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Elastic Buckling Load Prediction of Thin Conical Caps with Edge Ring Constraint under Uni-axial Compression

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Abstract
This study is devoted to the effects of edge ring thickness on the critical buckling load of conical caps having different shell thicknesses and cone angles based on geometrical nonlinearity. Additionally, different buckling failure mode shapes which may occur were investigated. The material behaviours of both shell and edge ring were assumed to be elastic and linear. Finite element simulations for the selected geometries were performed in COSMOSM package program. The models were subjected to compressive loads which starts from zero to a maximum value (buckling load) with a certain load step increments. The results were obtained from the simulations for different cone angles, edge ring and shell thickness. Moreover, an empirical relationship, which estimates buckling load, was established. The paper concludes with suggestions for implementing edge ring and possibility of this application according to finite element method results.
Keywords: Edge ring, Conical shell, Elastic buckling, Axial compression, Finite element method

Nomenclature

b_{ring}	Length of the edge ring	R_1	Upper radius
E	Modulus of elasticity	R_2	Bottom radius
F	Axial force	t_{ring}	Edge Ring thickness
F_{cr}	Critical load	t_{shell}	Shell thickness
h	Height of the pipe	θ	Cone angle
k	Ratio of the shell and the edge ring thicknesses	v	Poisson's ratio

1. Introduction

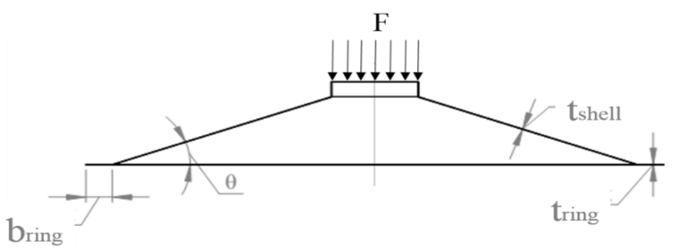
The demands in the conical shells are quite prevalent such as; space craft, robots, shelters, domes, tanks, in machinery and devices like belleville washers. Therefore, there has been a great concern for the designers to achieve maximum strength with a cost efficient solution for conical shells. In practical applications, limit loads of a structure computed by simply derived eigen-buckling formulations is not a reliable way in terms of safety aspects and generally gives higher loads than what it is in reality. Accordingly, many proposed analytical expressions of buckling load are based on linear buckling principle and not capable of estimating mode shapes. Therefore, implementation of non-linear buckling approach is a critical issue to stimulate real system response under axial compression. Effects of edge ring on the load carrying capacity of a conical cap plays important role depending on different geometrical parameters to specify critical range of edge ring geometry. It is quite indispensable to determine contribution of edge ring only by itself, as an alternative to commonly used stiffeners and similar methods against system collapse in practical applications, on the buckling load for ease of manufacture scheme. It is a substantial process to optimize a structure by understanding the individual influence of each design criteria. Numerical studies (Finite Element Method) are a leading guide in developing empirical relationship to estimate limit states of a buckling member as a function of basic parameters (i.e. overall geometry, slenderness, type of loading and boundary conditions) instead of performing numerous time and cost consuming experiments for each geometry configuration. This paper also provides the advantage of how to find out an empirical formula and system response concerning the above mentioned statements. Stability of conical shells under axial load has been studied by many prominent authors.

