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Spline Interpolations besides Widely Used Growth Models for the Fork Length of Flathead Trout Called *Salmo Platycephalus* Population from Zamanti Stream of Seyhan River in Turkey

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Highlights

For growth models, It was given spline interpolations besides classical models.
Spline interpolations were used for consecutive two data points.
Spline interpolations have the best fit according to the criteria of SSE.
The possible measurement error will not affect the entire data set.
With spline interpolations, the estimates of intermediate values could be made more precise.

Abstract

In this study, the fork length growth of the flathead trout (*Salmo platycephalus*) found in Zamanti Stream of Seyhan River were investigated. For this purpose, for growth curve, spline interpolation functions, alternative modeling passing to exactly all data points with respect to widely used models such as von Bertalanffy, logistic and Gompertz models, applied to growth data set were used. These interpolation functions and growth models used were examined separately for female, male and combined sex. These interpolation models used are linear spline, quadratic spline and cubic spline functions. The fork length curves of these spline interpolation functions and growth models were shown on the same graph. Thus, the differences in these models have been observed. It could be say that different spline interpolation functions especially cubic spline interpolation functions, used for each consecutive two data points, compared with other classical models have the best fit according to the criteria of the error sum of squares (SSE). The estimates for some intermediate values were done by using these spline interpolations and growth models.
Keywords: Spline Interpolations, Growth models.

1. Introduction

In this study, alternative models besides widely used growth models such as von Bertalanffy, logistic, Gompertz models were investigated. An alternative model could be polynomial interpolation. But, piecewise polynomial interpolations will be presented in this study.

Growth parameters of fish populations are important data for fisheries management and vary among populations (Alp et al., 2011). In this study, the fork length growth of the flathead trout (*Salmo platycephalus*) found in Zamantı Stream of Seyhan River were investigated. For that reason, some growth models were used.

The data given such as (t_0,y_0), (t_1,y_1), …, (t_n,y_n) can be interpolated by a polynomial $P_n(t)$ of degree n or less so that the curve of $P_n(t)$ passes through these n+1 points. If n is a large, there may be trouble: $P_n(t)$ may tend to oscillate for t between the nodes $t_0,t_1,\ldots,t_n$. Hence we must be prepared for numeric instability (Kreyszig, 2006). Although polynomial interpolation is valid on non-equally spaced discrete grids, it may develop a polynomial wiggle. There exists an alternative method to overcome the limitations of polynomial interpolation. If the entire interpolation interval is decomposed into smaller intervals connected at the given data points, the degree of interpolating polynomials can be reduced to avoid the polynomial wiggle. This idea leads to the spline interpolation (Grasselli and Pelinovsky, 2008). For that reason, a polynomial between each consecutive two data points can be found. So, piecewise polynomials for all data set will be obtained. As known, if the lower degree polynomials are independent of each other, a piecewise approximation is obtained. An alternate approach is to fit a lower degree polynomial to connect each pair of data points and to require the set of lower degree polynomials to be consistent with each other in some sense. This type of polynomial is called a spline interpolation function or simply a spline (Hoffman, 2001). Spline interpolation is frequently preferred over polynomial interpolation because the interpolation error could be made small even when low degree polynomials are used for the spline interpolation.

Since the interpolation error can be made small even when using low degree polynomials for the spline, spline interpolation is preferred over polynomial interpolation.

Splines can be of any degree. Linear splines straight line segments connected each pair of data points. Linear splines are independent of each other from interval to interval. Linear splines yield first order approximating polynomials. The slopes (i.e., first derivatives) and curvature (i.e., second derivatives) are discontinuous at every data point. Quadratic splines yield second order approximating polynomials. The slopes of the quadratic splines can be forced to be continuous at each data point, but the curvatures are still discontinuous. A cubic spline yields a third degree polynomial connecting each pair of data points. The slopes and curvatures of the cubic splines can be forced to be continuous at each data point. In fact, these requirements are necessary to obtain the additional conditions required to fit a cubic polynomial to two data points. Higher degree splines can be defined in a similar manner. However, cubic splines have proven to be a good comprise between accuracy and complexity (Hoffman, 2001). Cubic spline functions are the most popular spline functions, for a variety of reasons. They are a smooth function with which to fit data; and when used for interpolation, they do not have the oscillatory behavior that characterizes high degree polynomial interpolation. Splines are also fairly simple to calculate and use (Atkinson, 1978). Consequently, cubic spline functions with linear and quadratic spline functions are given in this manuscript.

Instead of applying one polynomial to all data set, in order to minimize errors, this model is based on applying many polynomials (linear, quadratic or cubic) to the data in corresponding intervals (Türker and Can, 1997).

By using these spline interpolation functions, the estimate could be made for the intermediate values between 1 year and 10 years. Actually, cubic spline gave us the best estimate value among the spline interpolation functions.

Some authors used cubic spline interpolation by using certain knot points in their study about lactation. These authors determined several knot points instead of all lactation days and then they used cubic spline function (Şahin and Efe, 2010). But that cubic spline function did not pass the observed average daily milk yield for lactation days**.** Nowadays, it is not difficult to construct spline functions which pass through all the data set, since it is easy to make calculations using computers. Therefore, in this study, the spline functions passing through all the data set were investigated. Korkmaz (2010) was used Lagrange and Newton interpolation functions passing through all the data set in his study. But, in this study, spline interpolation functions were used for growth models.

2. Materials and Methods

In this study, the mean fork lengths of the growth features of the flathead trout found in Zamantı Stream of Seyhan River were used (Kara et al., 2011). All procedures involving fish were approved by the University of Kahramanmaras¸ Animal Care and Use Committee. For the presentation of the spline interpolation functions, the values of the fork lengths (cm) in the age groups with female, male and combined sex of *S. platycephalus* from Zamantı Stream of the River Seyhan in their articles were used in this study in Table 1. In this study, some classical growth models used are given in Table 2.

The conditions for linear, quadratic and cubic spline interpolation fit are that a set of linear, quadratic and cubic spline interpolation through the data points was passed by using new linear, quadratic and cubic interpolation in each interval, respectively (Gerald and Wheatley, 1989).

