

Lessons from Quantum Logic and Ijiomah's Logic of Harmonious Monism: Towards the Axiomatization of African Logic

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Abstract
In this paper, lessons are drawn from quantum logic and Ijiomah's logic of Harmonious monism to show that even in Africa, logic can as well be presented with mathematical features and that the African lived-experiences of reality can generate context-propositions that can as well be axiomatized or mathematized in accordance to the tolerance principle of Rudolf Carnap. However, this is not to say that whatever should be regarded as African logic must always be in formal and syntactic mode; but rather, that we must also look at perspectives in which the mathematization of logic can be relevant to African lived experiences. Here, an attempt is made to mathematize Ijiomah's logic of harmonious monism borrowing from the developments made in quantum logic as a dimension of non-classical logic.

On the Mathematization of Logic as a Definitive Condition
The definition of logic as the science of good reasoning or correct thinking, or as the study of logical consequence or valid inference is now being challenged by different developments in the areas of mathematics and mathematical physics. Sonja Smets does well to observe that,

> Current research in logic is no longer confined to the traditional study of logical consequence or valid inference…the subject matter of logic encompasses several kinds of informational processes ranging from proofs and inferences to dialogues, observations, measurements, communication and computation (1).

In supporting Smets' position, Roy Cook notes that although the primary purpose of logic is to evaluate real arguments in natural languages, the term has grown to encompass much more than this. For Cook, the reason for this development is that over the past two or three centuries, mathematical features have been developed for studying the structure of arguments, leading to the development of the field of mathematical logic and computer science (174).Thus, in virtue of the foregoing, it can be said that logic has evolved in the modern period to the point of having a mathematical identity. According to Irvine, this evolution process began in the works of George Boole and Gottlob Frege (11-15). Today, the mathematical features of logic constitute its identity, at least for many professional logicians. This identity is hardly challenged by logicians of the Western academic culture. In the book *The Logical Syntax of Language,* Rudolf Carnap proposed what is regarded as the tolerance principle, which is stated as follows:

> In logic, there are no morals. Everyone is at liberty to build his own logic, i.e (sic)., his own form of language, as he wishes. All that is required of him is that, if he wishes to discuss it, he must state his methods clearly, and give syntactic rules instead of philosophical arguments (17).

The idea underlying Rudolf's tolerance principle is the need to retain the features of mathematization in what should be regarded as logic. This idea is very clear in two of the three conditions he mentions as the requirements for relativism in logic, namely, the provision of a formal or mathematical language and the statement of syntactic rules instead of philosophical arguments. Carnap's tolerance principle, which can be seen as an attempt at the criterion of logic gives room for pluralism in logic as well as relativism; however, it is clear about the position that pluralism and relativism is not to be sought in the direction of replacing formal language and syntactic axioms with argumentative essays. In other words, there may be different logics but these logics should have their own formal language and syntactic rules. Sustaining formal language and axioms in logic was also the position of Willard Quine in his *Philosophy of Logic;* however, for him, the formal language of logic should be that of classical first order logic (66). One need not be surprised therefore at the observation of Engesser, Gabbay and Lehmann in the book *A New Approach to Quantum Logic* that "logic nowadays means formal logic" (12).

So the question arises as to whether what we may call African logic should also conform to the tradition of mathematization or whether a formal system of logic that is African in origin can be developed. This question becomes very relevant today because (i) many African philosophers are very slow to present what they call African logic as a formal system of probability-calculus and (ii) because of the need for African intellectual properties to contribute to the growth and development of indigenous technology. The author deems it necessary to draw some lessons from quantum logic and Ijiomah's logic of Harmonious monism to show that the presentation of African logic as a system of probability-calculus generated from context-proposition indicating the African ontological worldview is actually possible.