Weingarten et al. [1, 3] investigated experimentally the elastic stability of conical and cylindrical shells not only under axial load but also under combined internal pressure and axial compression. Singer [2] assumed Poisson's Ratio to be zero and investigated axisymmetrical buckling in conical shells under axial load. Tani and Yamaki, [4] by taking the model which was applied to solve the problem under torsion before, studied buckling of truncated conical shells under axial compression in different boundary conditions. Tong et al. [5] provided a simple solution with linear buckling analysis for the orthotropic conical shell which was exposed to axial compressive load and external pressure by using Donnell type shell theory. Pariatmono and Chryssanthopoulos [6] investigated the elastic buckling of the conical shell which has different boundary conditions with operating axial load. The different mode shapes under the same critic load were found in this study. Tavares [7] studied to determine the stresses, strains and displacements of a truncated or complete thin conical shell with constant thickness and axisymmetric load, which was distributed or concentrated along the meridian. Teng and Barbagallo [8] examined buckling strength in rings attached to cone cylinder intersection. Chryssanthopoulos and Spagnoli [9] studied on the influence of the edge constrain to stability in non-linear behaviour via finite element analysis of stiffened conical shell subjected to compressive load. Spagnoli and Chryssanthopoulos [10] investigated the effect of the different shell and stiffening parameters on the response under the axial load. A linear and non-linear elastic buckling analysis was conducted by means of generated FEM in this study. Chryssanthopoulos and Poggi [11] examined experimentally strength of unstiffened conical shells subjected to axial load. In addition, the effect of plasticity was derived with theoretical approaches. Xue et al. [12] researched flat topped conical shells under the axial load through either experimental or theoretical approach. Different buckling modes, which can be formed under axial load, were investigated with linear eigenvalue finite element analysis in thin walled stiffened conical shells by Spagnoli [13]. Another paper presented by Spagnoli [14] defined locus and formation of koiter ellipse in cones by using linear eigenvalue finite element analysis. Thinvongpituk and El-Sobky [15, 16] examined the effects of different end conditions on buckling load of conical shells under the axial load via either experimental or finite element method. Ifayefunmi [17] analysed the influence of the material modelling behaviour in the elastic-plastic buckling of thick unstiffened steel cones under compression.

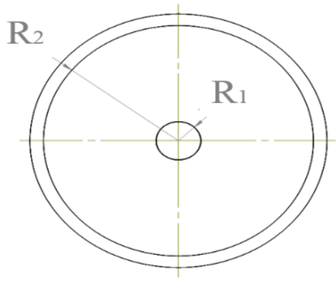
In this study, mode shapes and the effect of ring stiffness for different geometrical parameter on the non-linear elastic buckling load of conical cap subjected to axial load were investigated via finite element analysis. Moreover, an empirical relationship, which estimates buckling load, was established.

2. Materials and Methods

Numerical models and simulations have been performed by using finite element package programme COSMOS/M [18]. For the numerical analysis, large displacement module and Quadrilateral thick Shell element (SHELL4T) were assigned [19]. Models were generated for three cone angles "θ" (10°, 20° and 30°). Basic sketch of the structure is illustrated in Figure 1 with dimension parameters.



a) Front view



b) Top view

Figure 1. Front and top view of conical cap.

Upper "R_1" and bottom "R_2" radius was defined 25 and 150 mm respectively. The length of edge ring "b_{ring}" was chosen as a constant value which is 15 mm. The height of the relatively stiff pipe "h", located at the top of conical cap was assigned as10 mm. Shell thickness, ratio of shell and edge ring thickness "k" and cone angle values are presented in Table 1. In addition to these parameters, the model was performed without edge ring in order to find edge ring effect on the buckling load.

Table 1. Geometrical parameters of the structure.

Cone angle (θ)	k (t_{ring}/t_{shell})		t_{shell} [mm]
10°	0.2	1.2	0.5
	0.4	1.4	0.75
20°	0.6	1.6	1.0
	0.8	1.8	1.25
30°	1.0	2.0	1.5

In practical applications, the conical caps are used as a connector or an auxiliary component. The conical cap must be operated together an extra structure alongside it, in accordance of its usage. In order to conform boundary conditions in real application, ring slewing and rotation at hook direction were considered to be zero because of the connection between the conical cap and the adjacent structure. Referred constrains were observed on a full-scale model which is not investigated in this paper in detail (with adjacent structure). Vertical translation of the bottom side was also restricted.
Material employed in the numerical models was decided to S235 with linear, elastic and isotropic material behaviour. Mechanical properties such as modulus of elasticity "E" and Poisson's ratio "v" are 200GPa and 0.3, respectively.