Linear, quadratic and cubic spline interpolations are to be constructed to interpolate the Equation (1). On each interval $[t_0,t_1],[t_1,t_2],\dots,[t_{n-1},t_n]$, these spline interpolations are given by a different linear, quadratic and cubic polynomials, respectively.

$$S(t)=\begin{cases} S_0(t) & t\in[t_0,t_1] \\ S_1(t) & t\in[t_1,t_2] \\ \dots\dots\dots\dots\dots\dots \\ \dots\dots\dots\dots\dots\dots \\ \dots\dots\dots\dots\dots\dots \\ S_{n-1}(t) & t\in[t_{n-1},t_n] \end{cases} \qquad (1)$$

The polynomials S_{i-1} and S_i interpolate the same value at the point t_i and therefore the Equation (2) is given

$$S_{i-1}(t_i)=y_i=S_i(t_i) \qquad (1\le i\le n-1) \qquad (2)$$

Hence, S is automatically continuous (Kincaid and Cheney, 1996).

3. Results and Discussion

The functions of linear, quadratic and cubic splines and von Bertalanffy, logistic and Gompertz models were tested for the fork length growth of the flathead trout in Zamanti Stream of Seyhan River. For that reason, by using Table 1, linear, quadratic and cubic splines were given in Tables 3, 4 and 5, respectively and then the functions of von Bertalanffy, logistic and Gompertz models were given in Table 6. It is known that the functions of linear, quadratic and cubic splines are piecewise polynomials. So the functions of linear, quadratic and cubic splines can be separately investigated for each interval. Furthermore, how to change these functions on each interval can be separately observed. But, the functions of von Bertalanffy, logistic and Gompertz models do not give us separate results on each interval. These models give only general result.

By using Tables 3, 4 and 5, the rates of the functions of linear, quadratic and cubic splines could be found for each interval. In Table 3, the rates of the linear splines for female, male and combined sex are the coefficients of the time, t, on each interval, respectively. Because of that, these rates are seen easily in Table 3. So the rates of linear splines were not given separately. For example, the rates of linear splines for female, male and combined sex were found as 4.35, 3.57 and 4.91 on the first interval. These rates have the largest values for the linear splines for combined sex and female. While the rate of linear spline for female has the largest value, the rate of linear spline for male has the minimum value on the first interval. Similarly, by using Tables 4 and 5, the rates of the quadratic and cubic splines for female, male and combined sex could be found by taking the first derivative of the piecewise functions on each interval. While the rates of linear splines are constant on each interval, the rates of the quadratic and cubic splines are not constant on each interval.

The rates of quadratic and cubic splines were given in Tables 7 and 8. For example, the rates of quadratic splines for female, male and combined sex were found as 6.3554, 5.4771 and 7.4516 on the time 1.5 years, respectively. Furthermore; the rates of cubic splines for female, male and combined sex were found as 4.5958, 4.1114 and 5.5132 on the time 1 year.

von Bertalanffy, logistic and Gompertz models are given with their parameters in Table 9. According to von Bertalanffy, logistic and Gompertz models, the parameter a gives the average asymptotic value when time goes to infinity. The parameters, b and c show the coefficients about the growth. The found values for these parameters are given in Table 9. For example, according to von Bertalanffy, the average asymptotic values of fork length for female, male and combined sex were found as 58.8761, 62.9475 and 56.5416 cm, respectively. The coefficients about the growth, b and c for female, male and combined sex were found as 0.0968, -1.7546; 0.0792, -2.3989; 0.1042, -1.6131, respectively. However, any asymptotic value could not find for spline interpolation functions since spline interpolation functions are not valid generally outside the range of the data.

The estimates of spline interpolation functions for each data are exactly same with the observed value. So, the error sum of squares (SSE) of linear spline, quadratic spline and cubic spline functions are always zero. However, estimates of other models except Lagrange, Spline and Newton interpolation functions for each data are not exactly same with the observed value. So, the error sum of squares of other models except Lagrange, Spline and Newton interpolation functions are not zero.

Table 1. The mean values of fork lengths (cm) of *S. platycephalus* from Zamanti Stream of the River Seyhan

Sex / Age (year)	1	2	3	4	5	6	7	8	9	10
Female	13.68	18.03	21.63	25.55	28.29	30.85	33.37	36.03	38.30	40.0
Male	15.15	18.72	20.91	25.11	27.97	30.88	33.13	35.94	37.04	39.20
Combined sex	13.26	18.17	21.21	25.33	28.16	30.86	33.28	35.79	37.88	39.73

Table 2. Models used in the growth curves

Models	Equation
Von Bertalanffy	y=a(1-exp(-b(t-c)))
Logistic	y=a/(1+exp(b-ct))
Gompertz	y=a.exp(-exp(b-ct))
where: y = the fork length (cm), exp(1)= e (the base of natural logarithm) a = the average asymptotic length, b and c = the coefficients about growth	

Table 3. Linear splines for female, male and combined sex

Splines	$S_{1f}(t)$ for female	$S_{1m}(t)$ for male	$S_{1c}(t)$ for combined sex
Linear	$\begin{cases} 9.33+4.35t, & 1\le t\le 2 \\ 10.83+3.60t, & 2\le t\le 3 \\ 9.87+3.92t, & 3\le t\le 4 \\ 14.59+2.74t, & 4\le t\le 5 \\ 15.49+2.56t, & 5\le t\le 6 \\ 15.73+2.52t, & 6\le t\le 7 \\ 14.75+2.66t, & 7\le t\le 8 \\ 17.87+2.27t, & 8\le t\le 9 \\ 23.00+1.70t, & 9\le t\le 10 \end{cases}$	$\begin{cases} 11.58+3.57t, & 1\le t\le 2 \\ 14.34+2.19t, & 2\le t\le 3 \\ 8.31+4.20t, & 3\le t\le 4 \\ 13.67+2.86t, & 4\le t\le 5 \\ 13.42+2.91t, & 5\le t\le 6 \\ 17.38+2.25t, & 6\le t\le 7 \\ 13.46+2.81t, & 7\le t\le 8 \\ 27.14+1.10t, & 8\le t\le 9 \\ 17.60+2.16t, & 9\le t\le 10 \end{cases}$	$\begin{cases} 8.35+4.91t, & 1\le t\le 2 \\ 12.09+3.04t, & 2\le t\le 3 \\ 8.85+4.12t, & 3\le t\le 4 \\ 14.01+2.83t, & 4\le t\le 5 \\ 14.66+2.70t, & 5\le t\le 6 \\ 16.34+2.42t, & 6\le t\le 7 \\ 15.71+2.51t, & 7\le t\le 8 \\ 19.07+2.09t, & 8\le t\le 9 \\ 21.23+1.85t, & 9\le t\le 10 \end{cases}$
where t is the age (year), $S_{1f}(t)$, $S_{1m}(t)$ and $S_{1c}(t)$ are the values of the fork length (cm) for linear spline.			

