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Precise Computation of Energy Levels and Radiative Lifetimes in the *s, p, d,* and *f* Sequence of Hydrogen Isotope, with Natural Line Widths

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Abstract:
Energy levels and Radiative lifetimes in Deuterium for the following: $ns\ ^2S_{1/2}(n\geq2)$, $np^2P^o_{(1/2,3/2)}(n\geq2)$, $nd\ ^2D_{(3/2,5/2)}(n\geq3)$, and $nf\ ^2F^o_{(5/2,7/2)}(n\geq4)$ sequence have been evaluated with uncertainties in energies caused due to uncertainty principal. Theoretical calculations performed utilizing the Weakest Bound Electron Potential Model Theory (WBEPMT). Both sets of data show quite an excellent agreement with the experimental data listed at NIST. This theoretical computation is also a continuation of the work by Raza. S. *et al.* in Neutral Hydrogen. The high *'n'* (principal quantum number) values for both sets of data are presented very first time by utilizing WBEPMT.

Keywords: Energy levels, Radiative lifetimes, Quantum defects, Weakest bound electron, Natural line width.
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1. Introduction

Deuterium has many commercial and scientific applications. Like it is used as heavy water moderator in fission reactors, commonly used in nuclear magnetic resonance spectroscopy (NMR), in environmental sciences it is being used as a tracer, in nuclear weapons, and in drugs etc[1]. The need of its spectroscopic data is highly demanded. So, in our work we are going to utilize a computational method called WBEPMT for the computation of spectroscopic data of Deuterium [2]. Recently In 2018 Raza. S *et.al.* [3-4] computed the Rydberg energies series and radiative lifetimes of H I and In I by utilizing the same technique WBEPMT. And in past few years many elements in their atomic and ionic states were studied by this theory because of its semi-empirical nature. All these studies show remarkably good agreement in comparison with experimental results[5-22].

In this work first we obtained data from National Institute of Standards and Technology (NIST)[23] and rearranged them in an increasing order of quantum numbers *'n'* and then by Utilizing the first few energy levels and radiative lifetimes of lowest *'n'* values (experimental values of radiative lifetimes were obtained from transition probabilities listed at NIST) we computed energy levels sequence with quantum defects and radiative lifetimes with natural line widths for the *s, p, d* and *f* sequence of Deuterium up to $n=60$. Both sets of data show excellent agreement with experimental data [23].

2.Theory and Computational Approach:

The WBEPMT used is very effective and simplest model for calculating energy level sequence of Rydberg atoms and ions. Unlike previously published theoretical methods, it relies on a simple concept of non-weakest bound electrons (NWBE's) and weakest bound electron (WBE). The ionic-core of +(Z-1) charge formed with nucleus and NWBE's. The WBE moves under the influence of effective potential of an ion-core. During consecutive ionization step by step all WBE's separates from the ion-core, in each step only one weakest bound electron (WBE) ionize and rest non-weakest bound electrons (NWBE's) forming a new ion-core with effective potential (the effective potential of ion-core is due to penetration, polarization and shielding effect). This effective potential in which WBE is residing can be given as[2]:

$$V(r) = -\frac{Z}{r} + \frac{Y}{r^2} \tag{1}$$

where, $Y = \frac{m(m+1)+2ml}{2}$

In equation (1) the first and second terms at right hand side represent coulombic potential and polarization potential respectively. The polarization potential created by the dipole formed between ion-core and WBE.
In first term, the separation distance between nucleus and WBE's is 'r', the effective nucleus charge is Z and in Y, the angular quantum number of WBE's is 'l', and m is an unknown coefficient not necessarily an integer.
So, the energy formula for WBE's is:

$$T = -\frac{1}{2}\left(\frac{Z^*}{n^*}\right)^2 \tag{2}$$

In equation (2), n^* is the effective quantum number, n^* and Z^* are unknown parameters, but the problem is solved by the transformation between Eigen-values of Quantum defect theory (QDT) and WBE potential model theory, given as:

$$\frac{Z^*}{n^*} = \frac{Z_o}{n-\delta_n} \tag{3}$$

In equation (3) Z_o is the atomic kernel net charge number (Z_o = 1, 2, and 3... for the ion-core charge in (QDT) and δ_n is the quantum defect of nth energy level.

Now, the energy levels of Rydberg states of an atomic system in the WBEPMT written as: [24]

$$E = T_{lim} + T \tag{4}$$

In equation (4) the first term in right hand side is the ionization limit T_{lim} and 2nd term is the energy T of WBE's.

Now, by combining equations (3) and (4), we can rewrite (4) as:

$$E = T_{lim} - \frac{1}{2}\left(\frac{Z_o}{n-\delta_n}\right)^2 \tag{5}$$

The quantum defect 'δ_n' in equation (5) are computed by using Martin's formula for energy levels sequence, can be given as: [25].

$$\delta_n = a_0 + a_1(n-\delta_0)^{-2} + a_2(n-\delta_0)^{-4} + a_3(n-\delta_0)^{-6} \tag{6}$$

In equation (6) δ_0 is the lowest energy level quantum defect of the series, coefficients (a_i's, i=0,1,2,3) in (6) are obtained by the method of least-square fitting, by using the first few given experimental values of the energy levels sequence. [26].

By utilizing the Quantum defects of energy levels sequence, the radiative lifetimes of the energy levels can be found by Rykova's expression [27].

$$\tau = \tau_0 (n*)^{\alpha} \tag{9}$$

In which

$$n^* = n - \delta_n$$

In equation (9), τ_o and α are the parameter of Rykova's expression. A least square method can be used to calculate values of the parameters with the help of known radiative lifetimes of first few levels.

utilizing The Radiative lifetimes 'τ', the uncertainty in energy levels due to principal of uncertainty can be computed by using expression [28].

$$\Gamma = \frac{h}{2\Pi\tau} \tag{10}$$

In equation (10), Γ is natural line width and h is Planck's constant.

3.Result and Discussion:

3.1.Energy Levels Sequence with Quantum defects:

In this work first we computed energy levels sequence with quantum defects for the following: $ns\ ^2S_{1/2}(n\geq2)$, $np^2P^o_{(1/2,3/2)}(n\geq2)$, $nd\ ^2D_{(3/2,5/2)}(n\geq3)$, and $nf\ ^2F^o_{(5/2,7/2)}(n\geq4)$ sequence of deuterium. For the quantum defects are computed by equation 6 in text (Martin's expression) for *s, p, d* and *f* sequence. The coefficients of equation 6 are listed in Table I: computed by first few experimental values of energy levels obtained from NIST[23].