3. Results and Discussion

Effects of edge ring thickness on the critical buckling load of conical caps having different shell thicknesses and cone angles were investigated regarding to geometrical nonlinearity. The material behaviour of both shell and edge ring was assumed to be elastic and linear. Finite element simulations for the selected geometries were performed in COSMOSM package program. The models were subjected to compressive loads which starts from zero to a maximum value (buckling load) with a certain load step increments. The results obtained from the simulations for different cone angles, edge ring and shell thickness are presented below.
Buckling mode shapes of the conical cap with an angle of 10° are given in Figure 2. As a result of simulations, four different mode shapes depending upon various shell and edge ring thicknesses were observed. In the first mode shape, buckling occurred around edge ring (zone 2) due to higher radial deformation at lower edge ring thickness that causes to collapse and instability of the model. Therefore, it is pointed out that high amount of radial deformation creates an eccentric bending moment over the Zone 2, thereafter the cone surface deflected to the downward direction as depicted in Figure 2.a. It can be said that the effect of bending moment is much more dominant than compressive force at lower cone angles with relatively thin edge ring sections. In the first mode shape, there is no sign of deflection around the hole on the top surface called Zone 1, which is a result of lower geometrical stiffness of Zone 2 under this type of loading. A slightly increase in edge ring thickness was introduced a new mode shape (Mode 2) because of a decrease in radial deformation. It is clearly seen that the deflection of Zone 1 is initiated to appear in the buckling mode as seen in the cross sectional view. Particularly, some buckles are first appeared on cone surface although the buckling shape is similar to mode 1 which becomes unstable at Zone 2. Buckles are mainly formed in relatively thinner shell thicknesses as a result of low compressive force carrying capacity of conical cap.
In mode shape 3, no buckles are observed on the cone surface and the system instability is found out because of both zone 1 and zone 2 so that the geometric stiffness becomes higher. However, it is observed that due to high stiffness of edge ring in mode shape 4, there is solely deflection on zone 1 and conical cap reaches the limit state. Beyond this point, edge ring effect is not dominant as a result of proximity of radial deformations with increasing thicknesses. Hence, the mode shape remains similar which yields slight differences in critical load as it is seen in Figure 2.

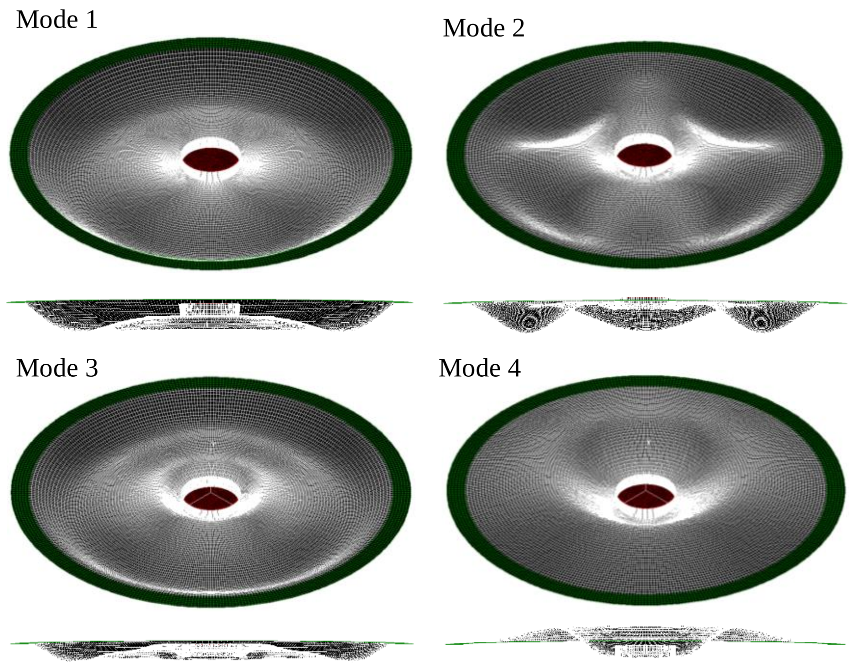


Figure 2. Buckling mode shapes of conical cap for 10°.