Lessons from Quantum Logic

In recent times, there have been arguments in support of the position that relativism in logic arises from different ontological contexts. As Ijiomah notes in *Sapientia: Journal of Philosophical Logic*, different logics arise from different ontological contexts (20). For instance, Łukasiewicz's three-valued logic arose from the ontological situation of not being able to make a precise decision at the present as to whether he will be in Warsaw next year. Łukasiewicz's argument is captured by Kneale and Kneale in the following words:

> I can assume without contradiction that my presence in Warsaw at a certain moment of next year e.g at noon on 21 December is at the present time determined neither positively nor negatively. Hence it is possible but not necessary that I shall be present at Warsaw at the given time. On this assumption, the proposition "I shall be in Warsaw at noon 21 December of next year" can at the present time be neither true or false. For if it were true, my future presence in Warsaw will have to be necessary which is contradictory to the assumption. If it were false on the other hand, my future presence in Warsaw would have to be impossible which is also contradictory to the assumption. Therefore, the proposition considered is at the moment neither true or false and must possess a third value different from "O" (i.e) falsity and "I"(which is) truth. These values we can designate by "½". It represents the possible and joins the true and false as a third value. The three value system of propositional logic owes its origin to this line of thought (569-570).

For Łukasiewicz, this real ontological situation of suspension of judgement has a formal truth-value called the "indeterminate" in propositional calculus. The indeterminate truth-value (I) of Łukasiewicz's three-valued logic is a finite set containing two truth-values signifying that he may possibly be at Warsaw or not, that is, I = {T/F}. To this extent, one could say that his three-valued logic arises from a real ontological background that has to do with the metaphysics of subjectivity – that is, inability of the mind to assign at the present, truth-values for contingent events in the future. Moreover, it shows that it is simply a reflection of the suspension of judgement between the two possible outcomes (T/F) in a probability-calculus. Nevertheless, because the ontological foundation of Łukasiewicz's three-valued modal logic somehow lies in the direction of metaphysics of subjectivity, it does not suffice as a very strong argument to debunk the thesis that logic is pure forms of thought – that is, that logic is fundamentally a priori. On the contrary, whereas quantum logic also has a realist ontological context from which it arises, yet this context points in the direction of the empirical.

Quantum logic arose from the need to give empirical interpretation s to the mathematical probability-expression of quantum experiments using the functions of classical logic. In classical mechanics, it is possible to determine by mathematical measurement, different properties of the system like position and momentum simultaneously; the empirical interpretation of the mathematical expression of the probability-outcome of classical mechanics does not conflict with classical logic. For instance, as shown by Hilary Putnam in *Mathematics, Matter and Method,* in the mathematical expression of the probability-outcome of classical stream of particles in a double-slit experiment, the probability that the particles get through the slit A_1 is equal to the probability that they get through the slit A_2 because classical particles in motion behave in a deterministic way such that even where the position of the slits are interchanged, their behaviour at the receiving side of the plane remains the same. In the formal language of classical probability, the statement: "the probability that the particles pass through slit A_1" is ½ A_1 and the statement: "the probability that the photon passes through slit A_2 is ½ A_2. With respect to the conjunction of both statements indicating the probability that the particles go through the slits A_1 and A_2, their mathematical expression in the language of classical probability is ½ A_1 × ½ A_2. Given that the particles behave in the same way even after the positions of the slits are interchanged, then the slits A_1 and A_2 are symmetric such that the behaviour of the particles at slit A_1 is equal to their behaviour at slit A_2 (180-183). Therefore, ½ A_1 = ½ A_2.

Thus, where $A_1 = A_2$; by virtue of the similar behavioural property of the particles in the slit A_1 and A_2, the probability that the particles passes through the both slits is represented as ½ A_1× ½ A_2 . The probability that the particles pass through either slits is represented as ½ A_1 + ½ A_2.

If the same experiment is conducted using quantum particles, symmetry does not preserve equality of the behaviour of quantum particles at the two slits. Moreover, the behaviour of quantum realities are found to be indeterministic. Thus, the probability-expression of the behaviour of quantum realities appears paradoxical when we use classical logic as the tool of explanation. On this note, quantum mechanics calls for a new probability-expression as well as a non-classical interpretation of the conjunction-connective. This is why many quantum logicians have adopted the angles and lines of the projective geometry of Hilbert space as the mathematical structure for expressing the probability-outcome of quantum experiments thereby giving rise to the presentation of quantum logic as non-distributive lattice. At the same time, quantum logic does not interpret the conjunction-connective in the way classical logic does (Holik 4). For instance, if *P* represents one property of a quantum system *S* and *Q* represents another property of the quantum system *S;* then, the formal expression of classical logic $P \wedge Q$ is in quantum logic not interpreted as the simultaneous measurement of two properties of a quantum system. In other words, as Jonathan Baines observers, the empirical interpretation of $P \wedge Q$ in quantum logic is not equivalent to the empirical interpretation of $P \wedge Q$ in classical logic (*www. is.poly.edu*).