Table 4. Quadratic splines for female, male and combined sex

Splines	Quadratic
$S_{2f}(t)$ for female	$\begin{cases} 13.6800+6.3554(t-1)^2, & 1\le t\le 1.5 \\ 8.6514+4.6892t-1.6661(t-2)^2, & 1.5\le t\le 2.5 \\ 10.6372+3.6643t+0.6410(t-3)^2, & 2.5\le t\le 3.5 \\ 11.9294+3.4052t-0.9001(t-4)^2, & 3.5\le t\le 4.5 \\ 15.5659+2.5448t+0.0398(t-5)^2, & 4.5\le t\le 5.5 \\ 15.6942+2.5260t-0.0586(t-6)^2, & 5.5\le t\le 6.5 \\ 15.0344+2.6194t+0.1520(t-7)^2, & 6.5\le t\le 7.5 \\ 16.2074+2.4778t-0.2936(t-8)^2, & 7.5\le t\le 8.5 \\ 18.1967+2.2337t+0.0495(t-9)^2, & 8.5\le t\le 9.5 \\ 40.0+0.t-2.2832(t-10)^2, & 9.5\le t\le 10 \end{cases}$
$S_{2m}(t)$ for male	$\begin{cases} 15.15+5.4771(t-1)^2, & 1\le t\le 1.5 \\ 12.0683+3.3259t-2.1512(t-2)^2, & 1.5\le t\le 2.5 \\ 11.6556+3.0848t+1.9102(t-3)^2, & 2.5\le t\le 3.5 \\ 10.2090+3.7252t-1.2697(t-4)^2, & 3.5\le t\le 4.5 \\ 13.9514+2.8037t+0.3482(t-5)^2, & 4.5\le t\le 5.5 \\ 15.6857+2.5324t-0.6196(t-6)^2, & 5.5\le t\le 6.5 \\ 14.6360+2.6420t+0.7292(t-7)^2, & 6.5\le t\le 7.5 \\ 21.0947+1.8557t-1.5155(t-8)^2, & 7.5\le t\le 8.5 \\ 20.2635+1.8641t+1.5239(t-9)^2, & 8.5\le t\le 9.5 \\ 39.2+0.t-3.3880(t-10)^2, & 9.5\le t\le 10 \end{cases}$
$S_{2c}(t)$ for combined sex	$\begin{cases} 13.26+7.4516(t-1)^2, & 1\le t\le 1.5 \\ 8.6964+4.7368t-2.7148(t-2)^2, & 1.5\le t\le 2.5 \\ 11.0721+3.3793t+1.3573(t-3)^2, & 2.5\le t\le 3.5 \\ 10.8201+3.6274t-1.1091(t-4)^2, & 3.5\le t\le 4.5 \\ 14.8806+2.6559t+0.1376(t-5)^2, & 4.5\le t\le 5.5 \\ 15.5166+2.5572t-0.2362(t-6)^2, & 5.5\le t\le 6.5 \\ 15.9153+2.4807t+0.1596(t-7)^2, & 6.5\le t\le 7.5 \\ 17.5601+2.2787t-0.3616(t-8)^2, & 7.5\le t\le 8.5 \\ 17.6581+2.2469t+0.3297(t-9)^2, & 8.5\le t\le 9.5 \\ 39.73-2.5766(t-10)^2, & 9.5\le t\le 10 \end{cases}$
where t is the age (year), $S_{2f}(t)$, $S_{2m}(t)$ and $S_{2c}(t)$ are the values of the fork length (cm) for quadratic spline.	