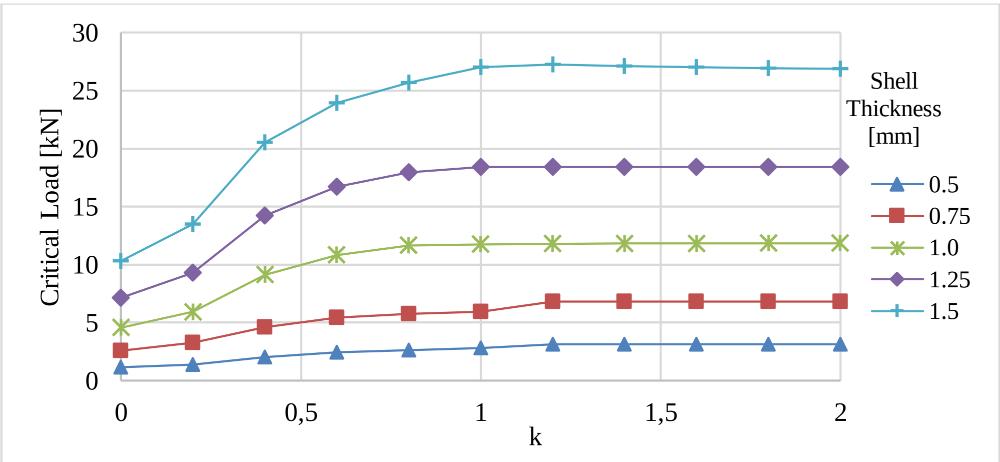


Figure 3. Trends of critical load for cone angle 10°.

The variation of critical buckling load for increasing edge ring and shell thickness are illustrated in Figure 3 at a cone angle of 10°. It is clearly seen that for all analysis, the buckling load is observed to be higher as either edge ring thickness or shell thickness increases. Edge ring has a more positive effect on buckling load for higher shell thickness up to the point where edge ring and shell thicknesses become equal to each other. This is an evidence of mode shape transition from mode shape 3 to 4.

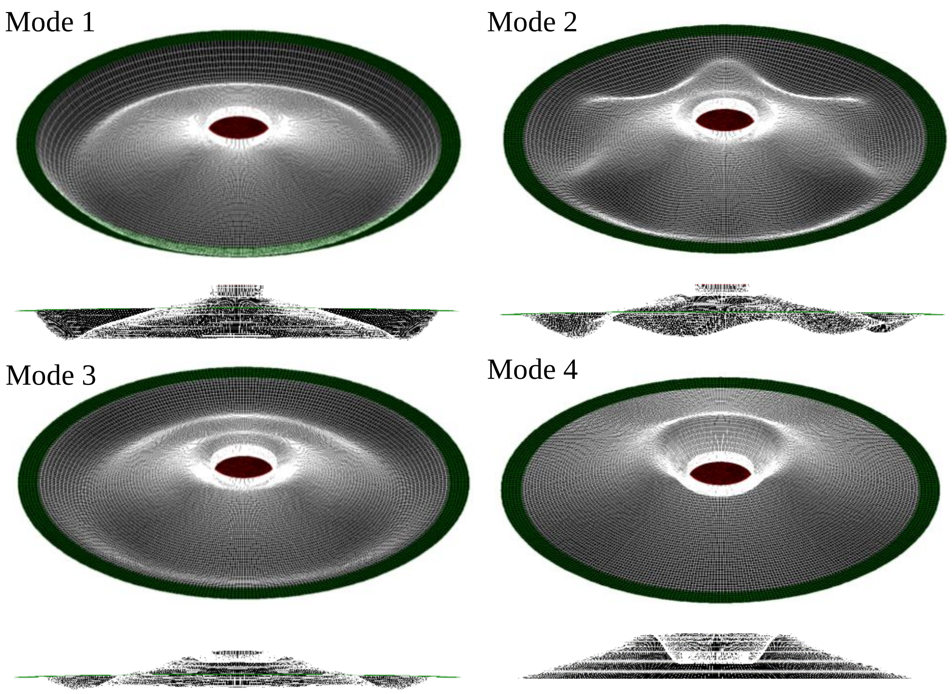


Figure 4. Buckling mode shapes of conical cap for 20°.