Thus, conjunction does not mean simultaneous measurement or measurability in quantum logic. Since quantum logicians see the need to preserve this empirical interpretation in quantum propositional calculus, conjunction does not distribute over disjunction thereby leading to the failure of the distributive principle. It was because of these findings that Birkhoff and Neumann began searching for another mathematical structure for quantum logic (*Annals* 823). So, the important lessons to take home here are that:

i. The empirical context of measurement due to the indeterministic behavioural nature of quantum realities is the basis for the axiomatization of quantum logic. Thus, as Guido Bacciagaluppi notes in the *Handbook of Quantum Logic and Quantum Structures,* these findings in quantum mechanics creates logical rooms for the supposition that logic is empirical (22), quantum logic elevates the necessity for formal expressions to correspond to realist experiences during quantum experiments.
ii. This means that empirical circumstances can lead to unique non-degenerate context-propositions that may necessitate a violation of any of the laws of classical logic and classical mechanics.
iii. Moreover, the logical functions of classical logic can be interpreted in other ways to give rise to a failure of at least one of the principles of classical logic. In Łukasiewicz's case, the indeterminate truth-value does not allow the classical law of excluded middle to hold. In the case of quantum logic, the failure of conjunction to mean simultaneous measurement does not allow the classical law of distributivity to hold. This is because where conjunction does not have the same interpretation as in classical logic, it is paradoxical to conceive it as distributing over disjunction.
iv. That truth-values in logical calculus is basically a mathematical expression of possibility-outcomes and not necessarily and ontological truth.
v. That i, ii, iii and iv can all be achieved without violating the tolerance principle of Carnap. In other words, a calculus of propositions can also be generated without some of the traditional laws of classical logic.

With these in view, it can be said that an African logic can also be generated as a propositional calculus without necessarily throwing away the tolerance principle of Carnap. All that is needed is a context-proposition, the traditional connectives with their specific interpretation which may be classical or non-classical, and syntax and semantics.

Lessons from Ijiomah's Logic of Harmonious Monism

Ijiomah does well to articulate the lived-experience of Africans with respect to their ontological worldview of reality as the necessary interrelations and interactions of matter with spirit or physical and spiritual. Here, we go straight to what informs the non-degenerate context propositions of the logic of harmonious monism. It is Ijiomah's statement in *African Journal of Religion Culture and Society* that every material thing has a spiritual dimension and that every spiritual thing has a material dimension (5-9). Ijiomah attempts to validate the tolerance principle by choosing symbols, and geometry-features like Venn diagrams and mathematical functions of set theory like *union* and *intersection* to capture the compositional relation of matter *M* and spirit *S* as binary complements of the set reality *I,* which he calls the universe of discourse (*Harmonious Monism: A Philosophical Logic* 128-129). We can draw logical implications from Ijiomah's ideas in the latter book as follows:

i. Reality is a finite, that is, a non-degenerate set composed of only two types of elements: matter and spirit. In other words, reality is a binary set. $I = M + S$; where *M* and *S* are the only possible judgement-outcomes Africans make about the ontological status of something that exists around them.
ii. By proposing that material substances can complement spirit or be complemented by spirit to give a spiritual substance , and that spiritual substances can also complement or be complemented by matter to produce the universe of discourse (*Harmonious Monism:*128), logically entails that matter and spirit have aspects of each other.
iii. By representing matter *M* and spirit *S* with Venn diagrams, and using the mathematical functions of set theory corresponding to the traditional logical connectives (*or, and*) and the *complement* corresponding the unary operator *not*; Ijiomah shows that the tradition of the mathematization of logic explicated in the tolerance principle must somewhat be preserved.

iv. By saying that all complements can unite without multiplying the number of elements and that the conjunction of *M* and *S* is a null set, Ijiomah shows that the conjunction operator is given a specific interpretation in the logic of harmonious monism, different from its usual understanding in classical propositional calculus. Recall that re-interpreting the conjunction-operator is also the major differentiating factor between classical logic and quantum logic.