Table I. Shows the Coefficients of Martin's Expression & Sequence Convergence (Limit:109708.61455299 cm^{-1})

Energy Sequence	a_0	a_1	a_2	a_3	δ_0	δ_{60}	Convergence Nature
$ns\ ^2S_{1/2}\ (n\geq2)$	-0.00105	0.01291	-0.08122	0.16919	-0.00025	-0.00105	Core-polarization
$nP\ ^2P^o_{1/2}\ (n\geq2)$	-0.00106	0.01291	-0.08121	0.16919	-0.00027	-0.00106	Core-polarization
$nP\ ^2P^o_{3/2}\ (n\geq2)$	-0.00105	0.01291	-0.08121	0.16918	-0.00025	-0.00104	Core-polarization
$nd\ ^2D_{3/2}\ (n\geq3)$	-0.00135	0.02716	-0.30029	1.19460	-0.00040	-0.00134	Core-polarization
$nd\ ^2D_{5/2}\ (n\geq3)$	-0.00135	0.02716	-0.30029	1.19459	-0.00040	-0.00134	Core-polarization
$nf\ ^2F^o_{5/2}\ (n\geq4)$	-0.00127	0.02079	-0.14486	0.00000	-0.00054	-0.00126	Core-polarization
$nf\ ^2F^o_{7/2}\ (n\geq4)$	-0.00127	0.02079	-0.14486	0.00000	-0.00054	-0.00127	Core-polarization

From above Table I. we can clearly see by the comparison of 'δ_{60}' the quantum defects of the highest energy level up to which energy level sequence computed in this work with 'a_0'the first coefficient of equation 6 in text, *s, p, d* and *f* sequence of quantum defects converge toward 'a_0'. The negative sign of 'a_0' indicates that core-polarization potential dominates the '*nl*' $_{(l=s,\ p,\ d\ \text{and}\ f)}$ energy levels sequence.

Using these quantum defects energy levels sequence of all '*nl*' sequence of Deuterium are determined precisely by utilizing equation 4 in text up to $n=60$ principal quantum number, listed in Tables: II-VIII. The results are in good accuracy with NIST values[23]. The energy levels and Quantum Defects for high '*n*' values are presented very first time.

Table II: Energy Level sequence in cm^{-1} and Quantum Defects of *ns* $^2S_{1/2}$ $(2 \le n \le 60)$ in Deuterium.

n	*Quantum defects*		*Energy Levels*		*n*	*Quantum defects*		*Energy Levels*	
	δ_{NIST}	δ_{Cal}	E_{NIST}	E_{Cal}		δ_{NIST}	δ_{Cal}	E_{NIST}	E_{Cal}
2	-0.00025	-0.00025	82281.34	82281.34	32		-0.00104		109601.46
3	-0.00039	-0.00039	97518.75	97518.75	33		-0.00104		109607.85
4	-0.00052	-0.00052	102851.83	102851.83	34		-0.00104		109613.69
5	-0.00065	-0.00065	105320.28	105320.28	35		-0.00104		109619.04
6	-0.00079	-0.00075	106661.16	106661.13	36		-0.00104		109623.95
7	-0.00092	-0.00082	107469.67	107469.61	37		-0.00104		109628.46
8	-0.00106	-0.00087	107994.43	107994.35	38		-0.00104		109632.62
9	-0.00119	-0.00090	108354.19	108354.11	39		-0.00104		109636.47
10	-0.00132	-0.00093	108611.54	108611.45	40		-0.00104		109640.03
11	-0.00146	-0.00095	108801.94	108801.85	41		-0.00104		109643.34
12	-0.00159	-0.00096	108946.75	108946.67	42		-0.00104		109646.41
13		-0.00098		109059.38	43		-0.00104		109649.27
14		-0.00099		109148.81	44		-0.00104		109651.93
15		-0.00099		109220.96	45		-0.00104		109654.43
16		-0.00100		109280.01	46		-0.00104		109656.76
17		-0.00101		109328.95	47		-0.00104		109658.94
18		-0.00101		109369.96	48		-0.00104		109660.99
19		-0.00101		109404.67	49		-0.00104		109662.91
20		-0.00102		109434.30	50		-0.00104		109664.72
21		-0.00102		109459.80	51		-0.00104		109666.43
22		-0.00102		109481.91	52		-0.00105		109668.03
23		-0.00103		109501.19	53		-0.00105		109669.55
24		-0.00103		109518.12	54		-0.00105		109670.98
25		-0.00103		109533.05	55		-0.00105		109672.34
26		-0.00103		109546.29	56		-0.00105		109673.62
27		-0.00103		109558.10	57		-0.00105		109674.84
28		-0.00103		109568.65	58		-0.00105		109675.99
29		-0.00103		109578.14	59		-0.00105		109677.09
30		-0.00104		109586.69	60		-0.00105		109678.13
31		-0.00104		109594.43					

Table III: Energy Level sequence in cm^{-1} and Quantum Defects of $np\ ^2P^o_{1/2}$ $(2\le n\le 60)$ in Deuterium.

n	*Quantum defects*		*Energy Levels*		n	*Quantum defects*		*Energy Levels*	
	δ_{NIST}	δ_{Cal}	E_{NIST}	E_{Cal}		δ_{NIST}	δ_{Cal}	E_{NIST}	E_{Cal}
2	-0.00025	-0.00025	82281.30	82281.30	32		-0.00104		109601.46
3	-0.00038	-0.00038	97518.74	97518.74	33		-0.00104		109607.85
4	-0.00052	-0.00052	102851.83	102851.83	34		-0.00104		109613.69
5	-0.00065	-0.00065	105320.28	105320.28	35		-0.00104		109619.04
6	-0.00079	-0.00075	106661.16	106661.13	36		-0.00104		109623.95
7	-0.00092	-0.00082	107469.67	107469.61	37		-0.00104		109628.46
8	-0.00105	-0.00087	107994.43	107994.35	38		-0.00104		109632.62
9	-0.00119	-0.00090	108354.19	108354.11	39		-0.00104		109636.47
10	-0.00132	-0.00093	108611.53	108611.45	40		-0.00104		109640.03
11	-0.00146	-0.00095	108801.94	108801.85	41		-0.00104		109643.34
12	-0.00159	-0.00096	108946.75	108946.67	42		-0.00104		109646.41
13		-0.00097		109059.38	43		-0.00104		109649.27
14		-0.00098		109148.81	44		-0.00104		109651.93
15		-0.00099		109220.96	45		-0.00104		109654.43
16		-0.00100		109280.01	46		-0.00104		109656.76
17		-0.00100		109328.95	47		-0.00104		109658.94
18		-0.00101		109369.96	48		-0.00104		109660.99
19		-0.00101		109404.67	49		-0.00104		109662.91
20		-0.00102		109434.30	50		-0.00104		109664.72
21		-0.00102		109459.80	51		-0.00104		109666.43
22		-0.00102		109481.91	52		-0.00104		109668.03
23		-0.00102		109501.19	53		-0.00104		109669.55
24		-0.00103		109518.12	54		-0.00104		109670.98
25		-0.00103		109533.05	55		-0.00104		109672.34
26		-0.00103		109546.29	56		-0.00104		109673.62
27		-0.00103		109558.10	57		-0.00104		109674.84
28		-0.00103		109568.65	58		-0.00104		109675.99
29		-0.00103		109578.14	59		-0.00104		109677.09
30		-0.00103		109586.69	60		-0.00104		109678.13
31		-0.00104		109594.43					

Table IV: Energy Level sequence in cm^{-1} and Quantum Defects of *nd* $^2P^o_{3/2}$ $(2\leq n\leq 60)$ in Deuterium.