Table 5. Cubic splines for female, male and combined sex

Splines	Cubic
$S_{3f}(t)$ for female	$\begin{cases} 9.0842+4.5958t-0.2458(t-1)^3, & 1\le t\le 2 \\ 10.3134+3.8583t-0.7375(t-2)^2+0.4792(t-2)^3, & 2\le t\le 3 \\ 10.1673+3.8209t+0.7001(t-3)^2-0.6010(t-3)^3, & 3\le t\le 4 \\ 11.8775+3.4181t-1.1029(t-4)^2+0.4247(t-4)^3, & 4\le t\le 5 \\ 15.8570+2.4866t+0.1713(t-5)^2-0.0979(t-5)^3, & 5\le t\le 6 \\ 15.6370+2.5355t-0.1224(t-6)^2+0.1069(t-6)^3, & 6\le t\le 7 \\ 15.0901+2.6114t+0.1983(t-7)^2-0.1497(t-7)^3, & 7\le t\le 8 \\ 15.5592+2.5589t-0.2509(t-8)^2-0.0380(t-8)^3, & 8\le t\le 9 \\ 20.8113+1.9432t-0.3648(t-9)^2+0.1216(t-9)^3, & 9\le t\le 10 \end{cases}$
$S_{3m}(t)$ for male	$\begin{cases} 11.0386+4.1114t-0.5414(t-1)^3, & 1\le t\le 2 \\ 13.7457+2.4871t-1.6243(t-2)^2+1.3272(t-2)^3, & 2\le t\le 3 \\ 11.2499+3.2200t+2.3572(t-3)^2-1.3772(t-3)^3, & 3\le t\le 4 \\ 9.8989+3.8028t-1.7744(t-4)^2+0.8316(t-4)^3, & 4\le t\le 5 \\ 14.2257+2.7489t+0.7205(t-5)^2-0.5593(t-5)^3, & 5\le t\le 6 \\ 15.8092+2.5118t-0.9575(t-6)^2+0.6947(t-6)^3, & 6\le t\le 7 \\ 14.3423+2.6840t+1.1297(t-7)^2-1.0037(t-7)^3, & 7\le t\le 8 \\ 20.4808+1.9324t-1.8813(t-8)^2+1.0489(t-8)^3, & 8\le t\le 9 \\ 25.1919+1.3165t+1.2653(t-9)^2-0.4218(t-9)^3, & 9\le t\le 10 \end{cases}$
$S_{3c}(t)$ for combined sex	$\begin{cases} 7.7468+5.5132t-0.6032(t-1)^3, & 1\le t\le 2 \\ 10.7628+3.7036t-1.8096(t-2)^2+1.1460(t-2)^3, & 2\le t\le 3 \\ 10.6426+3.5225t+1.6285(t-3)^2-1.0309(t-3)^3, & 3\le t\le 4 \\ 10.5836+3.6866t-1.4644(t-4)^2+0.6078(t-4)^3, & 4\le t\le 5 \\ 15.2542+2.5812t+0.3589(t-5)^2-0.2401(t-5)^3, & 5\le t\le 6 \\ 15.3873+2.5788t-0.3613(t-6)^2+0.2025(t-6)^3, & 6\le t\le 7 \\ 16.0339+2.4637t+0.2462(t-7)^2-0.2000(t-7)^3, & 7\le t\le 8 \\ 16.9396+2.3563t-0.3537(t-8)^2+0.0874(t-8)^3, & 8\le t\le 9 \\ 20.6805+1.9111t-0.0916(t-9)^2+0.0305(t-9)^3, & 9\le t\le 10 \end{cases}$
where t is the age (year), $S_{3f}(t)$, $S_{3m}(t)$ and $S_{3c}(t)$ are the values of the fork length (cm) for cubic spline.	

Table 6. The functions of von Bertalanffy, Logistic and Gompertz models for female, male and combined sex

Models	Sex	Functions (y) and Their parameters (a,b,c)
Von Bertalanffy	Female	$58.8761(1-\exp(-0.0968t-0.1699))$ a=58.8761, b=0.0968, c=-1.7546
	Male	$62.9475(1-\exp(-0.0792t-0.1901))$ a=62.9475, b=0.0792, c=-2.3989
	Combined sex	$56.5416(1-\exp(-0.1042t-0.1680))$ a=56.5416, b=0.1042, c=-1.6131
Logistic	Female	$44.0499/(1+\exp(1.0350-0.3220t))$ a=44.0499, b=1.0350, c=0.3220
	Male	$44.1921/(1+\exp(0.9349-0.2948t))$ a=44.1921, b=0.9349, c=0.2948
	Combined sex	$43.2500/(1+\exp(1.0481-0.3328t))$ a=43.2500, b=1.0481, c=0.3328
Gompertz	Female	$47.8328\exp(-\exp(0.4080-0.2100t))$ a=44.1921, b=0.9349, c=0.2948
	Male	$48.7476\exp(-\exp(0.3485-0.1868t))$ a=48.7476, b=0.3485, c=0.1868
	Combined sex	$46.7305\exp(-\exp(0.4152-0.2189t))$ a=46.7305, b=0.4252, c=0.2189
where: exp(1)=e (the base of natural logarithm), t=the age (year) and y=the value of fork length (cm)		

Table 7. The first derivatives of quadratic splines for female, male and combined sex

The first derivatives of Splines	Quadratic
$D_{2f}(t)$ for female	$\begin{cases} 12.7107t-12.7107, & 1\le t\le 1.5 \\ 11.3535-3.3321t, & 1.5\le t\le 2.5 \\ -0.1819+1.2821t, & 2.5\le t\le 3.5 \\ 10.6063-1.8003t, & 3.5\le t\le 4.5 \\ 2.1469+0.0796t, & 4.5\le t\le 5.5 \\ 3.2296-0.1173t, & 5.5\le t\le 6.5 \\ 0.4909+0.3041t, & 6.5\le t\le 7.5 \\ 7.1751-0.5872t, & 7.5\le t\le 8.5 \\ 1.3435+0.0989t, & 8.5\le t\le 9.5 \\ 45.6630-4.5663t, & 9.5\le t\le 10 \end{cases}$

$D_{2m}(t)$ for male	$\begin{cases} 10.9541t - 10.9541, & 1 \le t \le 1.5 \\ 11.9307 - 4..3024t, & 1.5 \le t \le 2.5 \\ -8.3761 + 3.8203t, & 2.5 \le t \le 3.5 \\ 13.8831 - 2.5395t, & 3.5 \le t \le 4.5 \\ -0.6784 + 0.6964t, & 4.5 \le t \le 5.5 \\ 9.9672 - 1.2391t, & 5.5 \le t \le 6.5 \\ -7.5665 + 1.4584t, & 6.5 \le t \le 7.5 \\ 26.1039 - 3.0310t, & 7.5 \le t \le 8.5 \\ -25.5664 + 3.0478t, & 8.5 \le t \le 9.5 \\ -6.7759t + 67.7594, & 9.5 \le t \le 10 \end{cases}$
$D_{2c}(t)$ for combined sex	$\begin{cases} 14.9032t - 14.9032, & 1 \le t \le 1.5 \\ 15.5961 - 5.4296t, & 1.5 \le t \le 2.5 \\ -4.7647 + 2.7147t, & 2.5 \le t \le 3.5 \\ 12.5007 - 2.2183t, & 3.5 \le t \le 4.5 \\ 1.2803 + 0.2751t, & 4.5 \le t \le 5.5 \\ 5.3916 - 0.4724t, & 5.5 \le t \le 6.5 \\ 0.2459 + 0.3192t, & 6.5 \le t \le 7.5 \\ 8.0636 - 0.7231t, & 7.5 \le t \le 8.5 \\ -3.6875 + 0.6594t, & 8.5 \le t \le 9.5 \\ -5.1531t + 51.5312, & 9.5 \le t \le 10 \end{cases}$
where t is the age (year), $D_{2f}(t)$, $D_{2m}(t)$ and $D_{2c}(t)$ are the first derivatives of fork length (cm) for quadratic spline.	

