Or $\sigma = \frac{\Delta y}{\Delta x}$

$$\epsilon_i = (T_{i-}\hat{T}_i)^2$$

Where "ϵ_i" is error, "T_i" equals the actual values, "$\hat{T}_i$" equals the values forecast by the model.
Significance of trend has been judged by using R^2 and it is calculated by

$$r = \frac{n\sum t_i T_i - (\sum t_i)(\sum T_i)}{\sqrt{[n\sum t_i^2 - (\sum t_i)^2][n\sum T_i^2 - (\sum T_i)^2]}}$$

It defines the relationship between the temperature and year, further we square the coefficient of correlation then we get the coefficient of determination (R^2) which describe how well trend represent the data and suitable to fit on the data.

2.1. Moving Average:

We used the technique of moving average (Running average) on the data (Moving average is an average of observation from several consecutive time periods. To compute moving average sequence, we compute successive averages of a given number of consecutive observations

$$\bar{M}(N) = \hat{t_n} = \frac{(t_n + t_{n-1} + t_{n-2} + \cdots t_{n-k+1})}{N}$$

$$\bar{M}(i) = \frac{1}{N}\sum_{J=i-(N-1)/2}^{i+(N-1)/2} t_J$$

Where N is the number of order in the moving average.

2.2. Polynomial curve fitting method:

We used the polynomial curve fitting on data form 2010-2014 on extreme temperature
A polynomial of degree "n" is a function of the form

$$f(t) = a_n t^n + a_{n-1}t^{n-1} + a_{n-2}t^{n-2} + \cdots + a_2 t^2 + a_1 t + a_0 = \sum_{k=0}^{n} a_k t^k$$

Where a_n, a_{n-1} ... a_1 is the coeffient of degree of polynomial ascending order and a_0 is the constant of polynomial.

2.3. Entropy Process:
As our next proposed model is Entropy principal, the entropy of a probability density function $f(x)$ as

$$W = -\int f(x) ln f(x) dx \qquad (1)$$

Maximizing "W" with some constraints.
Subject to $E\{\varphi_n(x)\} = \int \varphi_n(x) f(x) dx = \vartheta_n, \qquad n = 0, \dots, N \qquad (2)$
Where $\varphi_n(x) = x^n, n = 0, \dots, N$ and $\vartheta_n, n = 0, \dots, N$ are N moments.
$f(x) takes\ the\ form$

$$f(x) = e^{(-\sum_{n=0}^{N} \beta_n \varphi_n(x))} \qquad (3)$$

The $(N+1)$ Lagrangian parameters $\beta = [\beta_0, \dots, \beta_n]$ are obtained by $(N+1)$ nonlinear equations

$$Q_n(\beta) = \int \varphi_n(x) e^{(-\sum_{n=0}^{N} \beta_n \varphi_n(x))} dx = \vartheta_n, \qquad n = 0, \dots, N \quad (4)$$

Equation (3) gives various distribution, the one that maximizes W is obtained through moment constraints.
In this study we developed a program written in "PYTHON" to fit the distribution.

3. Result and discussion:
In this study we have used temperature data recorded at Quaid-e-Azam international Airport from 2010-2014, however the data is incomplete as some data for some months is missing. We extracted daily maximum and minimum temperature for each month and each year. We analyzed and modelled the maximum and minimum temperature, we used three different methods namely,

i- Moving Average method.
ii- Polynomial Curve fitting method.
iii- Maximum Entropy Principal method.

In the following section each of them is explained and the corresponding result is given and displayed in the form of graphs and tables.

For the moving average method, we tested various orders of moving average. It is found that 3rd order moving average fits well on recorded temperature data. The graphs of recorded data along with 3rd order moving average are given in the figure 3.1.1 – 3.1.5 for maximum temperature and 3.1.6 – 3.1.10 for minimum temperature.

The moving average is good in showing the variation of the recorded data of maximum and minimum temperature but fails to give any mathematical model, so we used 3rd order moving average for qualitative analysis only.

The second method is curve fitting method. In this method we fitted various degree of polynomial to model the maximum and minimum temperature. In general we saw that 6th degree polynomial fits well on both maximum and minimum recorded temperature, see the graphs 3.2.1 - 3.2.5 for maximum temperature and 3.2.6 – 3.2.10 for minimum temperature. The month-wise details of the polynomials for the years 2010 - 2014 are given in table 3.2.1 - 3.2.5 for maximum temperature and 3.2.6 - 3.2.10 for minimum temperature. This method is helpful in predicting any missing data and also useful for curves of subsequent data.

Our last used method is Maximum Entropy Principal Method. In this method we used methods of Maximum Entropy Principal, as defined in 2.3. The maximum and minimum recorded temperature were simulated using this method. The simulated data fits very well on the recorded maximum and minimum temperature data, except for a few cases where a slight deviation can be seen. The graphs given in figures 3.3.1 - 3.3.4 and 3.3.5 – 3.3.8 show the simulated and recorded data of the maximum and minimum temperature repectively. The details of coefficient of maximum entropy principal are shown in table 3.3.1 for maximum temperature and table 3.3.2 for minimum temperature.

3.1.a. Simulation of maximum recorded temperature data (January-December 2010-2014)
The simulated graphs using 3rd order moving average for maximum temperature are shown below. In each graph blue curve represents the maximum recorded temperature (January-December 2010-2014) and red curve represents the 3rd order moving average max. temperature (January-December 2010-2014).

Figure 3.1.1. 3rd order Moving average on maximum recorded temperature data (January-December 2010)

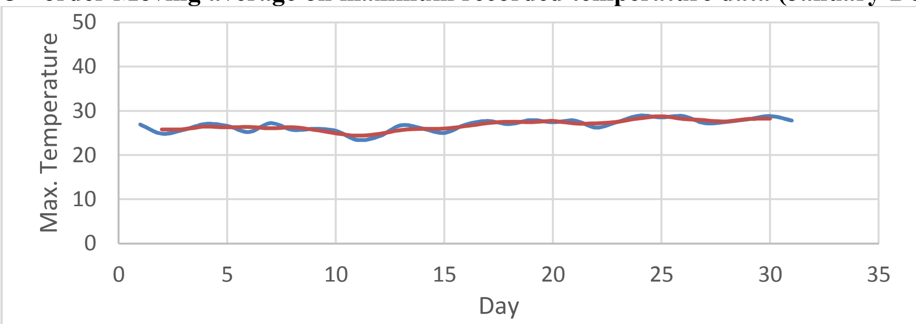

Figure.3.1.1. 3rd order Moving average on recorded temperature data (January) for 2010

Figure 3.1.2. 3rd order Moving average on maximum recorded temperature data (January-December 2011)

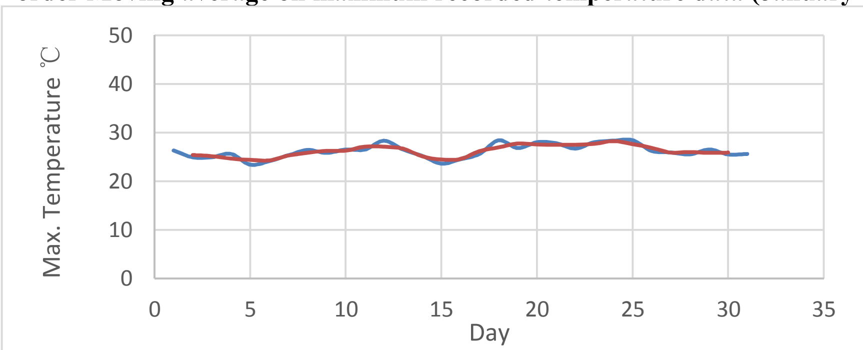

Figure.3.1.2. 3rd order Moving average on recorded temperature (January) for 2011

Figure 3.1.3. 3rd order Moving average on maximum recorded temperature data (January-December 2012)

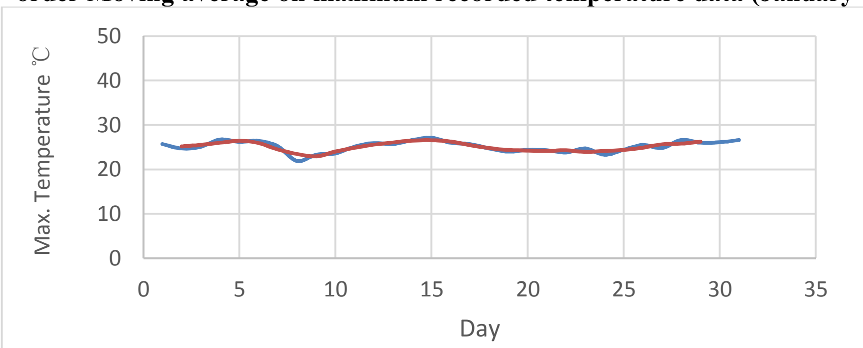

Figure.3.1.3. 3rd order Moving average on recorded temperature (January) for 2012