n	*Quantum defects*		*Energy Levels*		n	*Quantum defects*		*Energy Levels*	
	δ_{NIST}	δ_{Cal}	E_{NIST}	E_{Cal}		δ_{NIST}	δ_{Cal}	E_{NIST}	E_{Cal}
2	-0.00027	-0.00027	82281.67	82281.67	32		-0.00105		109601.46
3	-0.00040	-0.00040	97518.85	97518.85	33		-0.00105		109607.85
4	-0.00053	-0.00053	102851.87	102851.87	34		-0.00105		109613.69
5	-0.00066	-0.00066	105320.30	105320.30	35		-0.00105		109619.04
6	-0.00080	-0.00076	106661.18	106661.14	36		-0.00105		109623.95
7	-0.00093	-0.00083	107469.68	107469.62	37		-0.00105		109628.46
8	-0.00107	-0.00088	107994.43	107994.35	38		-0.00105		109632.62
9	-0.00120	-0.00091	108354.20	108354.11	39		-0.00105		109636.47
10	-0.00134	-0.00094	108611.54	108611.45	40		-0.00105		109640.03
11	-0.00147	-0.00096	108801.94	108801.86	41		-0.00105		109643.34
12	-0.00161	-0.00098	108946.76	108946.68	42		-0.00105		109646.41
13		-0.00099		109059.38	43		-0.00105		109649.27
14		-0.00100		109148.81	44		-0.00106		109651.93
15		-0.00101		109220.96	45		-0.00106		109654.43
16		-0.00101		109280.01	46		-0.00106		109656.76
17		-0.00102		109328.95	47		-0.00106		109658.94
18		-0.00102		109369.96	48		-0.00106		109660.99
19		-0.00103		109404.67	49		-0.00106		109662.91
20		-0.00103		109434.30	50		-0.00106		109664.72
21		-0.00103		109459.80	51		-0.00106		109666.43
22		-0.00104		109481.91	52		-0.00106		109668.03
23		-0.00104		109501.19	53		-0.00106		109669.55
24		-0.00104		109518.12	54		-0.00106		109670.98
25		-0.00104		109533.05	55		-0.00106		109672.34
26		-0.00104		109546.29	56		-0.00106		109673.62
27		-0.00104		109558.10	57		-0.00106		109674.84
28		-0.00105		109568.65	58		-0.00106		109675.99
29		-0.00105		109578.14	59		-0.00106		109677.09
30		-0.00105		109586.69	60		-0.00106		109678.13
31		-0.00105		109594.43					

Table V: Energy Level Sequence in cm^{-1} and Quantum Defects of *nd* $^2D_{3/2}$ *(3≤n≤60)* in Deuterium.

n	*Quantum defects*		*Energy Levels*		*n*	*Quantum defects*		*Energy Levels*	
	δ_{NIST}	δ_{Cal}	E_{NIST}	E_{Cal}		δ_{NIST}	δ_{Cal}	E_{NIST}	E_{Cal}
3	-0.00040	-0.00040	97518.85	97518.85	32		-0.00132		109601.46
4	-0.00053	-0.00053	102851.87	102851.87	33		-0.00132		109607.85
5	-0.00066	-0.00066	105320.30	105320.30	34		-0.00132		109613.69
6	-0.00080	-0.00080	106661.18	106661.18	35		-0.00133		109619.04
7	-0.00093	-0.00091	107469.68	107469.66	36		-0.00133		109623.95
8	-0.00107	-0.00099	107994.43	107994.40	37		-0.00133		109628.46
9	-0.00120	-0.00106	108354.20	108354.15	38		-0.00133		109632.62
10	-0.00134	-0.00110	108611.54	108611.49	39		-0.00133		109636.47
11	-0.00147	-0.00114	108801.94	108801.89	40		-0.00133		109640.03
12	-0.00161	-0.00117	108946.76	108946.70	41		-0.00133		109643.34
13		-0.00120		109059.40	42		-0.00133		109646.41
14		-0.00122		109148.83	43		-0.00133		109649.27
15		-0.00123		109220.97	44		-0.00133		109651.94
16		-0.00125		109280.02	45		-0.00133		109654.43
17		-0.00126		109328.96	46		-0.00133		109656.76
18		-0.00127		109369.97	47		-0.00133		109658.94
19		-0.00127		109404.67	48		-0.00134		109660.99
20		-0.00128		109434.31	49		-0.00134		109662.91
21		-0.00129		109459.81	50		-0.00134		109664.72
22		-0.00129		109481.91	51		-0.00134		109666.43
23		-0.00130		109501.20	52		-0.00134		109668.03
24		-0.00130		109518.12	53		-0.00134		109669.55
25		-0.00130		109533.05	54		-0.00134		109670.98
26		-0.00131		109546.30	55		-0.00134		109672.34
27		-0.00131		109558.10	56		-0.00134		109673.62
28		-0.00131		109568.66	57		-0.00134		109674.84
29		-0.00132		109578.14	58		-0.00134		109676.00
30		-0.00132		109586.70	59		-0.00134		109677.09
31		-0.00132		109594.43	60		-0.00134		109678.13

Table VI: Energy Level Sequence in cm^{-1} and Quantum Defects of *nd* $^2D_{5/2}$ *(3≤n≤60)* in Deuterium.

n	*Quantum defects*		*Energy Levels*		*n*	*Quantum defects*		*Energy Levels*	
	δ_{NIST}	δ_{Cal}	E_{NIST}	E_{Cal}		δ_{NIST}	δ_{Cal}	E_{NIST}	E_{Cal}
3	-0.00040	-0.00040	97518.88	97518.88	32		-0.00133		109601.46
4	-0.00054	-0.00054	102851.89	102851.89	33		-0.00133		109607.85
5	-0.00067	-0.00067	105320.31	105320.31	34		-0.00133		109613.69
6	-0.00080	-0.00080	106661.18	106661.18	35		-0.00133		109619.04
7	-0.00094	-0.00091	107469.68	107469.67	36		-0.00133		109623.95
8	-0.00107	-0.00100	107994.43	107994.40	37		-0.00133		109628.46
9	-0.00121	-0.00106	108354.20	108354.16	38		-0.00133		109632.62
10	-0.00134	-0.00111	108611.54	108611.49	39		-0.00133		109636.47
11	-0.00148	-0.00115	108801.94	108801.89	40		-0.00133		109640.03
12	-0.00161	-0.00118	108946.76	108946.70	41		-0.00134		109643.34
13		-0.00120		109059.40	42		-0.00134		109646.41
14		-0.00122		109148.83	43		-0.00134		109649.27
15		-0.00124		109220.97	44		-0.00134		109651.94
16		-0.00125		109280.02	45		-0.00134		109654.43
17		-0.00126		109328.96	46		-0.00134		109656.76
18		-0.00127		109369.97	47		-0.00134		109658.94
19		-0.00128		109404.67	48		-0.00134		109660.99
20		-0.00129		109434.31	49		-0.00134		109662.91
21		-0.00129		109459.81	50		-0.00134		109664.72
22		-0.00130		109481.91	51		-0.00134		109666.43
23		-0.00130		109501.20	52		-0.00134		109668.03
24		-0.00131		109518.12	53		-0.00134		109669.55
25		-0.00131		109533.05	54		-0.00134		109670.98
26		-0.00131		109546.30	55		-0.00134		109672.34
27		-0.00131		109558.10	56		-0.00134		109673.62
28		-0.00132		109568.66	57		-0.00134		109674.84
29		-0.00132		109578.14	58		-0.00134		109676.00
30		-0.00132		109586.70	59		-0.00134		109677.09
31		-0.00132		109594.43	60		-0.00134		109678.13

Table VII: Energy Level Sequence in cm^{-1} and Quantum Defects of $nf\,^2F^o_{5/2}$ $(4 \le n \le 60)$ in Deuterium.