Mode shapes for cone angle 20° are given in Figure 4. Radial load carried by edge ring decreases at higher values of cone angle. So that the deformation of edge ring becomes lower which concludes higher bending moment carrying capacity of Zone 2. In addition, cone surface is heavily subjected to compressive force making the conical cap stiffer against buckling. General pattern of mode shapes can be said similar to those at the cone angle of 10° excepting deformation responses. On the other, higher compressive loads are created more number of buckles on the cone surface and exhibit more reaction to buckling failure.

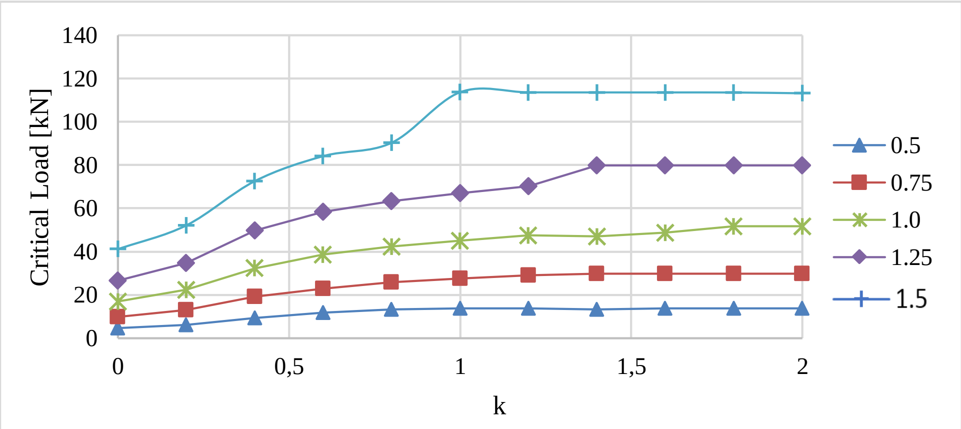


Figure 5. Trends of critical load for cone angle 20°.

Variation of critical load for different edge ring thickness and shell thickness are presented above Figure 5 regarding to cone angle of 20°. Critical load values exhibit same trends like 10 degrees of cone angle as expected considering compressive load carrying capacity. However, edge ring effect for shell thickness of 1.5mm is found out to be dominant up to k equals to 1. This can be explained as the small differences in transition region between mode 3 and mode 4 and with small fluctuations around edge ring value of 1.

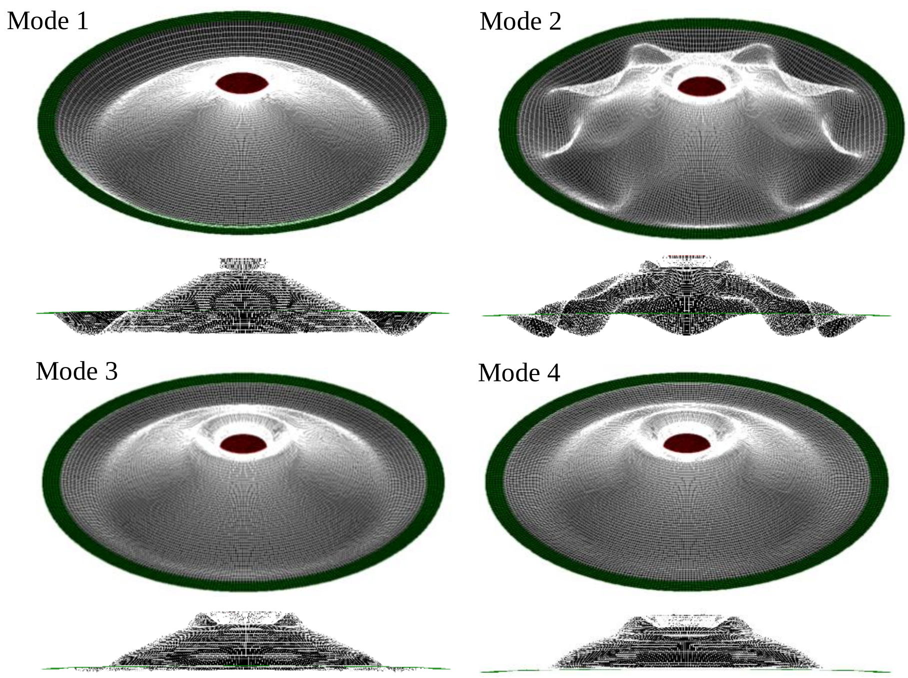


Figure 6. Buckling mode shapes of conical cap for 30°.