Figure 3.1.4. 3rd order Moving average on maximum recorded temperature data (January-December 2013)

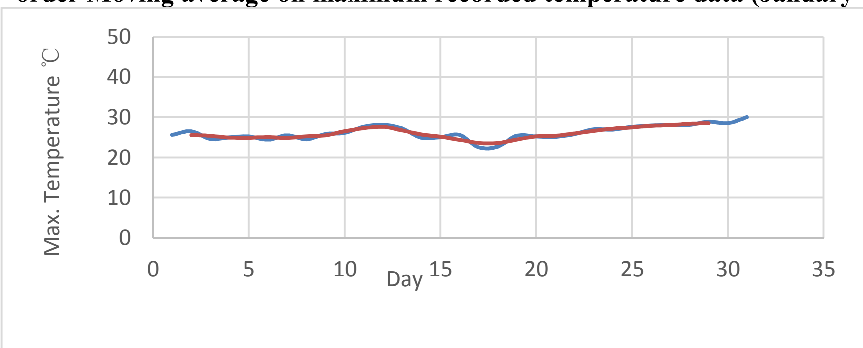

Figure.3.1.4. 3rd order Moving average on recorded temperature (January) for 2013

Figure 3.1.5. 3rd order Moving average on maximum recorded temperature data (January-December 2014)

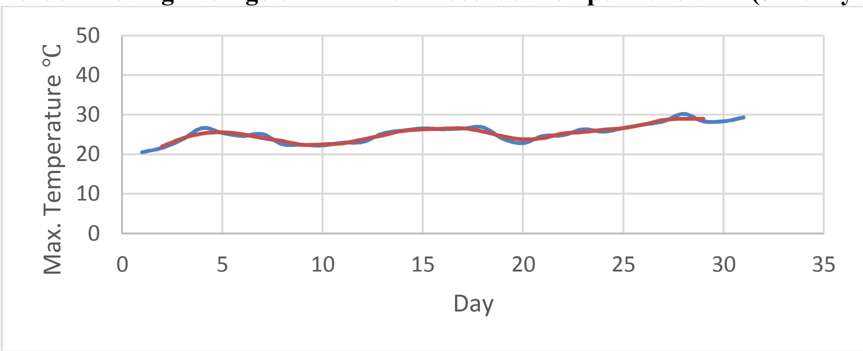

Figure.3.1.5. 3rd order Moving average on recorded temperature (January) for 2014

3.1.b. Simulation of minimum recorded temperature data (January-December 2010-2014)

The simulated graphs using 3rd order moving average for minimum temperature are shown below. In each graph blue curve represents the minimum recorded temperature (January-December 2010-2014) and red curve represents the 3rd order moving average min. temperature (January-December 2010-2014).

Figure 3.1.6. 3rd order Moving average on minimum recorded temperature data (January-December 2010)

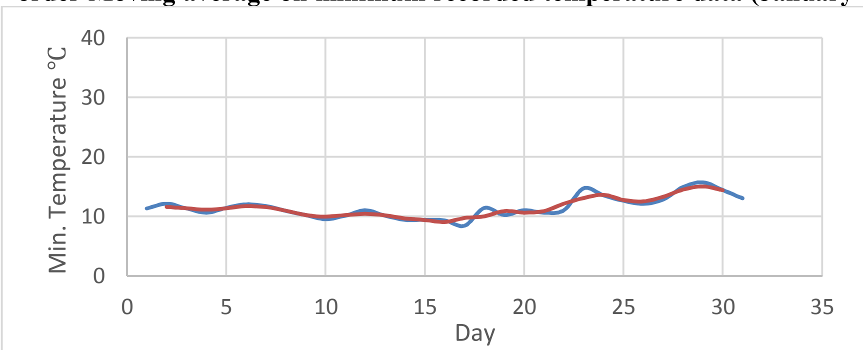

Figure.3.1.6. 3rd order Moving average on recorded temperature data (January) for 2010

Figure 3.1.7. 3rd order Moving average on minimum recorded temperature data (January-December 2011)

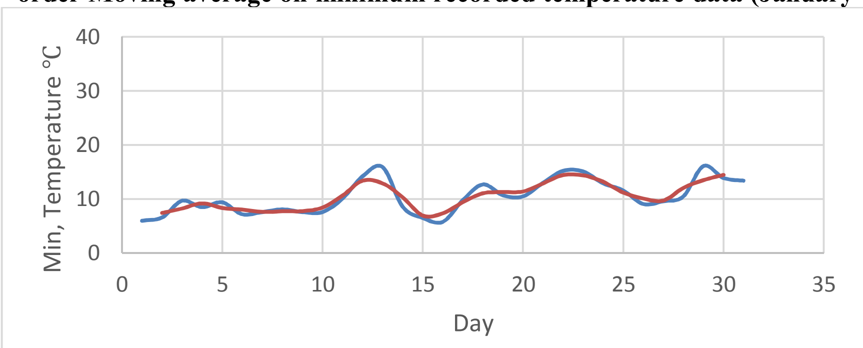

Figure.3.1.7. 3rd order Moving average on recorded temperature (January) for 2011

Figure 3.1.8. 3rd order Moving average on minimum recorded temperature data (January-December 2012)

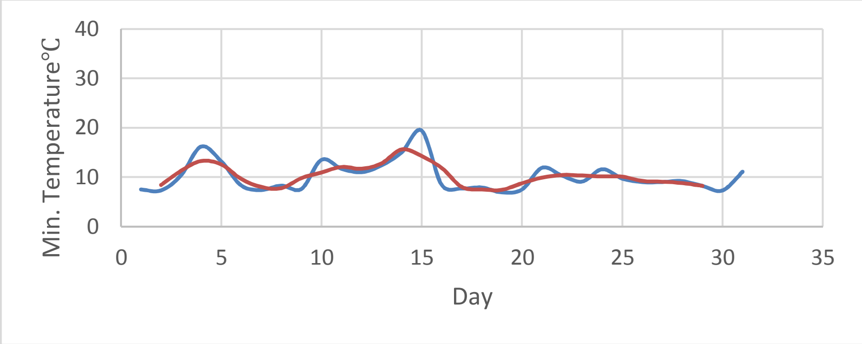

Figure.3.1.8. 3rd order Moving average on recorded temperature (January) for 2012

Figure 3.1.9. 3rd order Moving average on minimum recorded temperature data (January-December 2013)

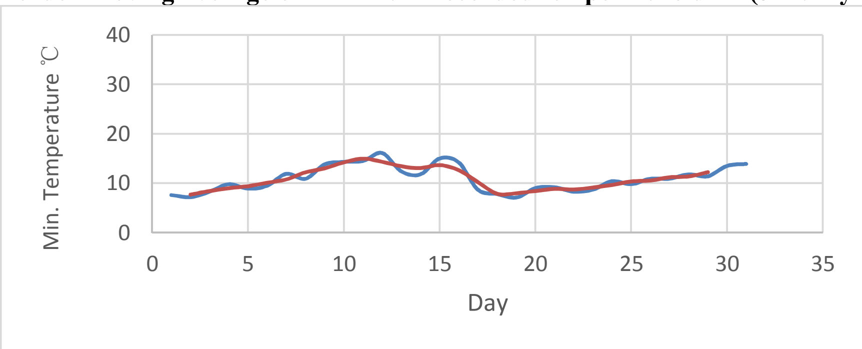

Figure.3.1.9. 3rd order Moving average on recorded temperature (January) for 2013

Figure 3.1.10. 3rd order Moving average on minimum recorded temperature data (January-December 2014)

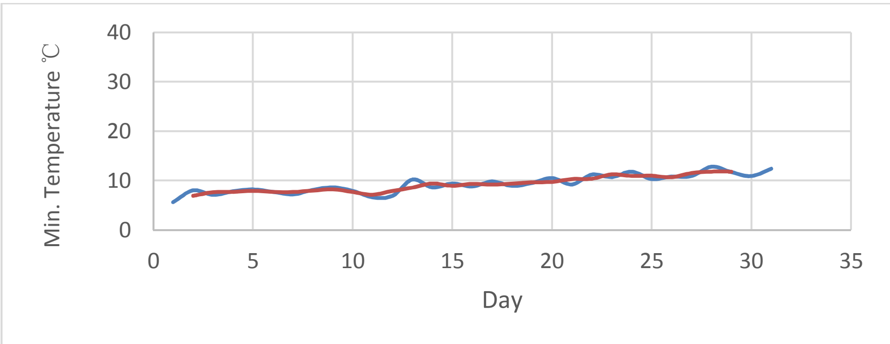

Figure.3.1.10. 3rd order Moving average on recorded temperature (January) for 2014