Table 8. The first derivatives of cubic splines for female, male and combined sex

Splines	Cubic
$D_{3f}(t)$ for female	$\begin{cases} 4.5958 - 0.7375(t-1)^2, & 1 \le t \le 2 \\ 6.8084 - 1.4750t + 1.4376(t-2)^2, & 2 \le t \le 3 \\ -0.3797 + 1.4002t - 1.8029(t-3)^2, & 3 \le t \le 4 \\ 12.2409 - 2.2057t + 1.2742(t-4)^2, & 4 \le t \le 5 \\ 0.7734 + 0.3426t - 0.2937(t-5)^2, & 5 \le t \le 6 \\ 4.0044 - 0.2448t + 0.3207(t-6)^2, & 6 \le t \le 7 \\ -0.1651 + 0.3966t - 0.4492(t-7)^2, & 7 \le t \le 8 \\ 6.5730 - 0.5018t - 0.1139(t-8)^2, & 8 \le t \le 9 \\ 8.5092 - 0.7296t + 0.3648(t-9)^2, & 9 \le t \le 10 \end{cases}$

$D_{3m}(t)$ for male	$\begin{cases} 4.1114-1.6243(t-1)^2, & 1\le t\le 2 \\ 8.9843-3.2486t+3.9815(t-2)^2, & 2\le t\le 3 \\ -10.9230+4.7144t-4.1316(t-3)^2, & 3\le t\le 4 \\ 17.9981-3.5488t+2.4949(t-4)^2, & 4\le t\le 5 \\ -4.4561+1.4410t-1.6780(t-5)^2, & 5\le t\le 6 \\ 14.0024-1.9151t+2.0872(t-6)^2, & 6\le t\le 7 \\ -13.1319+2.2594t-3.0110(t-7)^2, & 7\le t\le 8 \\ 32.0325-3.7625t+3.1466(t-8)^2, & 8\le t\le 9 \\ -21.4592+2.5306t-1.2653(t-9)^2, & 9\le t\le 10 \end{cases}$
$D_{3c}(t)$ for combined sex	$\begin{cases} 5.5132-1.8096(t-1)^2, & 1\le t\le 2 \\ 10.9421-3.6192t+3.4381(t-2)^2, & 2\le t\le 3 \\ -6.2485+3.2570t-3.0928(t-3)^2, & 3\le t\le 4 \\ 15.4014-2.9287t+1.8333(t-4)^2, & 4\le t\le 5 \\ -1.0079+0.7178t-0.7202(t-5)^2, & 5\le t\le 6 \\ 6.9142-0.7226t+0.6075(t-6)^2, & 6\le t\le 7 \\ -0.9836+0.4925t-0.5999(t-7)^2, & 7\le t\le 8 \\ 8.0149-0.7073t+0.2621(t-8)^2, & 8\le t\le 9 \\ 3.5596-0.1832t+0.0916(t-9)^2, & 9\le t\le 10 \end{cases}$
where t is the age (year), $D_{3f}(t)$, $D_{3m}(t)$ and $D_{3c}(t)$ are the first derivatives of fork length (cm) for cubic spline.	

Table 9. The first derivatives of Von Bertalanffy, Logistic and Gompertz models for female, male and combined sex

Models	Sex	The first derivatives (dy/dt)
Von Bertalanffy	Female	$5.7018\exp(-0.0968t-0.1699))$
	Male	$4.9877\exp(-0.0792t-0.1901))$
	Combined sex	$5.8894\exp(-0.1042t-0.1680))$
Logistic	Female	$14.1853\exp(1.0350-0.3220t)/(1+\exp(1.0350-0.3220t))^2$
	Male	$13.0257\exp(0.9349-0.2948t)/(1+\exp(0.9349-0.2948t))^2$
	Combined sex	$14.3932\exp(1.0481-0.3328t)/(1+\exp(1.0481-0.3328t))^2$
Gompertz	Female	$10.0437\exp(0.4080-0.2100t-\exp(0.4080-0.2100t))$
	Male	$9.1083\exp(0.3485-0.1868t-\exp(0.3485-0.1868t))$
	Combined sex	$10.2279\exp(0.4152-0.2189t-\exp(0.4152-0.2189t))$
where: exp(1)=e (the base of natural logarithm), t=the age (year) and dy/dt=the growth rate of fork length		

Table 10. The observed and estimated fork length (cm) according to linear spline, quadratic spline, cubic spline and Von Bertalanffy, Logistic and Gompertz models for female (F), male (M) and combined sex (C) with their Error Sum of Squares (SSE)