Apart from the buckling mode shapes for 10 and 20 degrees of cone angles, surfaces for all buckling modes tend to be a curved shape due to high stiffness of edge ring for the conical cap having 30 of cone angle which is given in Figure 6. In mode shape 2, number of buckles increase comparing to other cone angles due to high compressive load carrying capacity of members. In the same manner, those high compressive loads cause to deform edge ring continuously that mainly extends the effect of edge ring thickness on buckling load especially for relatively high shell thicknesses (Figure 7). The transition from mode shape 3 to 4 in 30 degrees of cone angle requires higher edge ring thickness than those in 10 and 20 degrees of cone angles.

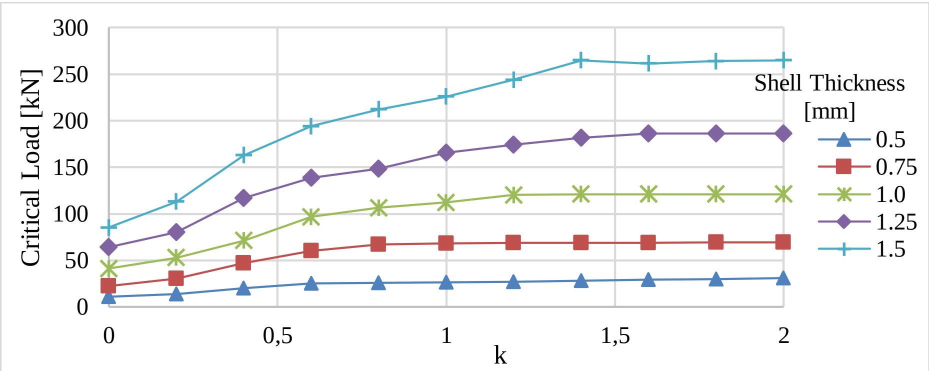


Figure 7. Trends of critical load for cone angle 30°.

According to buckling trends in Figure 7 for 30 degrees of cone angle, critical load was observed to be reached higher values with either increasing edge ring thickness or shell thickness as expected from previous results. It can be inferred from the graph that inflection points of distribution for relatively higher shell thicknesses are assessed to be mode shape transitions for 30° of cone angle. At lower shell thicknesses, number of modes observed are mainly limited to one or two whereas there is no sign of buckles for the higher shell thicknesses beyond 1.25 mm.

Critical load versus vertical displacement of top of cone are given in Figure 8 with respect to different shell and edge ring thicknesses at three cone angle of 10, 20 and 30 degrees respectively. Each curve on the graph is characterized by each mode shape of 1, 2, 3 and 4 for arbitrary selected edge and shell thickness values in order to examine geometrical stiffness behaviour of conical cap. In the first mode shape, sharp changes in stiffness were observed up to the limit state. Otherwise, second mode shape has some fluctuations due to buckle formation at buckling state. For all cone angles, third mode shape has nearly a linear stiffness value which leads to conical cap to exhibit more stable behaviour up to bifurcation point. The ideal stiffness variation is obtained for mode shape 4 having higher buckling limits comparing to the others. In this mode shape, there is a more smooth transition to post-buckling region so that it is possible to visualize displacement curve response after the limit state. Hence, the effect of geometrical stiffness change is getting lower as the mode shape approaches from 1 to 4 which is a desired system behaviour for the members under the buckling loading.