Table 3.2: Polynomial curve fitting Method:

Table 3.2.1: Fitted Polynomial Curve on Maximum recorded temperature 2010

Month	Trend model
January	$y = 5E\text{-}09x^6 + 7E\text{-}06x^5 - 0.0006x^4 + 0.0186x^3 - 0.2171x^2 + 0.8819x + 25.217$
February	$y = -3E\text{-}06x^6 + 0.0003x^5 - 0.0133x^4 + 0.2483x^3 - 2.2058x^2 + 7.7469x + 22.36$
March	$y = -1E\text{-}06x^6 + 0.0001x^5 - 0.0047x^4 + 0.07x^3 - 0.4035x^2 + 0.9124x + 29.569$
April	$y = 3E\text{-}07x^6 - 4E\text{-}05x^5 + 0.0021x^4 - 0.0519x^3 + 0.5879x^2 - 2.4705x + 36.821$
May	$y = -5E\text{-}07x^6 + 4E\text{-}05x^5 - 0.001x^4 + 0.0034x^3 + 0.1567x^2 - 1.5897x + 39.087$
June	$y = -1E\text{-}06x^6 + 0.0001x^5 - 0.004x^4 + 0.0584x^3 - 0.3346x^2 + 0.3318x + 35.128$
July	$y = 3E\text{-}07x^6 + 1E\text{-}05x^5 - 0.0022x^4 + 0.0725x^3 - 0.9382x^2 + 4.6211x + 29.705$
August	$y = -8E\text{-}07x^6 + 6E\text{-}05x^5 - 0.002x^4 + 0.0297x^3 - 0.2482x^2 + 1.2114x + 29.86$
September	$y = 2E\text{-}06x^6 - 0.0002x^5 + 0.0069x^4 - 0.1134x^3 + 0.9372x^2 - 3.7282x + 39.24$
October	$y = -2E\text{-}06x^6 + 0.0003x^5 - 0.0111x^4 + 0.2173x^3 - 1.967x^2 + 6.5579x + 33.048$
November	$y = -2E\text{-}06x^6 + 0.0002x^5 - 0.0052x^4 + 0.0849x^3 - 0.6722x^2 + 2.2855x + 31.976$
December	$y = 5E\text{-}07x^6 - 3E\text{-}05x^5 + 0.0001x^4 + 0.0115x^3 - 0.1674x^2 + 0.3934x + 28.93$

Table 3.2.2: Fitted Polynomial Curve on Maximum recorded temperature 2011

Month	Trend model
January	$y = 2E\text{-}06x^6 - 0.0002x^5 + 0.008x^4 - 0.1422x^3 + 1.2313x^2 - 4.5558x + 30.085$
February	$y = 7E\text{-}07x^6 - 8E\text{-}05x^5 + 0.0031x^4 - 0.0532x^3 + 0.4017x^2 - 1.2171x + 29.614$
March	$y = -2E\text{-}07x^6 - 7E\text{-}06x^5 + 0.0013x^4 - 0.0435x^3 + 0.5659x^2 - 2.1513x + 30.266$
April	$y = 3E\text{-}06x^6 - 0.0003x^5 + 0.0083x^4 - 0.1176x^3 + 0.7657x^2 - 2.3512x + 39.274$
May	$y = 9E\text{-}07x^6 - 7E\text{-}05x^5 + 0.002x^4 - 0.0196x^3 + 0.0053x^2 + 0.8624x + 32.252$
June	$y = -1E\text{-}06x^6 + 0.0001x^5 - 0.0065x^4 + 0.1381x^3 - 1.4572x^2 + 6.5605x + 28.688$
July	$y = -7E\text{-}07x^6 + 8E\text{-}05x^5 - 0.0033x^4 + 0.0675x^3 - 0.7029x^2 + 3.276x + 29.709$
August	$y = -3E\text{-}06x^6 + 0.0003x^5 - 0.0098x^4 + 0.1679x^3 - 1.3669x^2 + 4.5165x + 29.144$
September	$y = -2E\text{-}06x^6 + 0.0002x^5 - 0.0086x^4 + 0.1767x^3 - 1.7841x^2 + 7.5289x + 25.809$
October	$y = -3E\text{-}06x^6 + 0.0002x^5 - 0.0079x^4 + 0.1096x^3 - 0.6472x^2 + 1.5952x + 29.861$
November	$y = 3E\text{-}06x^6 - 0.0003x^5 + 0.0102x^4 - 0.1552x^3 + 1.0537x^2 - 2.5252x + 33.54$
December	$y = -3E\text{-}06x^6 + 0.0003x^5 - 0.0119x^4 + 0.2044x^3 - 1.6452x^2 + 4.9487x + 28.965$

Table 3.2.3: Fitted Polynomial Curve on Maximum recorded temperature 2012

Month	Trend model
January	$y = -2E\text{-}06x^6 + 0.0002x^5 - 0.0082x^4 + 0.1391x^3 - 1.0999x^2 + 3.5035x + 22.377$
February	$y = -6E\text{-}08x^6 + 4E\text{-}05x^5 - 0.0024x^4 + 0.0505x^3 - 0.3741x^2 + 0.3708x + 27.158$
March	$y = -1E\text{-}06x^6 + 1E\text{-}04x^5 - 0.0038x^4 + 0.0721x^3 - 0.6622x^2 + 2.6483x + 26.055$
April	$y = -2E\text{-}06x^6 + 0.0002x^5 - 0.008x^4 + 0.147x^3 - 1.3509x^2 + 5.4363x + 29.74$
May	$y = 2E\text{-}07x^6 - 1E\text{-}05x^5 + 0.0002x^4 - 0.0004x^3 + 0.038x^2 - 1.2895x + 42.093$
June	$y = 4E\text{-}07x^6 - 5E\text{-}05x^5 + 0.0027x^4 - 0.0639x^3 + 0.7467x^2 - 3.8794x + 41.189$
July	$y = 1E\text{-}06x^6 - 0.0001x^5 + 0.0042x^4 - 0.0765x^3 + 0.6581x^2 - 2.2999x + 36$
August	$y = -6E\text{-}07x^6 + 5E\text{-}05x^5 - 0.0016x^4 + 0.0231x^3 - 0.1745x^2 + 0.8496x + 30.222$
September	$y = -6E\text{-}07x^6 + 8E\text{-}05x^5 - 0.0038x^4 + 0.0821x^3 - 0.8049x^2 + 2.9572x + 31.748$
October	$y = 1E\text{-}06x^6 - 0.0001x^5 + 0.0038x^4 - 0.0614x^3 + 0.5061x^2 - 2.2697x + 38.887$
November	$y = 8E\text{-}08x^6 - 1E\text{-}05x^5 + 0.0008x^4 - 0.0223x^3 + 0.3121x^2 - 2.1285x + 38.065$
December	$y = 5E\text{-}06x^6 - 0.0005x^5 + 0.0159x^4 - 0.2559x^3 + 1.8482x^2 - 5.0068x + 32.296$

Table 3.2.4: Fitted Polynomial Curve on Maximum recorded temperature 2013

Month	Trend model
January	$y = 1E\text{-}06x^6 - 0.0001x^5 + 0.0057x^4 - 0.1127x^3 + 1.0569x^2 - 4.1052x + 29.866$
February	$y = 2E\text{-}06x^6 - 0.0002x^5 + 0.0097x^4 - 0.2061x^3 + 2.1988x^2 - 10.268x + 40.852$
March	$y = 2E\text{-}06x^6 - 0.0001x^5 + 0.0038x^4 - 0.0387x^3 + 0.0529x^2 + 0.9885x + 31.255$
April	$y = -6E\text{-}06x^6 + 0.0005x^5 - 0.0199x^4 + 0.3469x^3 - 2.8556x^2 + 9.7209x + 23.126$
May	$y = -2E\text{-}06x^6 + 0.0002x^5 - 0.0079x^4 + 0.1431x^3 - 1.1539x^2 + 2.889x + 38.202$
June	$y = -3E\text{-}06x^6 + 0.0003x^5 - 0.0099x^4 + 0.1565x^3 - 1.1837x^2 + 3.9818x + 32.102$
July	$y = 1E\text{-}06x^6 - 0.0001x^5 + 0.0041x^4 - 0.0734x^3 + 0.643x^2 - 2.5974x + 37.447$
August	$y = -1E\text{-}06x^6 + 0.0001x^5 - 0.0043x^4 + 0.0836x^3 - 0.7715x^2 + 2.9607x + 28.851$
September	$y = 2E\text{-}06x^6 - 0.0002x^5 + 0.0069x^4 - 0.1218x^3 + 1.0329x^2 - 3.4824x + 34.86$
October	$y = -1E\text{-}06x^6 + 9E\text{-}05x^5 - 0.0032x^4 + 0.0541x^3 - 0.4055x^2 + 0.7679x + 37.92$
November	$y = -2E\text{-}06x^6 + 0.0002x^5 - 0.0072x^4 + 0.136x^3 - 1.1705x^2 + 3.5684x + 30.892$
December	$y = -3E\text{-}06x^6 + 0.0003x^5 - 0.0087x^4 + 0.1381x^3 - 0.9931x^2 + 2.6453x + 28.657$