n	*Quantum defects*		*Energy Levels*		n	*Quantum defects*		*Energy Levels*	
	δ_{NIST}	δ_{Cal}	E_{NIST}	E_{Cal}		δ_{NIST}	δ_{Cal}	E_{NIST}	E_{Cal}
4	-0.00054	-0.00054	102851.89	102851.89	33		-0.00125		109607.85
5	-0.00067	-0.00067	105320.31	105320.31	34		-0.00125		109613.69
6	-0.00080	-0.00080	106661.18	106661.18	35		-0.00125		109619.04
7		-0.00090		107469.66	36		-0.00125		109623.95
8		-0.00098		107994.39	37		-0.00125		109628.46
9		-0.00103		108354.15	38		-0.00125		109632.62
10		-0.00108		108611.48	39		-0.00126		109636.47
11		-0.00111		108801.88	40		-0.00126		109640.03
12		-0.00113		108946.70	41		-0.00126		109643.34
13		-0.00115		109059.40	42		-0.00126		109646.41
14		-0.00117		109148.83	43		-0.00126		109649.27
15		-0.00118		109220.97	44		-0.00126		109651.94
16		-0.00119		109280.02	45		-0.00126		109654.43
17		-0.00120		109328.96	46		-0.00126		109656.76
18		-0.00121		109369.97	47		-0.00126		109658.94
19		-0.00121		109404.67	48		-0.00126		109660.99
20		-0.00122		109434.31	49		-0.00126		109662.91
21		-0.00122		109459.81	50		-0.00126		109664.72
22		-0.00123		109481.91	51		-0.00126		109666.43
23		-0.00123		109501.19	52		-0.00126		109668.03
24		-0.00123		109518.12	53		-0.00126		109669.55
25		-0.00124		109533.05	54		-0.00126		109670.98
26		-0.00124		109546.30	55		-0.00126		109672.34
27		-0.00124		109558.10	56		-0.00126		109673.62
28		-0.00124		109568.66	57		-0.00126		109674.84
29		-0.00124		109578.14	58		-0.00126		109675.99
30		-0.00125		109586.69	59		-0.00126		109677.09
31		-0.00125		109594.43	60		-0.00126		109678.13
32		-0.00125		109601.46					

Table VIII: Energy Level Sequence in cm^{-1} and Quantum Defects of *nf* $^2F^o_{7/2}$ *(4≤n≤60)* in Deuterium.

n	*Quantum defects*		*Energy Levels*		*n*	*Quantum defects*		*Energy Levels*	
	δ_{NIST}	δ_{Cal}	E_{NIST}	E_{Cal}		δ_{NIST}	δ_{Cal}	E_{NIST}	E_{Cal}
4	-0.00054	-0.00054	102851.89	102851.89	33		-0.00125		109607.85
5	-0.00067	-0.00067	105320.31	105320.31	34		-0.00125		109613.69
6	-0.00081	-0.00081	106661.18	106661.18	35		-0.00125		109619.04
7		-0.00091		107469.66	36		-0.00126		109623.95
8		-0.00098		107994.39	37		-0.00126		109628.46
9		-0.00104		108354.15	38		-0.00126		109632.62
10		-0.00108		108611.48	39		-0.00126		109636.47
11		-0.00111		108801.88	40		-0.00126		109640.03
12		-0.00113		108946.70	41		-0.00126		109643.34
13		-0.00115		109059.40	42		-0.00126		109646.41
14		-0.00117		109148.83	43		-0.00126		109649.27
15		-0.00118		109220.97	44		-0.00126		109651.94
16		-0.00119		109280.02	45		-0.00126		109654.43
17		-0.00120		109328.96	46		-0.00126		109656.76
18		-0.00121		109369.97	47		-0.00126		109658.94
19		-0.00121		109404.67	48		-0.00126		109660.99
20		-0.00122		109434.31	49		-0.00126		109662.91
21		-0.00122		109459.81	50		-0.00126		109664.72
22		-0.00123		109481.91	51		-0.00126		109666.43
23		-0.00123		109501.19	52		-0.00126		109668.03
24		-0.00124		109518.12	53		-0.00126		109669.55
25		-0.00124		109533.05	54		-0.00126		109670.98
26		-0.00124		109546.30	55		-0.00126		109672.34
27		-0.00124		109558.10	56		-0.00126		109673.62
28		-0.00124		109568.66	57		-0.00126		109674.84
29		-0.00125		109578.14	58		-0.00126		109675.99
30		-0.00125		109586.69	59		-0.00127		109677.09
31		-0.00125		109594.43	60		-0.00127		109678.13
32		-0.00125		109601.46					

3.2. Radiative Lifetimes with Natural line widths:

Transition probabilities listed at NIST [23], the reference values of Radiative lifetimes for Deuterium are obtained. Utilizing these reference values of radiative lifetimes, the coefficients of expression 9 in text (Rykova's Expression) can be conveniently determined by least square fitting, are listed in Table IX.

Table IX: Coefficients of Rykova's Expression.

Energy Sequence	$\tau_{o(nS)}$	α
ns $^2S_{1/2}$ *(n≥2)*	1.19564E-08	2.1146
nP $^2P^o_{(1/2,3/2)}$ *(n≥2)*	2.06236E-10	2.9500
nd $^2D_{(3/2,5/2)}$ *(n≥3)*	6.08515E-10	2.9449
nf $^2F^o_{(5/2,7/2)}$ *(n≥4)*	1.21321E-09	2.9510

Finally exploiting computed values of quantum defects listed in Tables: II-VIII and coefficients listed in Table IX. computed the radiative lifetimes up to *n=60,* with natural line widths for the following: *ns* $^2S_{1/2}$*(n≥2), np*$^2P^o_{(1/2,3/2)}$*(n≥2), nd* $^2D_{(3/2,5/2)}$*(n≥3),* and *nf* $^2F^o_{(3/2,5/2)}$*(n≥4)* sequence of deuterium Listed in Tables X-XIII.

Table X: Radiative Lifetimes in nS and Natural linewidths in cm^{-1}of *ns* $^2S_{1/2}$ $(2\leq n\leq 60)$ in Deuterium.

n	*Radiative Lifetimes*		*Natural line Widths*	*n*	*Radiative Lifetimes*		*Natural line Widths*
	τ_{NIST}	τ_{Cal}	Γ_{Cal}		τ_{NIST}	τ_{Cal}	Γ_{Cal}
2		51.793	2.036E-20	32		18214.614	5.790E-23
3		122.078	8.639E-21	33		19439.206	5.425E-23
4	226.489	224.303	4.702E-21	34		20705.868	5.093E-23
5	352.185	359.552	2.933E-21	35		22014.743	4.790E-23
6	534.972	528.681	1.995E-21	36		23365.972	4.513E-23
7		732.406	1.440E-21	37		24759.693	4.259E-23
8		971.345	1.086E-21	38		26196.038	4.026E-23
9		1246.044	8.464E-22	39		27675.138	3.811E-23
10		1556.988	6.773E-22	40		29197.122	3.612E-23
11		1904.619	5.537E-22	41		30762.112	3.428E-23
12		2289.341	4.607E-22	42		32370.232	3.258E-23
13		2711.525	3.889E-22	43		34021.600	3.100E-23
14		3171.517	3.325E-22	44		35716.333	2.953E-23
15		3669.640	2.874E-22	45		37454.546	2.816E-23
16		4206.197	2.507E-22	46		39236.351	2.688E-23
17		4781.473	2.206E-22	47		41061.857	2.568E-23
18		5395.738	1.954E-22	48		42931.174	2.456E-23
19		6049.249	1.743E-22	49		44844.405	2.352E-23
20		6742.250	1.564E-22	50		46801.656	2.253E-23
21		7474.973	1.411E-22	51		48803.028	2.161E-23
22		8247.642	1.279E-22	52		50848.622	2.074E-23
23		9060.469	1.164E-22	53		52938.536	1.992E-23
24		9913.661	1.064E-22	54		55072.867	1.915E-23
25		10807.415	9.758E-23	55		57251.710	1.842E-23
26		11741.920	8.981E-23	56		59475.159	1.773E-23
27		12717.361	8.293E-23	57		61743.306	1.708E-23
28		13733.914	7.679E-23	58		64056.242	1.646E-23
29		14791.752	7.130E-23	59		66414.057	1.588E-23
30		15891.042	6.636E-23	60		68816.837	1.532E-23
31		17031.943	6.192E-23				