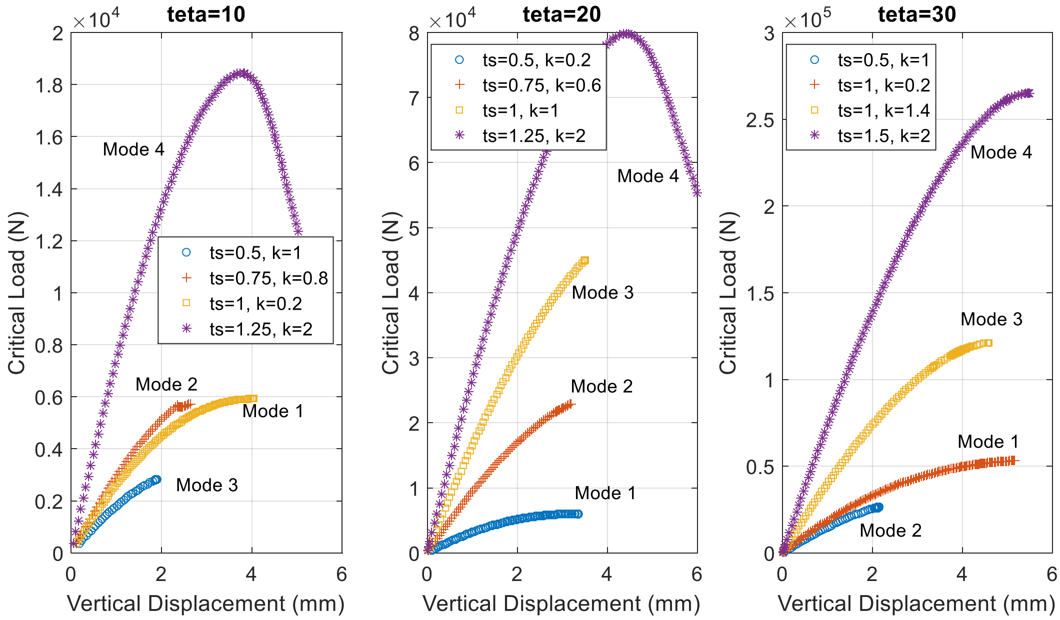


Figure 8. Critical load versus vertical displacement of top of conical cap for all mode shapes.

An empirical relationship for F_{cr} was established as a function relating to shell and edge ring thicknesses by using the data obtained from finite element method analysis. In consequence of trials, surface which gives the best result was found out by the following function.

$$F_{cr}(t_{ring}, t_{shell}) = \left(a + b\sqrt{t_{ring}/t_{shell}}\right)t_{shell}^2$$

Eq. 1.

For each cone angle, parameters, maximum and minimum amount of error and RMS values are shown in Table 2.

Table 2. Constants and statistical information for the equation.

Cone angle (θ)	a [Pa]	b [Pa]	R-square	Min. Error (%)	Max. Error (%)	RMSE (N)
10°	5037	5758	0.976	-2.7	12.2	1309
20°	16060	27250	0.984	-13.7	12.7	4288
30°	35100	64800	0.986	-12.1	10.7	9147

Coefficient of determination value is in an acceptable interval, therefore this equation can be introduced in terms of RMS and amount of error.