Table 3.2.5: Fitted Polynomial Curve on Maximum recorded temperature 2014

Month	Trend model
January	$y = -4E\text{-}06x^6 + 0.0004x^5 - 0.0133x^4 + 0.2418x^3 - 2.1585x^2 + 8.5871x + 12.9$
February	$y = -3E\text{-}06x^6 + 0.0002x^5 - 0.008x^4 + 0.127x^3 - 0.9058x^2 + 1.9951x + 28.012$
March	$y = 2E\text{-}06x^6 - 0.0002x^5 + 0.0072x^4 - 0.1245x^3 + 0.9661x^2 - 2.1897x + 27.65$
April	$y = 2E\text{-}06x^6 - 0.0002x^5 + 0.0057x^4 - 0.0857x^3 + 0.5637x^2 - 1.1216x + 33.058$
May	$y = 3E\text{-}06x^6 - 0.0003x^5 + 0.0097x^4 - 0.1478x^3 + 0.8866x^2 - 0.5187x + 28.981$
June	$y = -5E\text{-}08x^6 + 2E\text{-}05x^5 - 0.0015x^4 + 0.0472x^3 - 0.6467x^2 + 3.4969x + 32.537$
July	$y = 7E\text{-}07x^6 - 6E\text{-}05x^5 + 0.0017x^4 - 0.0234x^3 + 0.1312x^2 + 0.0377x + 32.089$
August	$y = 2E\text{-}06x^6 - 0.0002x^5 + 0.0068x^4 - 0.1351x^3 + 1.3276x^2 - 5.6921x + 40.051$
September	$y = -2E\text{-}06x^6 + 0.0001x^5 - 0.0047x^4 + 0.0678x^3 - 0.3536x^2 - 0.565x + 40.38$
October	$y = 2E\text{-}06x^6 - 0.0002x^5 + 0.0079x^4 - 0.1564x^3 + 1.4566x^2 - 5.2023x + 38.743$
November	Data missing
December	Data missing

Table 3.2.6: Fitted Polynomial Curve on Minimum recorded temperature 2010

Month	Trend model
January	$y = 1E\text{-}07x^6 - 1E\text{-}05x^5 + 0.0005x^4 - 0.0063x^3 + 0.0019x^2 + 0.1174x + 11.303$
February	$y = 3E\text{-}06x^6 - 0.0002x^5 + 0.0074x^4 - 0.0933x^3 + 0.4268x^2 - 0.3445x + 13.474$
March	$y = -7E\text{-}07x^6 + 8E\text{-}05x^5 - 0.0038x^4 + 0.0802x^3 - 0.7338x^2 + 2.0622x + 20.92$
April	$y = -1E\text{-}06x^6 + 9E\text{-}05x^5 - 0.0025x^4 + 0.0192x^3 + 0.1272x^2 - 1.8321x + 26.348$
May	$y = -8E\text{-}08x^6 + 1E\text{-}05x^5 - 0.0009x^4 + 0.024x^3 - 0.2978x^2 + 1.5754x + 24.713$
June	$y = -1E\text{-}06x^6 + 8E\text{-}05x^5 - 0.0027x^4 + 0.0386x^3 - 0.1939x^2 - 0.0889x + 29.048$
July	$y = 3E\text{-}07x^6 - 2E\text{-}05x^5 + 0.0003x^4 - 0.0021x^3 + 0.036x^2 - 0.4844x + 30.005$
August	$y = -8E\text{-}07x^6 + 8E\text{-}05x^5 - 0.003x^4 + 0.0498x^3 - 0.3723x^2 + 0.912x + 27.028$
September	$y = -4E\text{-}07x^6 + 5E\text{-}05x^5 - 0.0023x^4 + 0.0541x^3 - 0.6376x^2 + 3.3175x + 21.45$
October	$y = -1E\text{-}07x^6 + 7E\text{-}06x^5 + 2E\text{-}05x^4 - 0.0062x^3 + 0.0881x^2 - 0.1987x + 23.178$
November	$y = 3E\text{-}06x^6 - 0.0004x^5 + 0.0145x^4 - 0.2784x^3 + 2.5017x^2 - 8.9002x + 26.407$
December	$y = -2E\text{-}06x^6 + 0.0002x^5 - 0.0085x^4 + 0.1489x^3 - 1.2438x^2 + 4.0474x + 7.4875$

Table 3.2.7: Fitted Polynomial Curve on Minimum recorded temperature 2011

Month	Trend model
January	$y = 8E\text{-}07x^6 - 6E\text{-}05x^5 + 0.0017x^4 - 0.0192x^3 + 0.0491x^2 + 0.5579x + 5.9496$
February	$y = 2E\text{-}07x^6 - 2E\text{-}05x^5 + 0.0005x^4 - 0.0052x^3 - 0.0153x^2 + 0.5456x + 10.891$
March	$y = -2E\text{-}06x^6 + 0.0002x^5 - 0.0066x^4 + 0.1191x^3 - 0.9675x^2 + 2.6946x + 16.449$
April	$y = 7E\text{-}07x^6 - 0.0001x^5 + 0.0053x^4 - 0.1271x^3 + 1.3702x^2 - 5.4555x + 24.826$
May	$y = 7E\text{-}07x^6 - 7E\text{-}05x^5 + 0.0027x^4 - 0.05x^3 + 0.4693x^2 - 1.9228x + 28.711$
June	$y = 2E\text{-}07x^6 - 2E\text{-}05x^5 + 0.0008x^4 - 0.0155x^3 + 0.1588x^2 - 0.7111x + 29.556$
July	$y = 2E\text{-}07x^6 - 2E\text{-}05x^5 + 0.0007x^4 - 0.0112x^3 + 0.0793x^2 - 0.1871x + 28.394$
August	$y = 3E\text{-}08x^6 + 4E\text{-}06x^5 - 0.0005x^4 + 0.0193x^3 - 0.276x^2 + 1.4269x + 25.948$
September	$y = -1E\text{-}06x^6 + 0.0001x^5 - 0.0046x^4 + 0.0928x^3 - 0.9348x^2 + 3.966x + 22.754$
October	$y = 1E\text{-}06x^6 - 0.0001x^5 + 0.0038x^4 - 0.0509x^3 + 0.2517x^2 - 0.5029x + 25.527$
November	$y = 1E\text{-}06x^6 - 0.0001x^5 + 0.0039x^4 - 0.0613x^3 + 0.4424x^2 - 1.3817x + 23.034$
December	$y = -3E\text{-}06x^6 + 0.0003x^5 - 0.0102x^4 + 0.1652x^3 - 1.1769x^2 + 2.0139x + 19.51$

Table 3.2.8: Fitted Polynomial Curve on Minimum recorded temperature 2012

Month	Trend model
January	$y = -2E\text{-}06x^6 + 0.0002x^5 - 0.006x^4 + 0.1077x^3 - 0.9584x^2 + 4.0509x + 4.3787$
February	$y = 8E\text{-}07x^6 - 4E\text{-}05x^5 + 0.0003x^4 + 0.0015x^3 + 0.0623x^2 - 1.2035x + 11.53$
March	$y = -7E\text{-}06x^6 + 0.0007x^5 - 0.0257x^4 + 0.438x^3 - 3.4655x^2 + 10.493x + 10.933$
April	$y = 1E\text{-}07x^6 - 1E\text{-}05x^5 + 0.0006x^4 - 0.0134x^3 + 0.1209x^2 - 0.1022x + 21.511$
May	$y = 9E\text{-}07x^6 - 9E\text{-}05x^5 + 0.0032x^4 - 0.055x^3 + 0.4505x^2 - 1.3696x + 26.943$
June	$y = -3E\text{-}07x^6 + 3E\text{-}05x^5 - 0.0012x^4 + 0.0233x^3 - 0.2115x^2 + 0.8043x + 27.301$
July	$y = 6E\text{-}07x^6 - 6E\text{-}05x^5 + 0.0019x^4 - 0.0285x^3 + 0.1804x^2 - 0.3096x + 28.081$
August	$y = 3E\text{-}07x^6 - 3E\text{-}05x^5 + 0.0011x^4 - 0.0186x^3 + 0.155x^2 - 0.5459x + 27.436$
September	$y = -9E\text{-}07x^6 + 9E\text{-}05x^5 - 0.003x^4 + 0.0478x^3 - 0.325x^2 + 0.6122x + 27.401$
October	$y = -2E\text{-}07x^6 + 1E\text{-}05x^5 + 2E\text{-}05x^4 - 0.0141x^3 + 0.2983x^2 - 1.9587x + 25.548$
November	$Y = -7E\text{-}07x^6 + 6E\text{-}05x^5 - 0.0017x^4 + 0.0239x^3 - 0.2104x^2 + 1.266x + 15.987$
December	$y = -1E\text{-}06x^6 + 0.0001x^5 - 0.0034x^4 + 0.0585x^3 - 0.5534x^2 + 2.7943x + 9.943$