Attempting an Axiomatization of Ijiomah's Logic of Harmonious Monism

First: it should be pointed out here that Ijiomah's symbolic representation of matter and spirit is inadequate because it does not preserve in its formalism the idea of the continuum feature of matter and spirit in each symbolism. Given his position in the *African Journal of Religion, Culture and Society* that matter has spiritual dimensions and spirit has material dimensions (20), then by logical implication neither matter nor spirit is exclusively without the other. But Ijiomah's symbolization of matter and spirit rather portrays an exclusive elimination of the other. There is need for a re-symbolization of the binary elements of Ijiomah's reality-set *I*, to capture his idea of continuum. In this respect, the context-propositions that material things have spiritual dimensions should rather be symbolized as ***Ms***. Spiritual is here represented in lower case to show that the subject's ontological emphasis is the material. That spiritual things have material dimensions should be symbolized as ***Sm***, where material is here represented in lower case to show that the subject's ontological emphasis is spiritual.

Thus instead of $M + S = I$, we should rather express harmonious monism as $(\forall x)\ \{Ms \leftrightarrow_1 Sm\} = I$; where$\leftrightarrow_1$ is here introduced as the mathematical symbol for the continuum showing that the order of influence between matter and spirit can go in either directions from spirit to matter or from matter to spirit or even in both directions at the same time. In this regard, one can deduce well formed formulas from propositions signifying these dimensions of relations. To say that a thing is material is represented as ***Ms*** and to say that a thing is not material is $\neg Ms$. In the same way, the negation of ***Sm*** is $\neg Sm$. To say that a thing is either material or spiritual is represented as $Ms \vee Sm$. To say that a thing is material and spiritual is represented as $Ms \wedge Sm$. If a thing is not material $\neg Ms$, then it is spiritual ***Sm***, that is, $\neg Ms \rightarrow Sm$. Similarly if a thing is not spiritual $\neg Sm$ then it is material ***Ms*** that is, $\neg Sm \leftrightarrow Ms$. If a thing is neither immaterial nor spiritual $\neg Ms \vee \neg Sm$, then such a thing is not a member of the set $Ms \leftrightarrow_1 Sm = I$. In other words, such a thing does not exist, meaning that $\neg Ms \vee \neg Sm = 0$. Similarly, if a thing is not material and not spiritual $\neg Ms \wedge \neg Sm$, then it does not exist $\neg Ms \wedge \neg Sm = 0$. However, since the direction of a thing's emphasis is determined by other factors like cultural values, belief-systems, human inner and sensory faculties et cetera, it can be said that whereas a thing can have both material and spiritual dimensions, a thing cannot be both matter and spirit $MS = SM$ in the sense that they both have absolutely equal valued-emphasis per individual, context, period, culture, et cetera. This means that valued-emphases are relative to time, context, object, place, culture et cetera.

From the foregoing, some basic axioms for a propositional calculus of Harmonious monism, which is here denoted as H_C meaning calculus of Harmonious monism can be derived as follows:

1. Universe of Discourse $(\forall x)\ (Ms \leftrightarrow_1 Sm) = I$ or $(\forall x)\ (Sm \leftrightarrow_2 Ms) = I$
2. Double Negation $\neg\neg Ms = Ms$
   $\neg\neg Sm = Sm$
3. Disjunction: $Ms \vee Sm$
4. Conjunction: $Ms \wedge Sm$
5. Commutation $Ms \vee Sm = Sm \vee Ms$
   $Ms \wedge Sm = Sm \wedge Ms$
6. Implication: $\neg Ms \rightarrow Sm$
   $\neg Sm \rightarrow Ms$

One can deduce other axioms following the classical tradition of formal logic as a calculus-representation of the computation of possibilities. For instance, if a thing is judged as material; as a member of the set of reality which is a continuum of the material and spiritual, we may not also rule out the possibility that the thing is spiritual. This gives rise to the addition law. We can also infer one of the conjuncts from a conjunction of things that are conceived as material and spiritual. This informs the law of simplification. If a thing is immaterial, it should therefore be spiritual. If it is spiritual, then it is immaterial. This informs the law of modus tolens. If a thing is material, it should therefore not be spiritual. If it is not spiritual, then it is material. This informs the law of modus ponens. Thus, the following argument-forms can be considered to be well formed formulas in the attempt at a calculus of Harmonious monism.