Table XI: Radiative Lifetimes in nS and Natural linewidths in cm^{-1} of *np* $^2P^o_{(1/2,3/2)}$ $(2 \le n \le 60)$ in Deuterium.

n	*Radiative Lifetimes*		*Natural line Widths*	*n*	*Radiative Lifetimes*		*Natural line Widths*
	τ_{NIST}	τ_{Cal}	Γ_{Cal}		τ_{NIST}	τ_{Cal}	Γ_{Cal}
2	1.596	1.594	6.615E-19	32		5683.271	1.856E-22
3	5.270	5.273	2.000E-19	33		6223.301	1.695E-22
4	12.302	12.320	8.560E-20	34		6796.203	1.552E-22
5	23.783	23.795	4.432E-20	35		7402.922	1.425E-22
6	40.808	40.745	2.588E-20	36		8044.403	1.311E-22
7		64.203	1.643E-20	37		8721.587	1.209E-22
8		95.196	1.108E-20	38		9435.418	1.118E-22
9		134.744	7.827E-21	39		10186.834	1.035E-22
10		183.858	5.736E-21	40		10976.775	9.607E-23
11		243.547	4.330E-21	41		11806.179	8.933E-23
12		314.812	3.350E-21	42		12675.983	8.320E-23
13		398.652	2.645E-21	43		13587.121	7.762E-23
14		496.058	2.126E-21	44		14540.529	7.253E-23
15		608.021	1.734E-21	45		15537.139	6.788E-23
16		735.527	1.434E-21	46		16577.884	6.361E-23
17		879.558	1.199E-21	47		17663.695	5.970E-23
18		1041.095	1.013E-21	48		18795.503	5.611E-23
19		1221.113	8.636E-22	49		19974.236	5.280E-23
20		1420.588	7.424E-22	50		21200.823	4.974E-23
21		1640.490	6.429E-22	51		22476.190	4.692E-23
22		1881.789	5.604E-22	52		23801.265	4.431E-23
23		2145.451	4.915E-22	53		25176.973	4.189E-23
24		2432.442	4.336E-22	54		26604.238	3.964E-23
25		2743.722	3.844E-22	55		28083.984	3.755E-23
26		3080.254	3.424E-22	56		29617.135	3.561E-23
27		3442.996	3.063E-22	57		31204.611	3.380E-23
28		3832.903	2.751E-22	58		32847.334	3.211E-23
29		4250.932	2.481E-22	59		34546.224	3.053E-23
30		4698.036	2.245E-22	60		36302.202	2.905E-23
31		5175.165	2.038E-22				

Table XII: Radiative Lifetimes in nS and Natural linewidths cm^{-1} of $nd^2D_{(3/2,5/2)}$ $(3\leq n\leq 60)$ in Deuterium.

n	*Radiative Lifetimes*		*Natural line Widths*	*n*	*Radiative Lifetimes*		*Natural line Widths*
	τ_{NIST}	τ_{Cal}	Γ_{Cal}		τ_{NIST}	τ_{Cal}	Γ_{Cal}
3	15.463	15.471	6.817E-20	32		16475.555	6.401E-23
4	36.139	36.095	2.922E-20	33		18038.235	5.846E-23
5	69.651	69.637	1.514E-20	34		19695.779	5.354E-23
6	119.084	119.129	8.852E-21	35		21450.899	4.916E-23
7		187.571	5.622E-21	36		23306.306	4.525E-23
8		277.932	3.794E-21	37		25264.705	4.174E-23
9		393.160	2.682E-21	38		27328.797	3.859E-23
10		536.182	1.967E-21	39		29501.279	3.575E-23
11		709.907	1.486E-21	40		31784.845	3.318E-23
12		917.227	1.150E-21	41		34182.183	3.085E-23
13		1161.022	9.083E-22	42		36695.979	2.874E-23
14		1444.158	7.302E-22	43		39328.916	2.681E-23
15		1769.487	5.960E-22	44		42083.672	2.506E-23
16		2139.851	4.928E-22	45		44962.923	2.345E-23
17		2558.083	4.123E-22	46		47969.339	2.198E-23
18		3027.004	3.484E-22	47		51105.589	2.064E-23
19		3549.428	2.971E-22	48		54374.339	1.940E-23
20		4128.157	2.555E-22	49		57778.252	1.825E-23
21		4765.989	2.213E-22	50		61319.985	1.720E-23
22		5465.712	1.929E-22	51		65002.196	1.622E-23
23		6230.107	1.693E-22	52		68827.537	1.532E-23
24		7061.948	1.493E-22	53		72798.659	1.449E-23
25		7964.002	1.324E-22	54		76918.209	1.371E-23
26		8939.031	1.180E-22	55		81188.832	1.299E-23
27		9989.789	1.056E-22	56		85613.171	1.232E-23
28		11119.026	9.485E-23	57		90193.864	1.169E-23
29		12329.486	8.553E-23	58		94933.550	1.111E-23
30		13623.906	7.741E-23	59		99834.861	1.056E-23
31		15005.020	7.028E-23	60		104900.429	1.005E-23

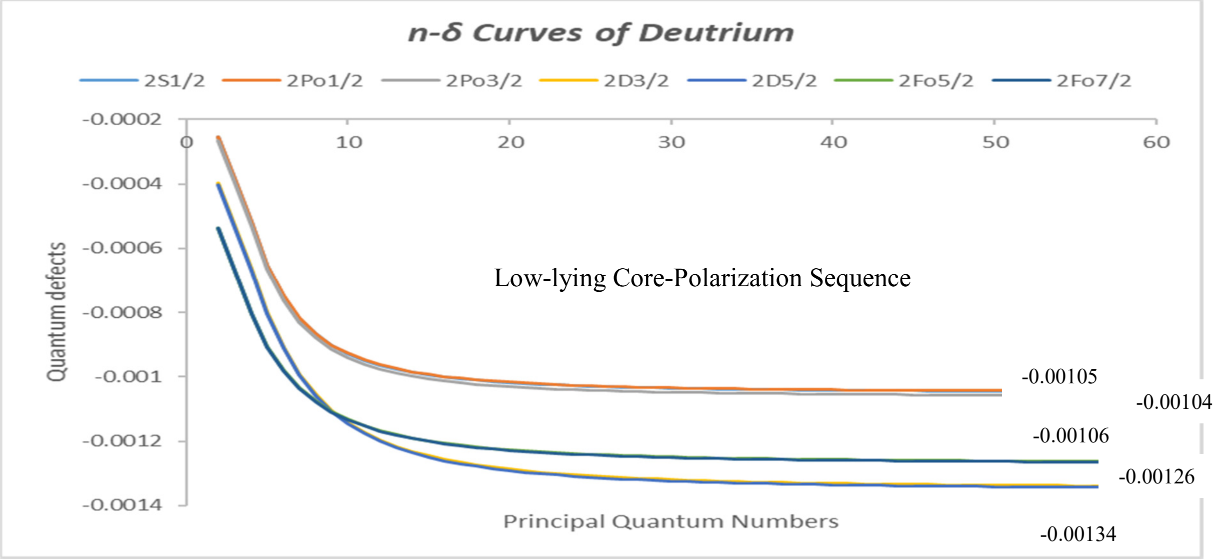
Table XIII: Radiative Lifetimes in nS and Natural linewidths cm^{-1} $nf\,^2F^o_{(5/2,7/2)}$ $(4 \leq n \leq 60)$ in Deuterium.