Table 3.2.9: Fitted Polynomial Curve on Minimum recorded temperature 2013

Month	Trend model
January	$y = 5E\text{-}07x^6 - 8E\text{-}05x^5 + 0.0043x^4 - 0.1006x^3 + 1.0123x^2 - 3.252x + 10.478$
February	$y = -3E\text{-}06x^6 + 0.0002x^5 - 0.0079x^4 + 0.1195x^3 - 0.7228x^2 + 0.5791x + 16.411$
March	$y = 8E\text{-}06x^6 - 0.0007x^5 + 0.0239x^4 - 0.4039x^3 + 3.3896x^2 - 12.402x + 29.277$
April	$y = -1E\text{-}06x^6 + 9E\text{-}05x^5 - 0.0028x^4 + 0.0391x^3 - 0.2076x^2 + 0.3085x + 21.32$
May	$y = -3E\text{-}07x^6 + 3E\text{-}05x^5 - 0.0012x^4 + 0.023x^3 - 0.2071x^2 + 0.8712x + 24.682$
June	$y = -7E\text{-}07x^6 + 6E\text{-}05x^5 - 0.002x^4 + 0.0298x^3 - 0.2108x^2 + 0.8337x + 27.076$
July	$y = 2E\text{-}07x^6 - 3E\text{-}05x^5 + 0.0011x^4 - 0.0198x^3 + 0.1757x^2 - 0.8344x + 30.323$
August	$y = 4E\text{-}07x^6 - 4E\text{-}05x^5 + 0.0015x^4 - 0.0293x^3 + 0.2889x^2 - 1.0851x + 27.316$
September	$y = -9E\text{-}07x^6 + 9E\text{-}05x^5 - 0.0032x^4 + 0.0554x^3 - 0.4582x^2 + 1.5241x + 25.069$
October	$y = -2E\text{-}06x^6 + 0.0002x^5 - 0.0063x^4 + 0.1083x^3 - 0.8773x^2 + 2.7576x + 25.196$
November	$y = 3E\text{-}06x^6 - 0.0002x^5 + 0.0083x^4 - 0.1375x^3 + 1.0761x^2 - 3.8554x + 23.76$
December	$y = -3E\text{-}07x^6 + 3E\text{-}05x^5 - 0.0008x^4 + 0.0078x^3 + 0.0289x^2 - 0.5891x + 15.894$

Table 3.2.10: Fitted Polynomial Curve on Minimum recorded temperature 2014

Month	Trend model
January	$y = -5E\text{-}07x^6 + 6E\text{-}05x^5 - 0.0024x^4 + 0.0488x^3 - 0.486x^2 + 2.2157x + 4.2228$
February	$y = -1E\text{-}06x^6 + 0.0001x^5 - 0.0049x^4 + 0.1024x^3 - 1.006x^2 + 3.416x + 12.684$
March	$y = -3E\text{-}07x^6 + 3E\text{-}05x^5 - 0.0014x^4 + 0.0346x^3 - 0.4432x^2 + 2.8171x + 10.242$
April	$y = -3E\text{-}06x^6 + 0.0003x^5 - 0.011x^4 + 0.1882x^3 - 1.4252x^2 + 3.7357x + 20.569$
May	$y = 1E\text{-}06x^6 - 0.0001x^5 + 0.005x^4 - 0.0896x^3 + 0.7615x^2 - 2.5428x + 27.984$
June	$y = -3E\text{-}08x^6 + 7E\text{-}06x^5 - 0.0004x^4 + 0.0106x^3 - 0.1099x^2 + 0.4623x + 28.455$
July	$y = -4E\text{-}07x^6 + 5E\text{-}05x^5 - 0.0019x^4 + 0.0362x^3 - 0.3129x^2 + 1.1404x + 27.02$
August	$y = -2E\text{-}07x^6 + 3E\text{-}05x^5 - 0.0014x^4 + 0.0341x^3 - 0.4247x^2 + 2.503x + 22.282$
September	$y = -8E\text{-}07x^6 + 8E\text{-}05x^5 - 0.0029x^4 + 0.0545x^3 - 0.494x^2 + 1.7395x + 26.772$
October	$y = -8E\text{-}07x^6 + 8E\text{-}05x^5 - 0.0027x^4 + 0.0466x^3 - 0.4171x^2 + 1.4991x + 24.095$
November	Data missing
December	Data missing

3.2.a. Simulation of maximum recorded temperature data (January-December 2010-2014)

The simulated graphs using polynomial fit for maximum temperature are shown below. In each graph blue curve represents the maximum recorded temperature (January-December 2010-2014), red curve represents the 3rd order moving average max. temperature(January-December 2010-2014) and doted curve represent the maximum simulated trend temperature (January-December 2010-2014).

Figure 3.2.1. Simulation of maximum recorded temperature data (January-December 2010)

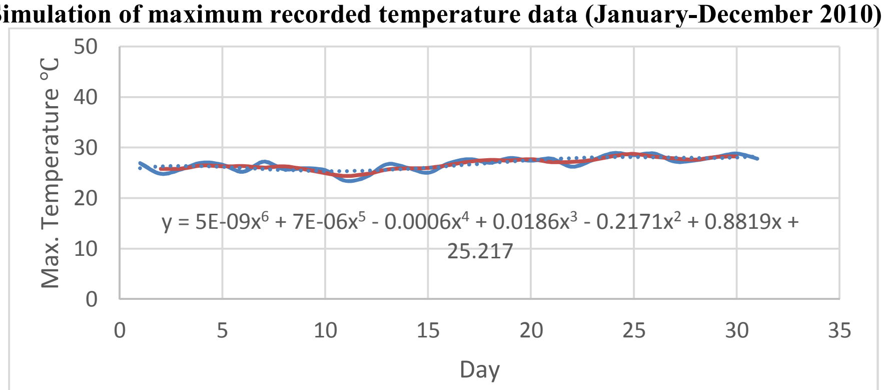

Figure.3.2.1. Simulation of max. temperature (January) for 2010

Figure 3.2.2. Simulation of maximum recorded temperature data (January-December 2011)

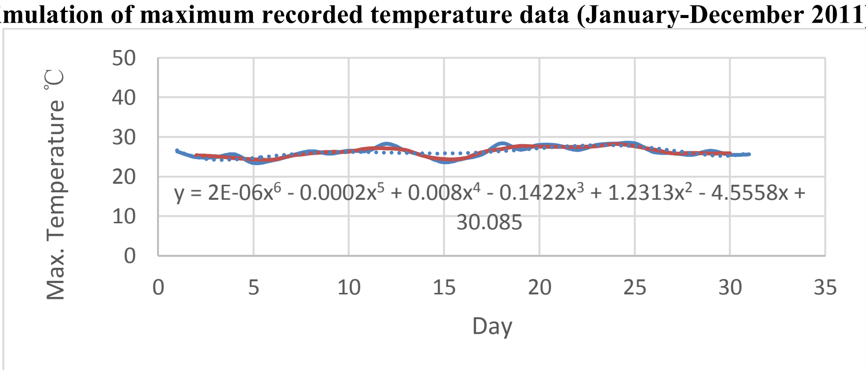

Figure.3.2.2. Simulation of max. temperature (January) for 2011

Figure 3.2.3. Simulation of maximum recorded temperature data (January-December 2012)