7. Disjunctive Syllogism: i. $\dfrac{\begin{array}{c}Ms \vee Sm\\ \neg Sm\end{array}}{Ms}$ ii.. $\dfrac{\begin{array}{c}Sm \vee Ms\\ \neg Ms\end{array}}{Sm}$
8. Addition: i. $\dfrac{Ms}{Ms \vee Sm}$ ii. $\dfrac{Sm}{Sm \vee Ms}$

9. Simplification: i. $\dfrac{Ms \wedge Sm}{Ms}$ ii. $\dfrac{Ms \wedge Sm}{Sm}$
10. Modus Tolens: i. $\dfrac{\neg Ms \to Sm \quad \neg Sm}{\neg Ms}$ ii. $\dfrac{\neg Sm \to Ms \quad \neg Ms}{\neg Sm}$

11. Modus Ponens: i. $\dfrac{\neg Ms \to Sm \quad \neg Ms}{Sm}$ ii. $\dfrac{\neg Sm \to Ms \quad \neg Sm}{Ms}$

12. Transposition: $(\neg Ms \to Sm) \leftrightarrow_2 (\neg Sm \to Ms)$
$(\neg Sm \to Ms) \leftrightarrow_2 (\neg Ms \to Sm)$

13. Material Implication: $\neg Ms \to Sm \leftrightarrow_2 (Ms \vee Sm)$

14. Idempotence: $P \leftrightarrow_2 (Ms \vee Ms)\ (Sm \vee Sm)$
$P \leftrightarrow_2 (Ms \wedge Ms)\ (Sm \wedge Sm)$

From the foregoing, in order to preserve the continuum-thesis, a thing is material only because its immaterial side is less emphasized. A thing is spiritual only because its material side is less emphasized. In this way, the identity of a thing is preserved with the continuum-thesis. By implication, the identity of a thing is somewhat determined by its epistemic or valued-emphasis. This immediately underscores the conscious effort to preserve in the symbolism, the two possibilities of the materiality and the immateriality of all that constitute reality. So if valued-emphasis is a product of human judgement, irrespective of what the basis of this judgement is, then truth-values are also to be seen as examples of valued-emphasis. In this context, they are epistemic judgmental emphasis. In this respect, since reality in the anthropocentric context is a continuum of unequalled valued-emphasis; then truth is just the valued-emphasis over falsehood. Thus, in symbolizing truth-functions, H_C requires new semantics.

Of course, the tolerance principle allows one to generate one's own syntax and semantics. The Lindenbaum Tarski semantics of truth-values is a good example in this regard. So, in order to preserve in the semantics of H_C, the supposition that reality is a continuum of unequalled two dimensional possibilities of valued-emphasis; Truth (*T*) is here symbolized as (T_f) and False is here symbolized as (F_t). The plausibility of this position lies in the fact that sometimes we assign truth-values to propositions and later it turns out that what we assigned to these propositions were in reality the contrary. For instance, Mr. *A* is convicted of a crime *C* and the value True (*T*) is assigned to the proposition: "Mr. *A* is guilty of the crime *C"*. If somehow evidence shows up proving that Mr. *A* is innocent; it means that we had earlier assigned a wrong truth-value to the proposition: "Mr. *A* is guilty of the crime *C"*. In other words, we had assigned *T* to what is actually *F.* This reflects a valued-emphasis that does not correspond to actuality.