Models	Sex	Age (year)										
		1	2	3	4	5	6	7	8	9	10	SSE
Observed Fork Legth (cm)	F	13.68	18.03	21.63	25.55	28.29	30.85	33.37	36.03	38.30	40.0	
	M	15.15	18.72	20.91	25.11	27.97	30.88	33.13	35.94	37.04	39.20	
	C	13.26	18.17	21.21	25.33	28.16	30.86	33.28	35.79	37.88	39.73	
Linear Spline	F	13.68	18.03	21.63	25.55	28.29	30.85	33.37	36.03	38.30	40.0	0
	M	15.15	18.72	20.91	25.11	27.97	30.88	33.13	35.94	37.04	39.20	0
	C	13.26	18.17	21.21	25.33	28.16	30.86	33.28	35.79	37.88	39.73	0
Quadratic Spline	F	13.68	18.03	21.63	25.55	28.29	30.85	33.37	36.03	38.30	40.0	0
	M	15.15	18.72	20.91	25.11	27.97	30.88	33.13	35.94	37.04	39.20	0
	C	13.26	18.17	21.21	25.33	28.16	30.86	33.28	35.79	37.88	39.73	0
Cubic Spline.	F	13.68	18.03	21.63	25.55	28.29	30.85	33.37	36.03	38.30	40.0	0
	M	15.15	18.72	20.91	25.11	27.97	30.88	33.13	35.94	37.04	39.20	0
	C	13.26	18.17	21.21	25.33	28.16	30.86	33.28	35.79	37.88	39.73	0
Von Bertalanffy	F	13.79	17.95	21.73	25.15	28.27	31.09	33.66	35.98	38.10	40.02	0.37
	M	14.86	18.52	21.91	25.03	27.92	30.59	33.06	35.33	37.44	39.38	1.79
	C	13.47	17.73	21.57	25.03	28.15	30.96	33.49	35.77	37.82	39.67	0.52
Logistic	F	14.49	17.77	21.27	24.80	28.19	31.29	34.00	36.28	38.13	39.60	2.27
	M	15.26	18.32	21.54	24.78	27.91	30.81	33.39	35.61	37.49	38.98	1.10
	C	14.20	17.54	21.09	24.67	28.08	31.18	33.85	36.07	37.85	39.24	2.48
Gompertz	F	14.14	17.81	21.47	24.99	28.26	31.22	33.85	36.14	38.11	39.79	1.06
	M	15.05	18.38	21.71	24.92	27.94	30.72	33.23	35.47	37.45	39.17	1.23
	C	**13.84**	**17.58**	**21.30**	**24.86**	**28.15**	**31.09**	**33.69**	**35.93**	**37.83**	**39.44**	**1.24**

Table 11. Some estimates of Fork length (cm) according to spline interpolation functions and Von Bertalanffy, Logistic and Gompertz models for female (F), male (M) and combined sex (C)

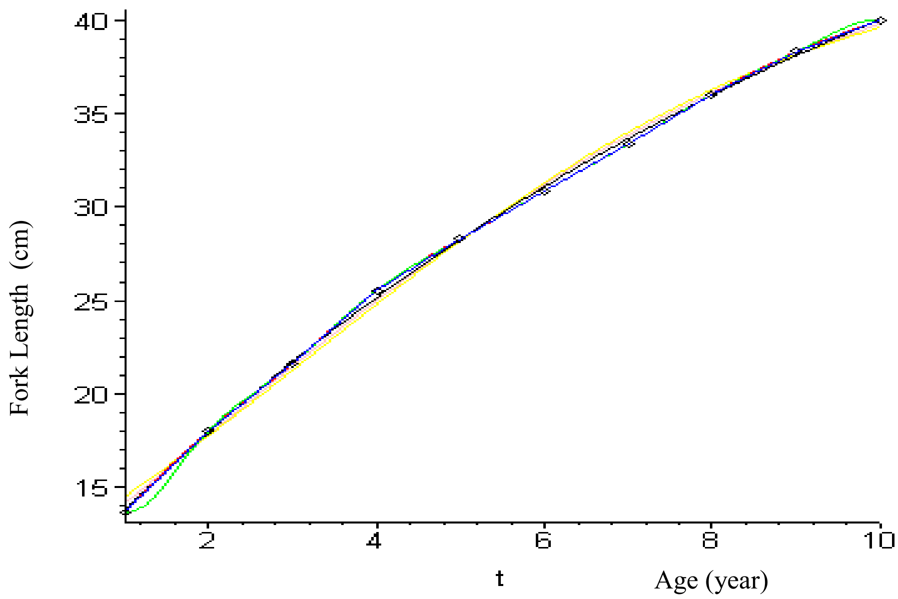
Models	Sex	Intermediate Age (years)								
		1.5	2.5	.53	4.5	5.5	6.5	7.5	8.5	9.5
Linear Spline	F	15.86	19.83	23.59	26.92	29.57	32.11	34.70	37.17	39.15
	M	16.94	19.82	23.01	26.54	29.43	32.01	34.54	36.49	38.12
	C	15.72	19.69	23.27	26.75	29.51	32.07	34.54	36.84	38.81
Quadratic Spline	F	15.27	19.96	23.62	27.03	29.57	32.10	34.72	37.20	39.43
	M	16.52	19.85	22.93	26.66	29.46	31.99	34.63	36.49	38.35
	C	15.12	19.86	23.24	26.87	29.52	32.08	34.56	36.84	39.09
Cubic Spline.	F	15.95	19.83	23.64	27.04	29.56	32.10	34.71	37.24	39.20
	M	17.14	19.72	22.94	26.67	29.45	31.98	34.63	36.57	37.96
	C	15.94	19.71	23.25	26.88	29.51	32.08	34.55	36.89	38.82

Von Bertalanffy	F	15.92	19.88	23.48	26.75	29.71	32.41	34.85	37.07	39.08
	M	16.73	20.25	23.50	26.51	29.28	31.85	34.22	36.41	38.43
	C	15.66	19.70	23.35	26.63	29.59	32.26	34.66	36.82	38.77
Logistic	F	16.10	19.50	23.04	26.52	29.78	32.70	35.20	37.26	38.91
	M	16.76	19.91	23.16	26.37	29.39	32.14	34.55	36.58	38.27
	C	15.83	19.30	22.89	26.40	29.68	32.57	35.02	37.01	38.59
Gompertz	F	15.96	19.65	23.26	26.66	29.78	32.58	35.03	37.16	38.98
	M	16.71	20.06	23.33	26.45	29.36	32.01	34.39	36.50	38.34
	C	15.70	19.45	23.11	26.54	29.66	32.44	34.85	36.92	38.67

Table 12. The observed and predicted mean fork length (cm) (bold font) on the missing age 4 years for female, 5 years for male and 6 years for combined sex, according to linear spline, quadratic spline, cubic spline and Von Bertalanffy, Logistic and Gompertz models for female (F), male (M) and combined sex (C) with their Error Sum of Squares (SSE)