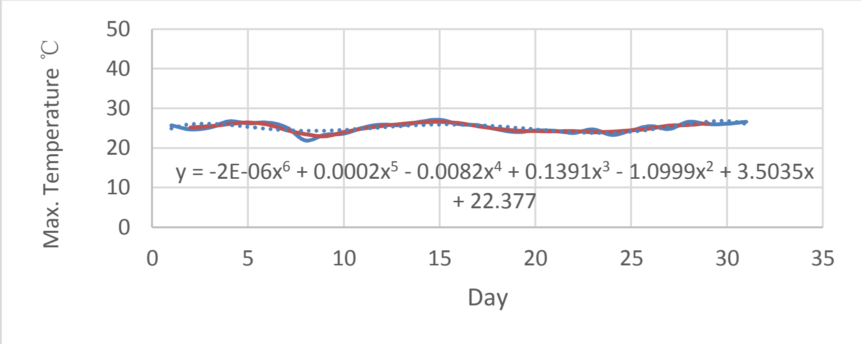

Figure.3.2.3. Simulation of max. temperature (January) for 2012

Figure 3.2.4. Simulation of maximum recorded temperature data (January-December 2013)

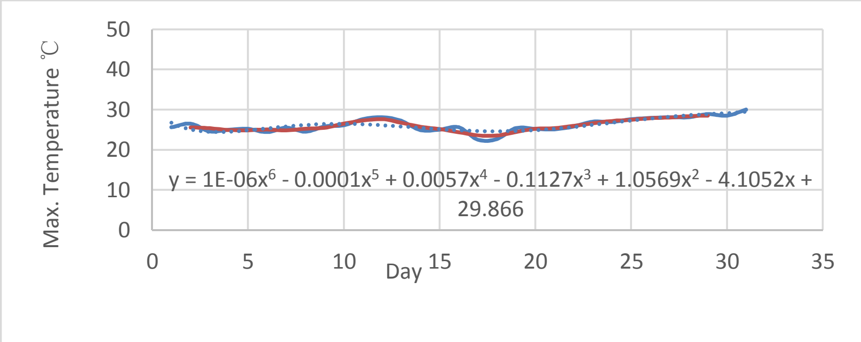

Figure.3.2.4. Simulation of max. temperature (January) for 2013

Figure 3.2.5. Simulation of maximum recorded temperature data (January-December 2014)

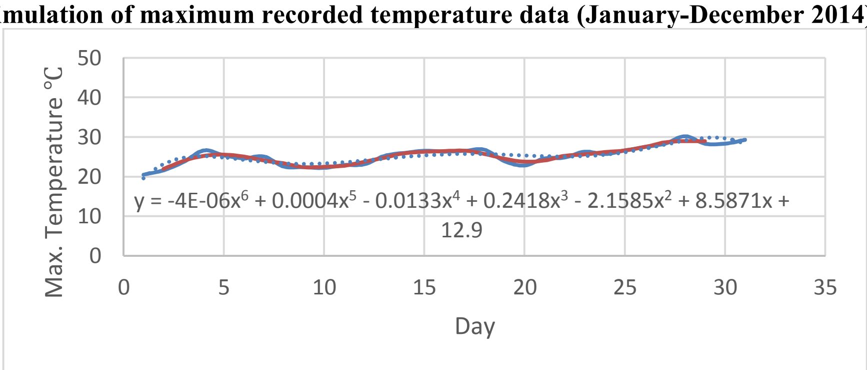

Figure.3.2.5. Simulation of max. temperature (January) for 2014

3.2.b. Simulation of minimum recorded temperature data (January-December 2010-2014)

The simulated graphs using polynomial fit for minimum temperature are shown below. In each graph blue curve represents the minimum recorded temperature (January-December 2010-2014), red curve represents the 3rd order moving average min. temperature (January-December 2010-2014) and doted curve represent the minimum simulated trend temperature (January-December 2010-2014).

Figure 3.2. 6. Simulation of minimum recorded temperature data (January-December 2010)

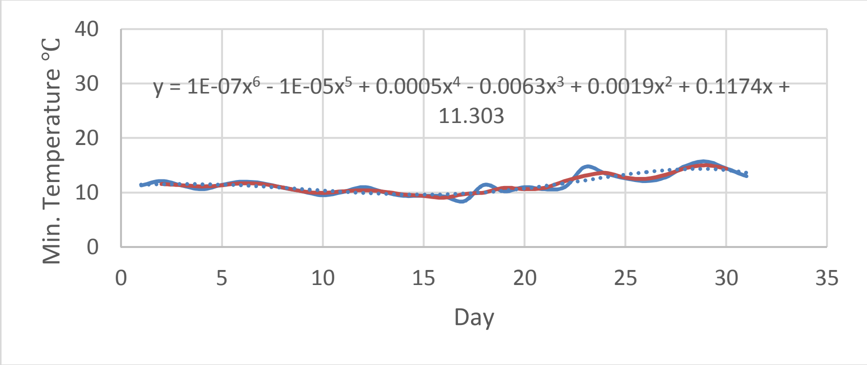

Figure.3.2.6. Simulation of min. temperature (January) for 2010

Figure 3.2.7. Simulation of minimum recorded temperature data (January-December 2010)

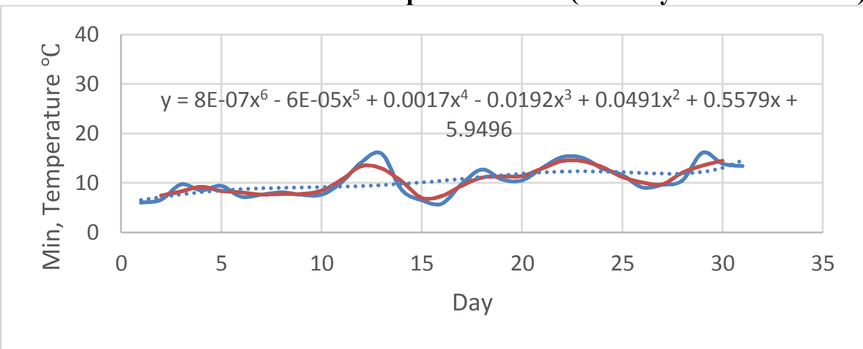

Figure.3.2.7. Simulation of min. temperature (January) for 2011

Figure 3.2.8. Simulation of minimum recorded temperature data (January-December 2012)

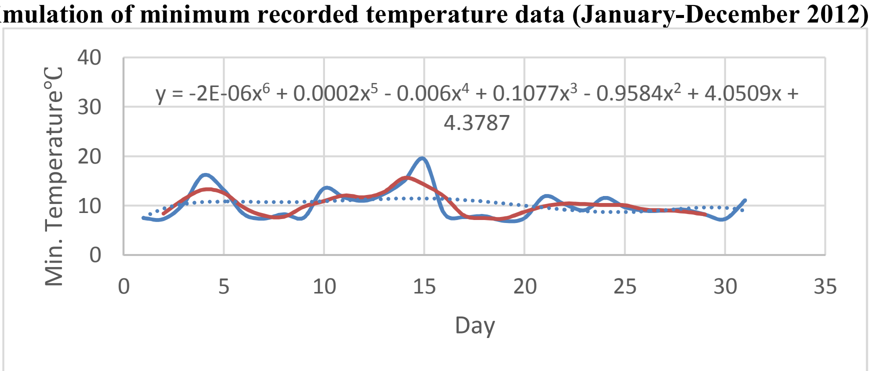

Figure.3.2.8. Simulation of min. temperature (January) for 2012

Figure 3.2.9. Simulation of minimum recorded temperature data (January-December 2013)

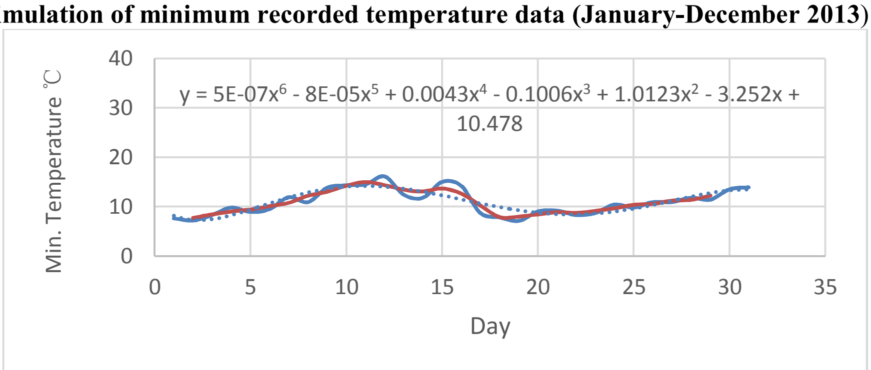

Figure.3.2.9. Simulation of min. temperature (January) for 2013

Figure 3.2.10. Simulation of minimum recorded temperature data (January-December 2014)

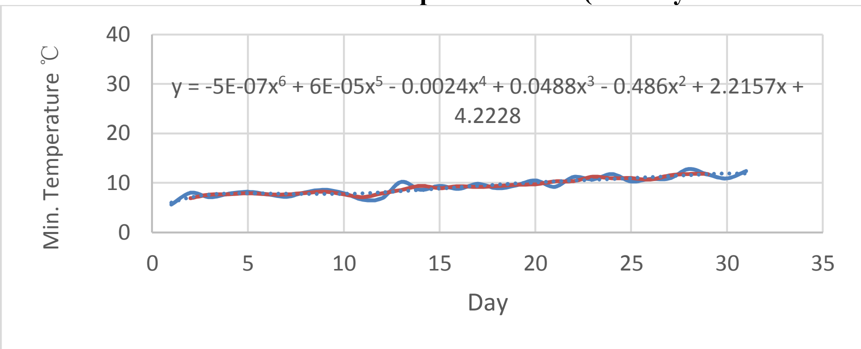

Figure.3.2.10. Simulation of min. temperature (January) for 2014

Table 3.3: Maximum Entropy Principal Method.