n	*Radiative Lifetimes*		*Natural line Widths*	n	*Radiative Lifetimes*		*Natural line Widths*
	τ_{NIST}	τ_{Cal}	Γ_{Cal}		τ_{NIST}	τ_{Cal}	Γ_{Cal}
4	72.509	72.575	1.453E-20	33		36738.165	2.871E-23
5	140.281	140.206	7.522E-21	34		40121.367	2.629E-23
6	239.880	240.121	4.392E-21	35		43704.376	2.413E-23
7		378.429	2.787E-21	36		47492.777	2.221E-23
8		561.190	1.879E-21	37		51492.147	2.048E-23
9		794.422	1.327E-21	38		55708.054	1.893E-23
10		1084.107	9.728E-22	39		60146.061	1.753E-23
11		1436.194	7.343E-22	40		64811.722	1.627E-23
12		1856.603	5.680E-22	41		69710.584	1.513E-23
13		2351.227	4.485E-22	42		74848.190	1.409E-23
14		2925.937	3.604E-22	43		80230.072	1.314E-23
15		3586.579	2.940E-22	44		85861.759	1.228E-23
16		4338.980	2.431E-22	45		91748.772	1.149E-23
17		5188.949	2.032E-22	46		97896.626	1.077E-23
18		6142.276	1.717E-22	47		104310.831	1.011E-23
19		7204.734	1.464E-22	48		110996.891	9.501E-24
20		8382.081	1.258E-22	49		117960.303	8.940E-24
21		9680.059	1.089E-22	50		125206.559	8.423E-24
22		11104.399	9.497E-23	51		132741.145	7.945E-24
23		12660.815	8.330E-23	52		140569.544	7.502E-24
24		14355.010	7.347E-23	53		148697.231	7.092E-24
25		16192.675	6.513E-23	54		157129.676	6.712E-24
26		18179.489	5.801E-23	55		165872.347	6.358E-24
27		20321.120	5.190E-23	56		174930.702	6.029E-24
28		22623.225	4.662E-23	57		184310.200	5.722E-24
29		25091.450	4.203E-23	58		194016.290	5.436E-24
30		27731.434	3.803E-23	59		204054.419	5.168E-24
31		30548.804	3.452E-23	60		214430.029	4.918E-24
32		33549.178	3.143E-23				

All *'nl'* sequence radiative lifetimes of Deuterium show a good agreement with refence values [23] and follows the simple scaling law $(n\text{-}\delta)^{\alpha}$, with their uncertainty in energies due to uncertainty principle. The uncertainties that is natural line width shows that all transitions have negligible natural broadening. In result the spectrum of Deuterium must have sharp spectral lines. The radiative lifetimes and Natural line widths for high *'n'* values are presented very first time.

3.3. Graphical Representation:

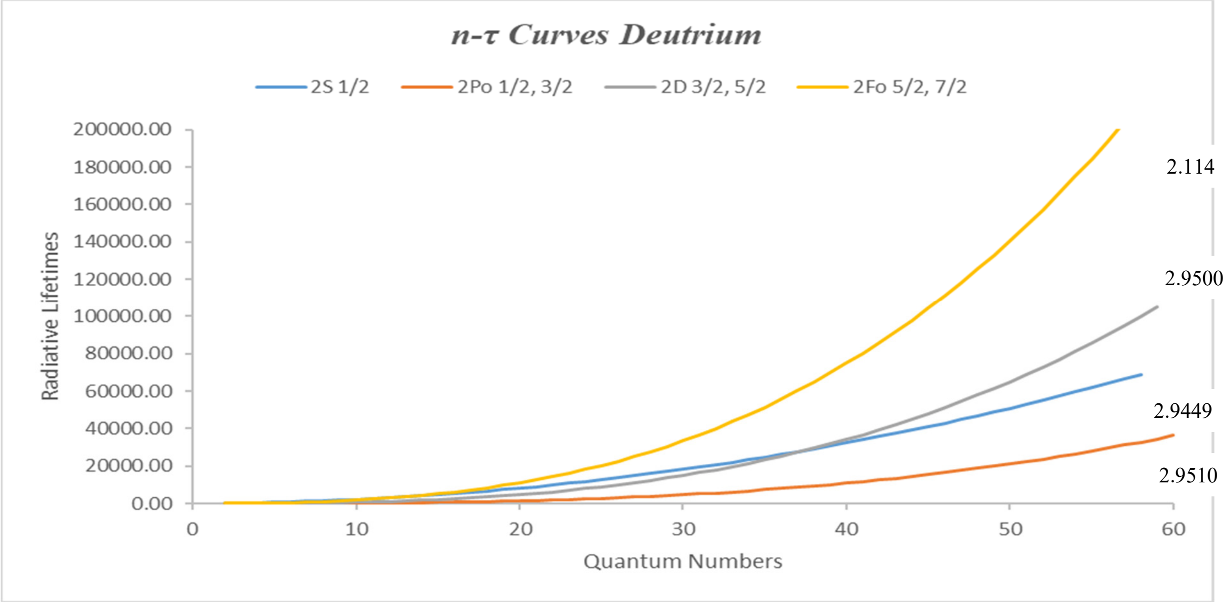
Graph I: Quantum defects as a function of Principal quantum numbers (*n-δ curves*).



From the comparison of coefficient of equation 6 in text ‘a_0’ with the quantum defects of lowest possible state of each series that is ‘δ_0’ and highest state up to which series is computed that is ‘δ_{60}’. We can clearly see that quantum defects of all *‘nl’* sequence of Deuterium converges towards the coefficient ‘a_0’. can also be clearly seen in Graph I. The core-polarization nature of all *‘nl’* sequence of Deuterium clearly displays in Graph I. Further analysation of the n-δ curves in Graph I, reveals that in all sequence, quantum defects ‘δ_n’, first exponentially decrease and then becomes constant and shows asymptotic behaviour due to the continuous change in quantum defects.