Models	Sex	Age (years)										
		1	2	3	**4**	5	6	7	8	9	10	SSE
Observed FL (cm)	F	13.68	18.03	21.63	**25.55**	28.29	30.85	33.37	36.03	38.30	40.0	
	M	15.15	18.72	20.91	25.11	**27.97**	30.88	33.13	35.94	37.04	39.20	
	C	13.26	18.17	21.21	25.33	28.16	**30.86**	33.28	35.79	37.88	39.73	
Linear Spline	F	13.68	18.03	21.63	**24.96**	28.29	30.85	33.37	36.03	38.30	40.0	**0.35**
	M	15.15	18.72	20.91	25.11	**28.00**	30.88	33.13	35.94	37.04	39.20	**0.001**
	C	13.26	18.17	21.21	25.33	28.16	**30.72**	33.28	35.79	37.88	39.73	**0.74**
Quadratic Spline	F	13.68	18.03	21.63	**25.08**	28.29	30.85	33.37	36.03	38.30	40.0	**0.22**
	M	15.15	18.72	20.91	25.11	**28.46**	30.88	33.13	35.94	37.04	39.20	**0.24**
	C	13.26	18.17	21.21	25.33	28.16	**30.71**	33.28	35.79	37.88	39.73	**0.02**
Cubic Spline.	F	13.68	18.03	21.63	**25.12**	28.29	30.85	33.37	36.03	38.30	40.0	**0.18**
	M	15.15	18.72	20.91	25.11	**28.55**	30.88	33.13	35.94	37.04	39.20	**0.34**
	C	13.26	18.17	21.21	25.33	28.16	**30.67**	33.28	35.79	37.88	39.73	**0.04**
Von Bertalanffy	F	13.79	17.89	21.63	**25.04**	28.15	31.00	33.59	35.95	38.11	40.08	**0.43**
	M	14.87	18.52	21.90	25.02	**27.91**	30.58	33.05	35.33	37.44	39.39	**1.77**
	C	13.47	17.73	21.57	25.03	28.15	**30.98**	33.49	35.77	37.82	39.67	**0.52**
Logistic	F	14.44	17.66	21.09	**24.59**	27.97	31.10	33.87	36.23	38.17	39.72	**2.84**
	M	15.26	18.32	21.54	24.78	**27.89**	30.81	33.39	35.61	37.49	38.98	**1.09**
	C	14.20	17.54	21.09	24.67	28.08	**31.29**	33.85	36.07	37.85	39.24	**2.55**
Gompertz	F	14.12	17.72	21.34	**24.83**	28.10	31.08	33.75	36.10	38.13	39.88	**1.17**
	M	15.05	18.38	21.71	24.92	**27.93**	30.72	33.23	35.47	37.45	39.17	**1.22**
	C	13.84	17.58	21.30	24.86	28.15	**31.17**	33.69	35.93	37.83	39.44	**1.27**

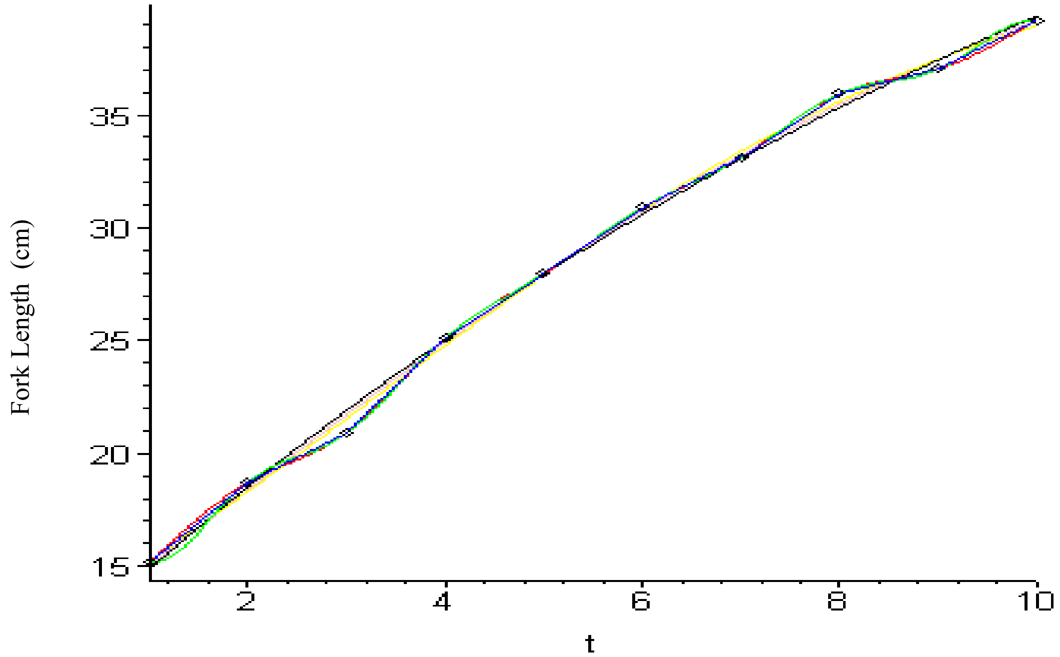
The rates of von Bertalanffy, logistic and Gompertz models for female, male and combined sex were given in Tables 9. So the rates of von Bertalanffy, logistic and Gompertz models for female, male and combined sex at any age could be calculated by using Table 9. For example, the rates of von Bertalanffy for female, male and combined sex were found as 3.5978, 3.2517 and 3.6424 on the age 3 years.
By using Table 1, the estimates of linear spline, quadratic spline, cubic spline and von Bertalanffy, logistic, Gompertz models for the fork lengths were investigated, respectively. In Table 10, the estimates of the average fork lengths of linear spline, quadratic spline and cubic spline are exactly same with the mean fork length observed. But for von Bertalanffy, logistic and Gompertz models, the difference between observed and estimated average fork lengths is quite high. So, the error sum of squares (SSE) of the spline interpolation functions for female, male and combined sex is equal to zero. However, von Bertalanffy, logistic and Gompertz models are slightly different from each other. Normally the curves of the functions or the models provide the best fit to the data in the sense that the sum of squares of the deviations is the smallest (Johnson and Bhattacharyya, 1992). So this shows that spline interpolation functions are important to use in regression. Actually, we could say that according to the criteria of SSE, the cubic spline interpolation function gave the best estimate value among the spline interpolation functions, von Bertalanffy, logistic and Gompertz models.
The fork length curves of spline interpolation functions and von Bertalanffy, logistic and Gompertz models for female (F) were given in the same graph (Figure 1).