Table 3.3.1: The coefficients of polynomial fitted on maximum temperature (January 2010- December 2014).

	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7
January	2.45E+04	-5.94E+03	5.95E+02	-3.16E+01	9.38E-01	-1.47E-02	9.59E-05	
February	4.39e+04	9.61e+03	8.72e+02	4.19e+01	1.13e+00	1.62e-02	9.63e05	
March	-1.80E+03	2.52E+02	-1.37E+01	3.66E-01	-1.80E+03	2.52E+02		
April	-2.41E+03	2.82E+02	-1.22E+01	2.32E-01	-2.41E+03			
May	9.25E+04	-1.17E+04	5.87E+02	-1.47E+01	9.25E+04	-1.17E+04		
June	-5.02E+04	6.83E+03	-3.69E+02	9.93E+00	-5.02E+04	6.83E+03		
July	-2.37E+03	2.80E+02	-1.22E+01	2.35E-01	-1.67E-03			
August	-3.85E+04	5.51E+03	-3.13E+02	8.85E+00	-3.85E+04	5.51E+03		
September	-3.26E+05	5.63E+04	-4.02E+03	1.53E+02	-3.26E+00	3.69E-02	-1.73E-04	
October	-2.10E+04	2.92E+03	-1.61E+02	4.43E+00	-6.06E-02	3.30E-04		
November	-7.59E+04	1.19E+04	-7.47E+02	2.33E+01	-3.63E-01	2.26E-03		
December	-1.63E+04	3.62E+03	-3.32E+02	1.62E+01	-4.39E-01	6.33E-03	-3.78E-05	

Table 3.3.2: The coefficients of polynomial fitted on minimum temperature (January 2010- December 2014).

	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7
January	1.06E+03	-6.64E+02	1.74E+02	-2.47E+01	2.05E+00	-9.98E-02	2.63E-03	-2.92E-05
February	-2.30E+03	1.32E+03	-3.17E+02	4.11E+01	-3.13E+00	1.40E-01	-3.39E-03	3.46E-05
March	2.88E+04	-1.12E+04	1.84E+03	-1.66E+02	8.88E+00	-2.82E-01	4.93E-03	-3.65E-05
April	1.66E+04	-4.30E+03	4.60E+02	-2.61E+01	8.28E-01	-1.39E-02	9.70E-05	
May	-3.17E+04	6.26E+03	-4.93E+02	1.93E+01	-3.78E-01	2.95E-03		
June	-1.49E+05	2.53E+04	-1.70E+03	5.73E+01	-9.64E-01	6.48E-03		
July	-7.96E+03	9.52E+02	-4.07E+01	7.17E-01	-4.10E-03			
August	-9.07E+03	1.13E+03	-4.95E+01	8.84E-01	-4.90E-03			
September	-5.07E+03	7.69E+02	-4.33E+01	1.07E+00	-9.88E+03			
October	-4.09E+04	1.05E+04	-1.11E+03	6.24E+01	-1.96E+00	3.26E-02	-2.24E-04	
November	1.29E+04	-5.15E+03	8.71E+02	-8.11E+01	4.48E+00	-1.47E-01	2.65E-03	-2.02E-05
December	-4.15E+02	2.44E+02	-5.82E+01	7.38E+00	-5.40E-01	2.29E-02	-5.21E-04	4.94E-06

Figure 3.3.a. Simulation of maximum temperature (January 2010 - December 2014)

The simulated graphs using Entropy method to fit the polynomial for maximum temperature are shown below. In each graph blue curve represents the simulated polynomial curve for maximum temperature (January-December 2010-2014) and the green Histogram is drawn from maximum recorded temperature (January-December 2010-2014).