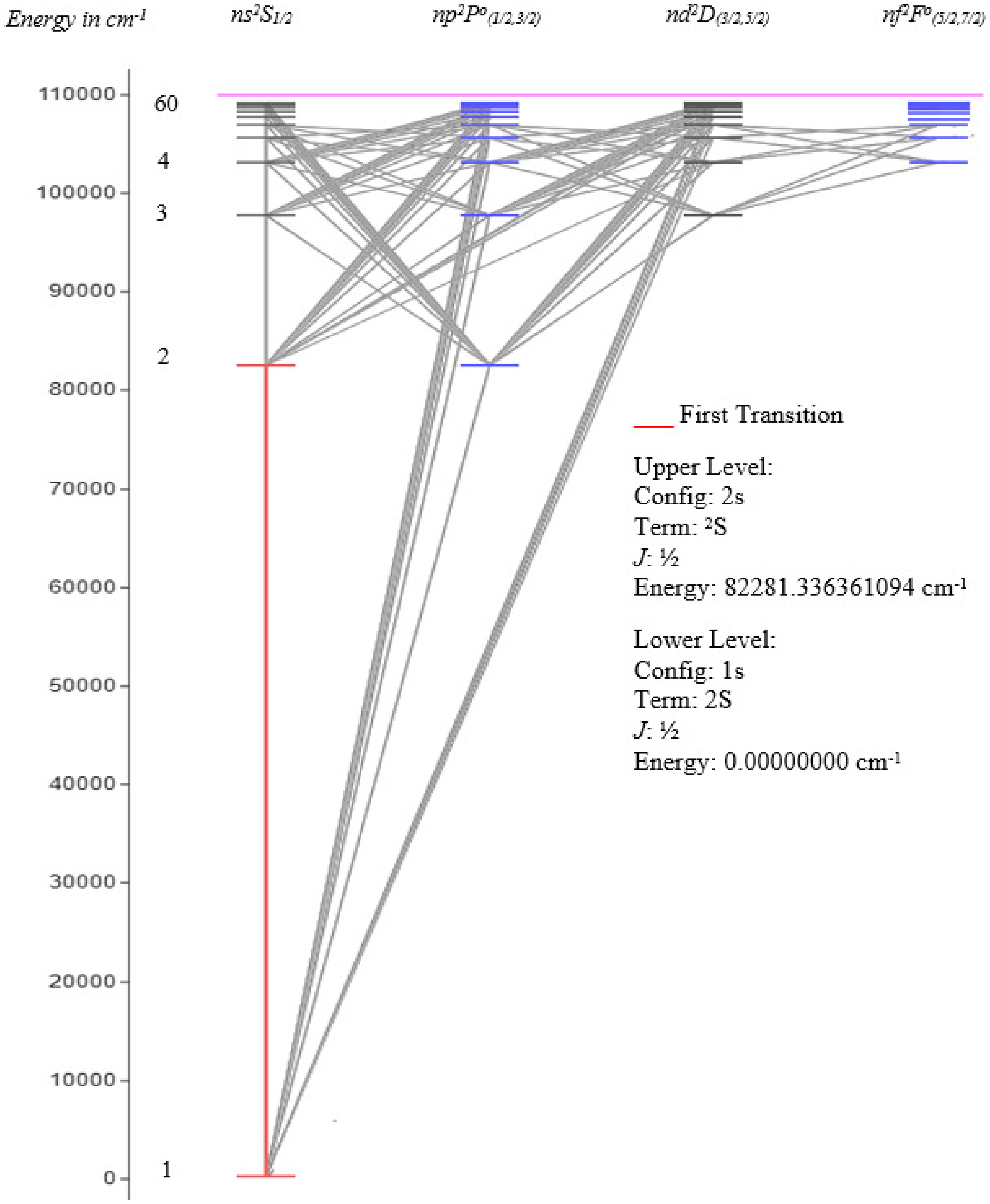
From above n-δ curves we can also see that $ns^2S_{1/2}$ sharply overlapped $np^2P^o{}_{1/2}$, *and in these* $nd^2D_{(3/2,5/2)}$, $nf^2F^o{}_{(5/2,7/2)}$ sequences $J= l\pm 1/2$ sharply overlaps each other and there is a fine splitting between $^2P^o{}_{1/2}$, and $^2P^o{}_{3/2}$. The quantum defects sequence of $nf\,^2F^o{}_{(5/2,7/2)}$ lies above $nd\,^2D_{(3/2,5/2)}$.

Graph II: Radiative Lifetimes as a function of principal quantum numbers (n-τ *curves*).



In n-τ curves in the following sequence $np\,^2P^o{}_{(1/2,3/2)}$, $nd\,^2D_{(3/2,5/2)}$, and $nf\,^2F^o{}_{(5/2,7/2)}$, $J=l\pm 1/2$ sharply overlaps each other and shows exponentially increasing behaviour. Among all sequence $nf\,^2F^o{}_{(5/2,7/2)}$ has the steepest curve but series: $ns^2S_{1/2}$ and, $nd\,^2D_{(3/2,7/2)}$ are closest and $np\,^2P^o{}_{(1/2,3/2)}$ has the lowest curve. The Graph II. also shows that *s, p, d* and *f* sequence follow the simple scaling law $(n-\delta_n)^\alpha$. The values of ‘α’ for *‘nl’* sequence of radiative lifetimes are explicitly shows in Graph II.

Graph III: Grotrian Energy Level Diagram for *s, p, d* and *f* Sequence of Deuterium.



A summarising view of all *'nl'* energy levels sequences of Deuterium are shown in Grotrian energy level diagram. From Graph III we can deduce that *s, p, d* and *f* energy levels sequence of Deuterium are same as hydrogen with bound and *l* degenerated states for higher *'n'* levels and all the transitions obeys the selection rule *J*=0, ±1, ±2. It also shows that for all *'nl'* energy levels sequence of Deuterium the quantum defect is approximately equal to zero.

4. Conclusion

The energy level sequence with Quantum defects and Radiative lifetimes with Natural line widths for the following sequence: $ns^2S_{1/2}$, $np^2P^o_{(1/2,3/2)}$, $nd^2D_{(3/2,5/2)}$ and, $nf^2F^o_{(5/2,7/2)}$ up to $n=60$, in deuterium are presented. Both sets of that compared with their experimental values obtained from NIST[23]. Quite an excellent agreement found between experimental and computed values in this work.

Conclusive remarks about theoretical computation of deuterium are as:

i. The deviation of this work is less than 0.1 cm^{-1} for energy levels sequence of Deuterium and 0.001 for quantum defects.

ii. Transition probabilities A_{ik} listed at NIST [23] were utilized to obtained first few experimental values of radiative Lifetimes of all *'nl'* sequence in deuterium. Then by least square fitting of data by using equation 9 in text, radiative lifetimes were computed for up to $n=60$ quantum number. The deviation of radiative lifetimes in this work is less than 0.1 nS in *p, d* and *f* sequence except for *s* sequence.

iii. From Table I and Graph I in text we can clearly infer that al *'nl'* sequence of Deuterium are converges towards the Martin's expression coefficient *'a_0'* and the negative sign of *'a_0'* indicates that all *'nl'* sequence are low lying core polarization sequence.

iv. In all *'nl'* sequence of quantum defects of: $ns^2S_{1/2}$ sharply overlaps $np^2P^o_{1/2}$, and in $nd^2D_{3/2,5/2}$, $nf^2F^o_{5/2,7/2}$ sequences quantum defects of $J=l\pm1/2$ sharply overlaps each other and there is a fine splitting between $np^2P^o_{1/2}$ *and* $np^2P^o_{3/2}$ (see Graph. I).

v. In all *'nl'* sequence of radiative lifetimes $J=l\pm1/2$ sharply overlaps each other except *ns* sequence and shows exponentially increasing behaviour. Among them the $nf^2F^o_{5/2,7/2}$ has the steepest curve but series: $ns^2S_{1/2}$ and, $nd^2D_{3/2,5/2}$ are closest (see Graph. III).

vi. All *'nl'* sequences of radiative lifetimes in Deuterium follows the simple scaling law $(n-\delta_n)^{\alpha}$.

vii. The Natural line widths computed based on principal of uncertainty show negligible values. Means natural broadening is approximately zero which shows the Deuterium spectrum is a sharp line spectrum (see Graph. III).

viii. The Grotrian diagram clearly displays the approximately zero quantum defects of deuterium *'nl'* sequence and the degeneracy of states for higher n values (see Graph. II).

ix. Finally, In Grotrian diagram for all the allowed transitions data available at NIST [23] are shown, following the selection rule J=0, ±1, ±2.

x. This theoretical computation is also a continuation of the work by Raza. S. *et al.* in Neutral Hydrogen [3].