Colors of Linear, Quadratic, Cubic splines, Von Bertalanffy, Logistic and Gompertz models for female: blue, green, red, black, yellow and pink.

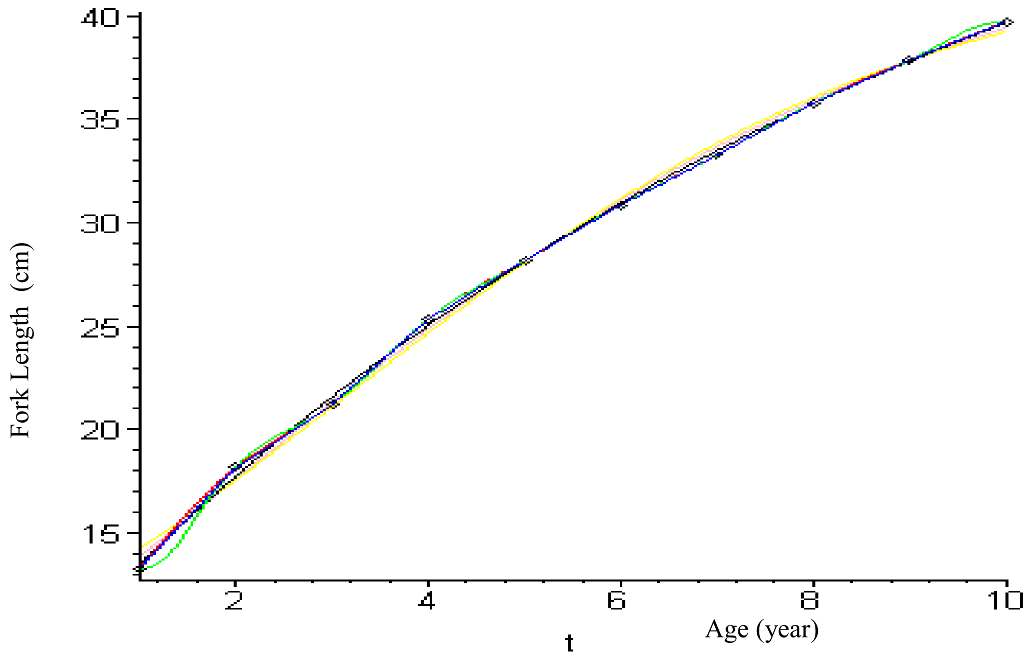
Figure 1. Fork length of *Salmo platycephalus* population for female according to the age

The Fork length curves of spline interpolation functions and von Bertalanffy, logistic and Gompertz models for male (M) were given in the same graph (Figure 2).
The Fork length curves of spline interpolation functions and von Bertalanffy, logistic and Gompertz models for combined sex (C) were given in the same graph (Figure 3).
Furthermore, by using the spline interpolation functions and von Bertalanffy, logistic and Gompertz models, the mean estimates for some intermediate ages of fork lengths were found in Table 11. Although the accuracy of these estimates could not be controlled, the estimates using especially cubic spline interpolation functions could be thought to be more accurate. For example, for the linear, quadratic, cubic spline interpolation functions and von Bertalanffy, logistic and Gompertz models, the estimated values of fork lengths for female at age 7.5 years were found as 34.70, 34.72, 34.71, 34.85, 35.20, 35.03 cm, respectively. While the values found in the linear, quadratic and cubic spline interpolation functions are close to each other, the values found in von Bertalanffy, logistic and Gompertz models is far away from those three values.



Colors of Linear, Quadratic, Cubic splines, Von Bertalanffy, Logistic and Gompertz models for male: blue, green, red, black, yellow and pink

Figure 2. Fork length of *Salmo platycephalus* population for male according to the age.



Colors of Linear, Quadratic, Cubic splines, Von Bertalanffy, Logistic and Gompertz models for combined sex: blue, green, red, black, yellow and pink.

Figure 3. Fork length of *Salmo platycephalus* population for combined sex according to the age.

Missing or incorrect observation researches could be also made by using spline interpolation functions. If the measurement has never been on any age or it is believed to be incorrect, it would be estimated by spline interpolations. For example, assume that the measured values of age 4, 5 and 6 years for female, male and combined sex are missing, respectively, and then these values were tried to be found using spline interpolation functions and von Bertalanffy, logistic and Gompertz models, then the results in Table 12 were obtained.

The estimated values of linear spline, quadratic spline and cubic spline are exactly same with the observed mean fork length. Only for the predicted fork length on the missing age 4 and 6 years for female and combined sex, respectively, slight differences were found for spline interpolation functions and von Bertalanffy model. But for logistic and Gompertz models, the difference between observed and estimated mean fork lengths is more different. However, only for the predicted fork length on the missing age 5 years for male, slight differences were found for linear spline interpolation function and von Bertalanffy, logistic and Gompertz models. But for quadratic and cubic spline interpolation functions, the difference between observed and estimated mean fork length is more different. These spline interpolation functions differ by a small amount on missing or incorrectly measured age but they have the same values on the other observed mean fork length. However, since other models, such as von Bertalanffy, logistic and Gompertz models, are obtained from all data set, values of missing or incorrectly measured age will change the whole estimate value. For that reason, it could be advised to use especially cubic spline interpolation for estimating the values of missing or incorrectly measured age.

4. Conclusions

It could be say that different spline interpolation functions especially cubic spline interpolation functions, used for each consecutive two data points, compared with other classical models have the best fit according to the criteria of SSE. Since in our study different spline interpolation functions were used for each consecutive two data points, the possible measurement error will not affect the entire data set. But, in the other classical models the possible measurement error will directly affect the all data set. With spline interpolations, the estimates of intermediate values could be made more precise. Furthermore, by using spline interpolations, some of the predicted values for missing or incorrect observation were very successful according to the values of von Bertalanffy, logistic and Gompertz models.

According to the study of literature, spline interpolation functions as in this study were not used in fisheries. By using spline interpolations, it is shown to obtain new ideas and interpretations in addition to the information of the well-known classical analysis to the investigators.

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