The lesson from this is that what is true today may be false tomorrow and what is false today may be true tomorrow. Therefore our ascription of truth-values to a proposition at any point in time does not nullify the possibility that the contrary might be the case. It is in this respect that the proposed semantics for H_C shows that truth-valuation is not necessarily a one-to-one correspondence of proposition to reality in the direction of naïve realism or that the truth-function of a proposition does not necessarily imply the real state of affairs. Moreover, if what was assigned T_f turns out to be F_t; in the semantics of H_C, it simply shows that the valued-emphasis is what was mistaken but reality was still preserved as the de-emphasized possibility. Given these semantics, it is also very easy to attempt a truth-table representation for the axioms 1 to 6. Given also that only material and spiritual dimensions inform the valued-emphasis that generates a finite binary set of reality *I*; then 2^n defines the number of rows in the truth table. Given that n = 2, then a typical truth table for H_C can be constructed as follows:

Ms	**Sm**	**¬ Ms**	**¬ Sm**	**Ms ∨ Sm**	**Ms ∧ Sm**	**¬Ms→ Sm**
T_f	T_f	F_t	F_t	T_f	T_f	T_f
T_f	F_t	F_t	T_f	T_f	F_t	T_f
F_t	T_f	T_f	F_t	T_f	F_t	T_f
F_t	F_t	T_f	T_f	F_t	F_t	F_t

It is also possible to derive rules of replacement for H_C using truth-table analysis. Example it can be shown by comparing columns 5 and 7 that:

15. **¬Ms→ Sm = Ms ∧ Sm**

It is hoped that an extension of the truth-table by the addition of the bi-conditional and other well formed formulas that are deducible from 1 to 14 can give rise to more rules of replacement. For want of space, this shall not be undertaken in this paper. Nevertheless, so far, it has been shown that the context proposition defining the ontological worldview of harmonious monism can be completely axiomatized like the experimental propositions of quantum mechanics or the future contingent proposition of Łukasiewicz. .

Conclusion

There have been cases where Africans have been denied certain rational abilities by their Western counterparts. For instance, Placide Tempels thought that whereas the West can separate Being from its attributes, the Bantus cannot. This is more like saying that the latter cannot separate substance from accidents. In a similar dimension, Levy Bruhl did say that third-world peoples have pre-logical mentality (Ijiomah *Harmonious Monism* 4-5). This is just like saying that third-world peoples like Africans have no logic. But if the failure of the African to present logic in the formal dimension was Bruhl's reason for such a fallacious denial, then it is high time Africans try to present logic also in the format that is generally recognized by professional logicians rather than dwell extensively on long argumentative essays.

The replacement of the distributive law with the orthomodularity law in quantum logic arises from the consistent effort to preserve in the formalism, the empirical meaning that conjunction does not stand for simultaneous measurement or measurability of two properties of a quantum system. Ijiomah also tries to show that in Harmonious monism, conjunction has a different interpretation from the classical understanding, thereby announcing the many-valued dimension of modality within the context of the African's worldview-understanding of the relations between matter and spirit . These two examples show us that any of the classical laws can be replaced by a non-classical one depending on the concrete situation that is represented in propositional form and also depending on how the mathematical operators are interpreted to preserve the ideas preserved in the context propositions. Nevertheless, the attempt to axiomatize the context proposition of African worldview in this paper does away with the tradition of introducing the third truth-value to manage the problem of regimentation. It rather preserves in the syntax and semantics, the continuum theory of relations such that two-valued character, which is always explicitly or implicitly preserved in logical calculi is somehow retained.

Just as a geometry structuration of quantum logic is used in designing the motherboards of micro-computers in their ever perfecting specificities like Android, laptops, palm-tops, et cetera; if Africans feel the need to develop an indigenous computer that will assist in the development of African science, they will have to develop a computational calculus that will assist in the development of African computer languages and applications. Of course binary semantics are still the bedrock of computer computations; hence, the indispensability of two-valued logic. Nevertheless, an African calculus need not be restricted to the world-view proposition of the relation between the material and the spiritual as this is not the only issue the African grapples with in his day-to-day strive to make sense out of life. Therefore, taking African logic into the formalized dimension may not resolve the problem of regimentation, but may offer Africans some theoretical breakthroughs in the science of computations, which may generate into the invention of African computational machines and technology. It is hoped that this paper with all its many limitations can stand as a clarion call to African academics to take up this challenge.

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