5. Appendix A. Supplementary Data:

Related to this work, all the supplementary Data can be easily available online at:
https://www.physics.nist.gov/PhysRefData/ASD/lines_form.html

6. Explanation of Tables:

Table I. **Shows the Coefficients of Martin's Expression & Sequence Convergence (Limit:109708.61455299 cm^{-1})**

Energy Sequence: Energy levels sequence of Deuterium with initial principal quantum number
a_0: *Coefficient of Martin's expression for 'nl' sequence of Deuterium.*
a_1: *Coefficient of Martin's expression for 'nl' sequence of Deuterium.*
a_2: *Coefficient of Martin's expression for 'nl' sequence of Deuterium.*
a_3: *Coefficient of Martin's expression for 'nl' sequence of Deuterium.*
δ_0: *Quantum defects of lowest possible state of each series of Deuterium.*
δ_{60}: *Quantum defects of highest possible state up to which sequence is computed.*
Convergence Nature: Shows the dominating potential in which electron revolves.

Table II **Energy Level sequence and Quantum Defects of $ns\ ^2S_{1/2}$ $(2\leq n\leq 60)$ in Deuterium.**

n: Principal Quantum number.
δ_{NIST}: *Quantum Defects computed by the Experimental values of energies listed at NIST*
δ_{Cal}: *Quantum Defects calculated in this work by WBEPMT*
E_{NIST}: *Energy levels Listed at NIST*
E_{cal}: *Energy levels calculated in this work by WBEPMT*

Table III **Energy Level sequence and Quantum Defects of $ns\ ^2P^o_{1/2}$ $(2\leq n\leq 60)$ in Deuterium.**

n: Principal Quantum number.
δ_{NIST}: *Quantum Defects computed by the Experimental values of energies listed at NIST*
δ_{Cal}: *Quantum Defects calculated in this work by WBEPMT*
E_{NIST}: *Energy levels Listed at NIST*
E_{cal}: *Energy levels calculated in this work by WBEPMT*

Table IV **Energy Level sequence and Quantum Defects of ns $^2P^o_{3/2}$ $(2\leq n\leq 60)$ in Deuterium**

n: Principal Quantum number.
δ_{NIST}: *Quantum Defects computed by the Experimental values of energies listed at NIST*
δ_{Cal}: *Quantum Defects calculated in this work by WBEPMT*
E_{NIST}: *Energy levels Listed at NIST*

$E_{cal:}$ Energy levels calculated in this work by WBEPMT

Table V **Energy Level Sequence and Quantum Defects of *ns* $^2D_{3/2}$ ($3\le n\le 60$) in Deuterium.**
n: Principal Quantum number.
$\delta_{NIST:}$ Quantum Defects computed by the Experimental values of energies listed at NIST
$\delta_{Cal:}$ Quantum Defects calculated in this work by WBEPMT
$E_{NIST:}$ Energy levels Listed at NIST
$E_{cal:}$ Energy levels calculated in this work by WBEPMT

Table VI **Energy Level Sequence and Quantum Defects of ns $^2D_{5/2}$ ($3\le n\le 60$) in Deuterium.**
n: Principal Quantum number.
$\delta_{NIST:}$ Quantum Defects computed by the Experimental values of energies listed at NIST
$\delta_{Cal:}$ Quantum Defects calculated in this work by WBEPMT
$E_{NIST:}$ Energy levels Listed at NIST
$E_{cal:}$ Energy levels calculated in this work by WBEPMT

Table VII **Energy Level Sequence and Quantum Defects of *ns* $^2F^o_{5/2}$ ($4\le n\le 60$) in Deuterium.**
n: Principal Quantum number.
$\delta_{NIST:}$ Quantum Defects computed by the Experimental values of energies listed at NIST
$\delta_{Cal:}$ Quantum Defects calculated in this work by WBEPMT
$E_{NIST:}$ Energy levels Listed at NIST
$E_{cal:}$ Energy levels calculated in this work by WBEPMT

Table VIII **Energy Level Sequence and Quantum Defects of *ns* $^2D_{7/2}$ ($4\le n\le 60$) in Deuterium**
n: Principal Quantum number.
$\delta_{NIST:}$ Quantum Defects computed by the Experimental values of energies listed at NIST
$\delta_{Cal:}$ Quantum Defects calculated in this work by WBEPMT
$E_{NIST:}$ Energy levels Listed at NIST
$E_{cal:}$ Energy levels calculated in this work by WBEPMT

Table IX. **Coefficients of Rykova's Expression.**
Energy Sequence: Energy levels sequence of Deuterium with initial principal quantum number
τ_o: Coefficients of Rykova's Expression for measuring radiative lifetimes.
α: Power of effective quantum number n^*, for which Radiative lifetimes are directly proportional.

Table X. **Radiative Lifetimes in nS and Natural linewidths in $cm^{-1}$of $^2S_{1/2}$ ($2\le n\le 60$) in Deuterium.**
n: Principal Quantum number.
τ_{NIIST}: Experimental values of radiative lifetimes obtained from Transition probabilities listed at NIST.
$\tau_{cal:}$ Radiative lifetimes computed by exploiting Rykova's expression and WBEPMT.
Γ_{Cal}: Natural line widths produce due to uncertainty principal.

Table XI. **Radiative Lifetimes in nS and Natural linewidths in cm^{-1} of $^2P^o_{(1/2,3/2)}$ ($2\le n\le 60$) in Deuterium.**
n: Principal Quantum number.
τ_{NIIST}: Experimental values of radiative lifetimes obtained from Transition probabilities listed at NIST.
$\tau_{cal:}$ Radiative lifetimes computed by exploiting Rykova's expression and WBEPMT.
Γ_{Cal}: Natural line widths produce due to uncertainty principal.

Table XII. **Radiative Lifetimes in nS and Natural linewidths cm^{-1} of $^2D_{(3/2,5/2)}$ ($2\le n\le 60$) in Deuterium.**
n: Principal Quantum number.
τ_{NIIST}: Experimental values of radiative lifetimes obtained from Transition probabilities listed at NIST.
$\tau_{cal:}$ Radiative lifetimes computed by exploiting Rykova's expression and WBEPMT.
Γ_{Cal}: Natural line widths produce due to uncertainty principal.

Table XIII. **Radiative Lifetimes in nS and Natural linewidths cm^{-1} of $^2F^o_{(5/2,7/2)}$ ($2\le n\le 60$) in Deuterium.**
n: Principal Quantum number.
τ_{NIIST}: Experimental values of radiative lifetimes obtained from Transition probabilities listed at NIST.
$\tau_{cal:}$ Radiative lifetimes computed by exploiting Rykova's expression and WBEPMT.
Γ_{Cal}: Natural line widths produce due to uncertainty principal.

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