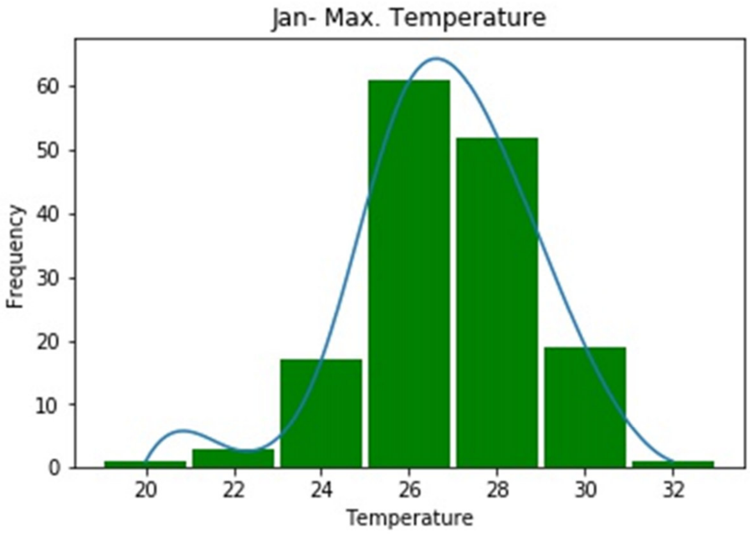

Figure.3.3.1. Simulation of max. temperature (January) for 2010-2014

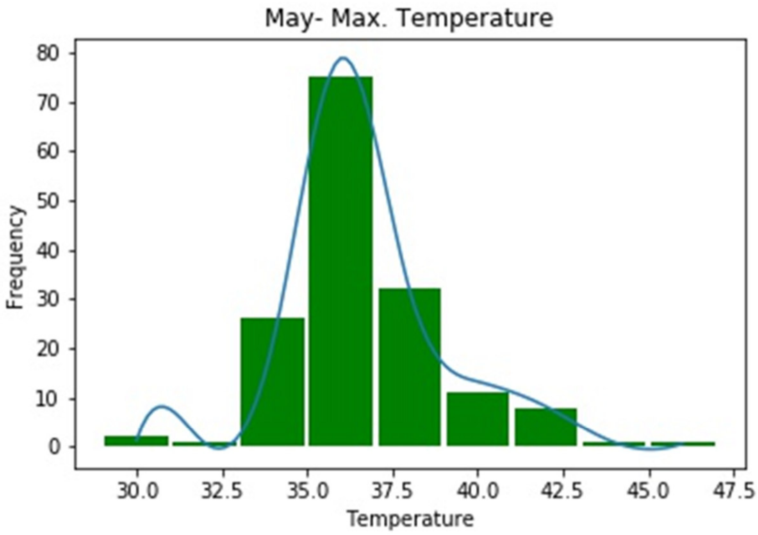

Figure.3.3.2. Simulation of max. temperature (May) for 2010-2014

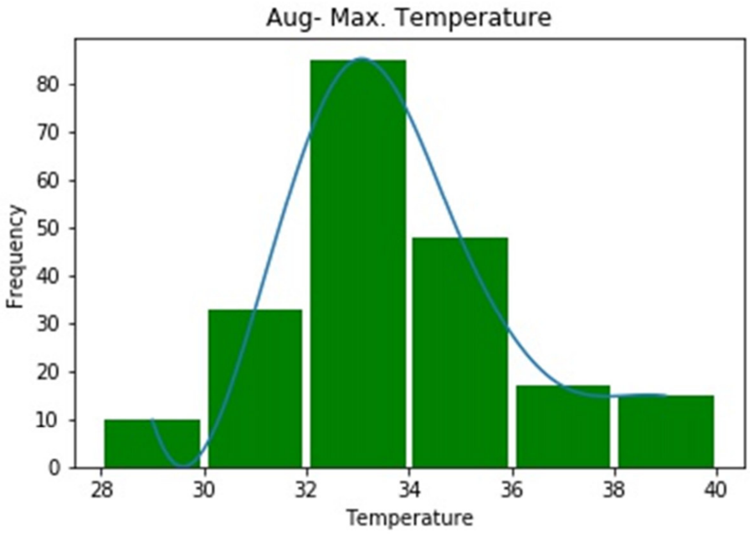

Figure.3.3.3. Simulation of max. temperature (August) for 2010-2014

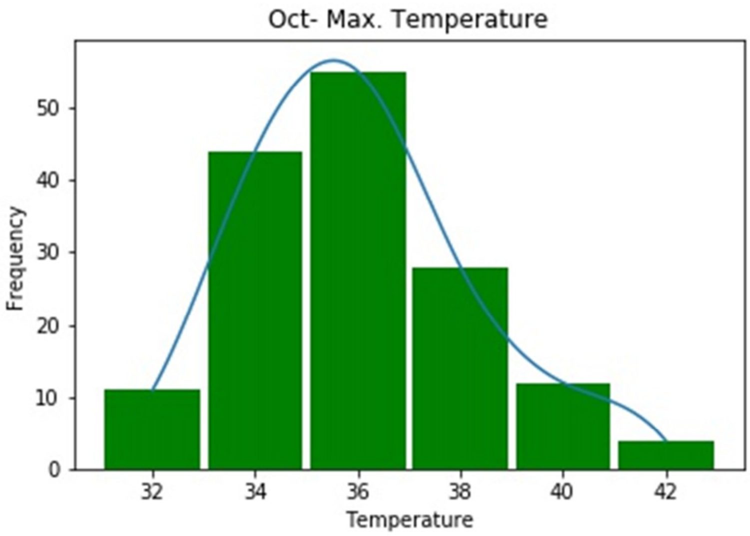

Figure.3.3.4. Simulation of max. temperature (October) for 2010-2014

Figure 3.3.b. simulation of minimum temperature (January 2010 – December 2014)

The simulated graphs using Entropy method to fit the polynomial for minimum temperature are shown below. In each graph blue curve represents the simulated polynomial curve for minimum temperature (January-December 2010-2014) and the green Histogram is drawn from minimum recorded temperature (January-December 2010-2014).

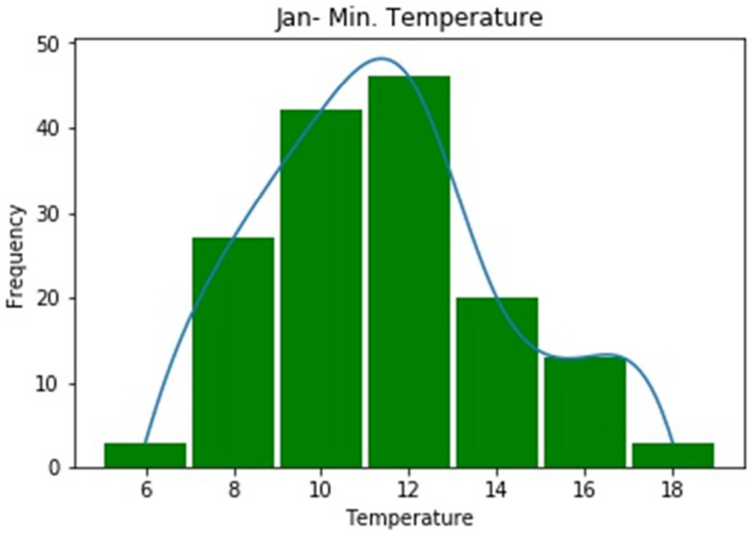

Figure.3.1.5. Simulation of min. temperature (January) for 2010-2014

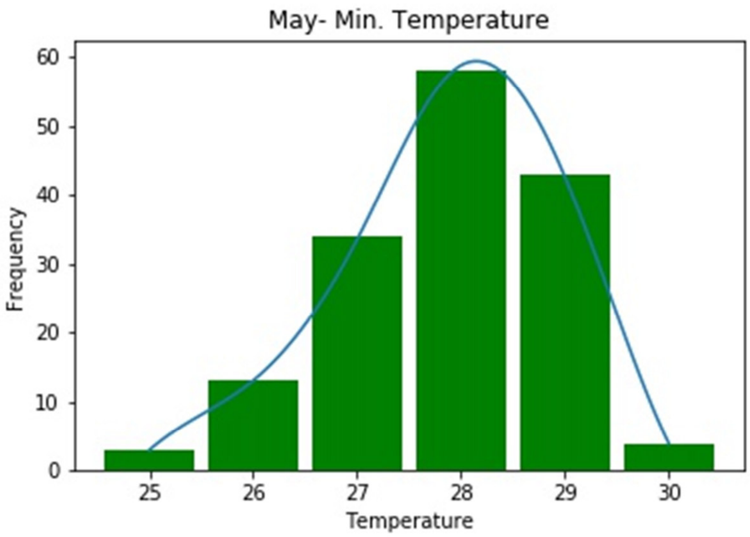

Figure.3.3.6. Simulation of min. temperature (May) for 2010-2014

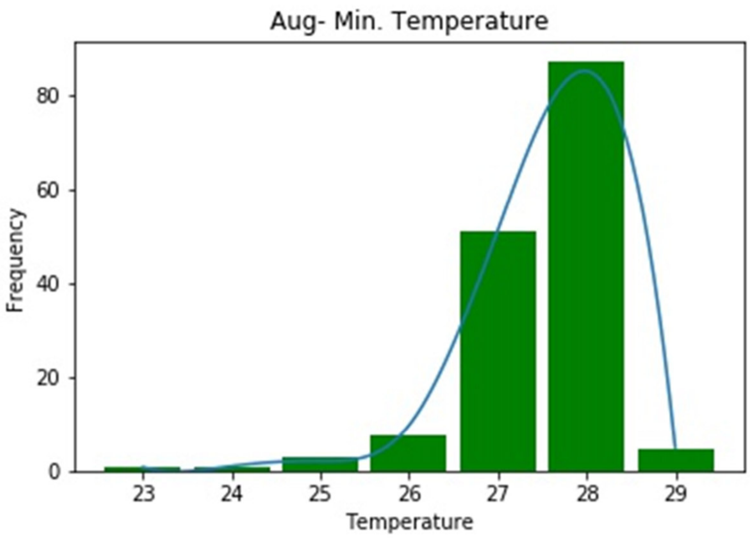

Figure.3.3.7. Simulation of min. temperature (August) for 2010-2014

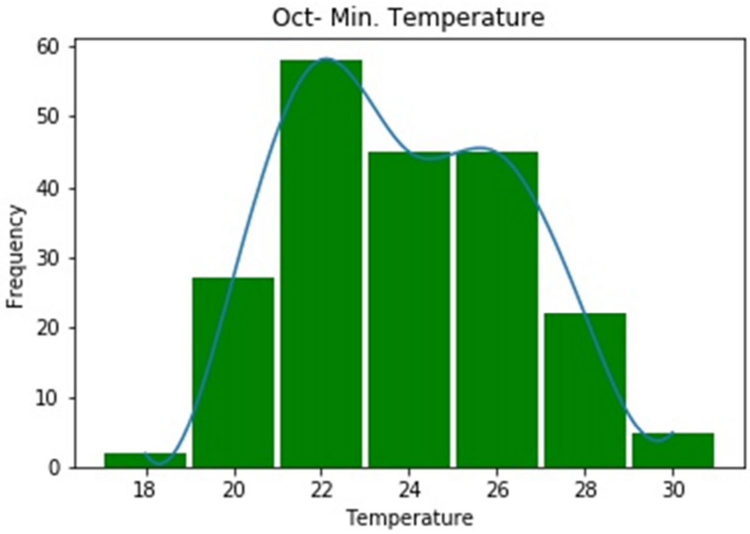

Figure.3.3.8. Simulation of min. temperature (October) for 2010-2014

Conclusion

We have used three methods to model data: Moving Average Method, Polynomial Curve Fitting Method and Maximum Entropy method. The data is modelled by third order moving averages and the trend lines overlap well on recorded data. The trend lines were also obtained by fitting recorded data of maximum and minimum temperature using polynomial fitting. The trend lines are shown for both maximum and minimum temperature data see tables 3.2.1-3.2.10. In general we see that six degree polynomial fits well on recorded data. The mathematical model obtained by curve fitting enables us to predict any missing temperature.

The maximum entropy method was also used to simulate the maximum and minimum temperature data. The graphs given in section 3.3.1-3.3.8 shown that the simulated graph gives relatively good fitting. The missing temperatures can also be calculated using this method.

These methods are in good agreement with the experimental data but to predict future temperatures we would need other parameters on which temperature variation depends. These parameters could be atmospheric temperature, humidity, precipitation, etc. We suggest future work to find a mathematical model based on the parameters mentioned above to predict future temperatures.

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