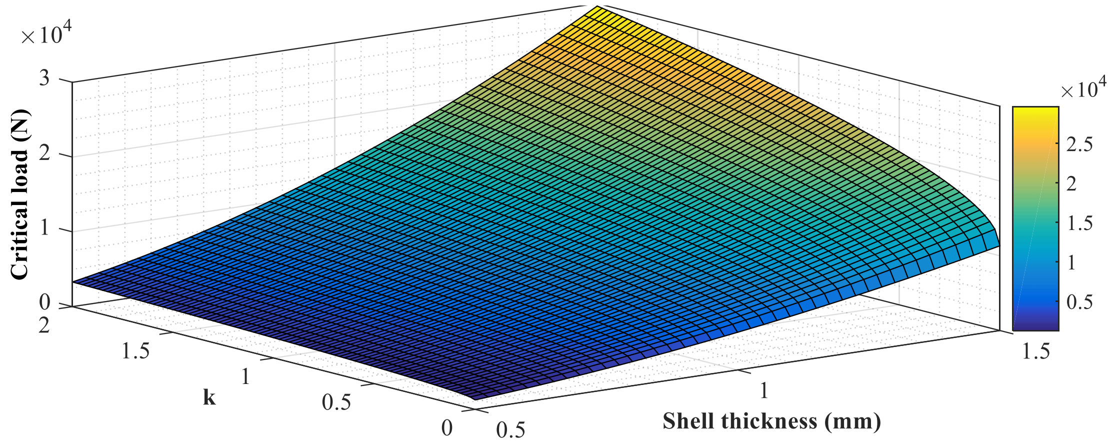


Figure 9. Surface graphs of critical load obtained from equation for cone angle 10°.

Equation is directly proportional to square of shell thickness and square root of edge ring thickness and the surface can be seen in the Figure 9.

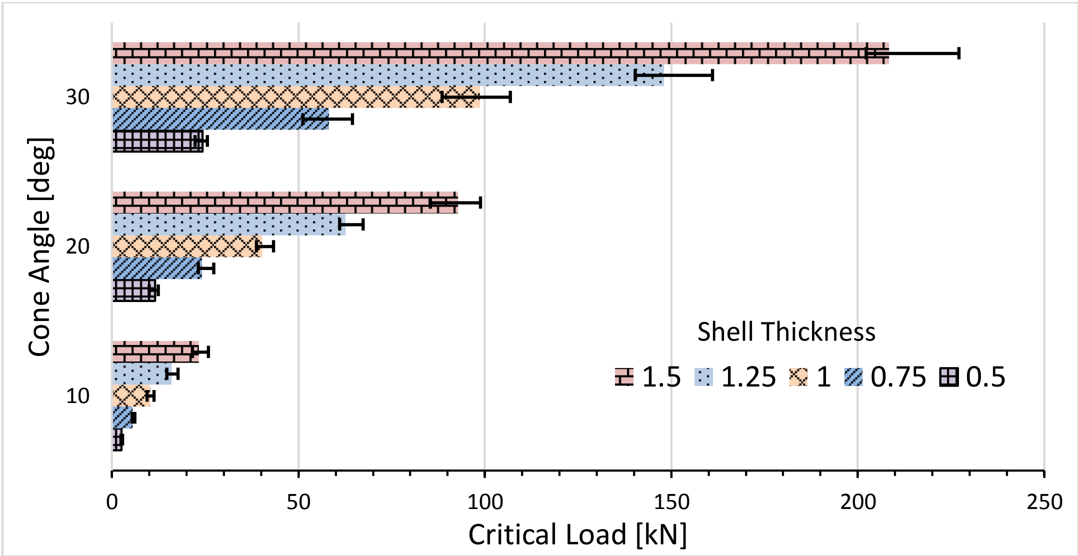


Figure 10. Average critical load of each shell thickness and error bands.

When the deviation of the data obtained from the equation and analysis results was examined, it was seen that there are more error possibilities in lower edge ring and shell thicknesses. In the same way, percentage error decreases on higher cone angles thus equation yields to more precise result in higher cone angles. For each shell thickness, average critical load values and error bands can be seen in Figure 10.

4. Concluding Remarks

In this study, in the case of axial load, the edge ring's influence on limit load of shell structures having different geometries was examined. Results which were obtained from the non-linear finite element analyses are stated below.

- Edge ring implementation and its thickness increases the limit load of the structure under the compressive loading conditions.
- Under axial loading, influence of edge ring on the buckling load increases in higher shell thickness of conical cap. These results show that application of edge ring with higher shell thickness is getting more advantageous.
- Critical buckling load is equated with a high precision empirical relation for each cone angle value as a function of edge ring and shell thickness.

- In the upcoming study, an optimization methodology (Taguchi Method) which was also implemented on some FEM results in Ref. [21], is being considered to examine effect of each geometrical parameters for establishing an analytical expression of dimensionless geometry parameters to evaluate elastic buckling load.

References

[1] Weingarten VI, Morgan EJ, Seide P. Elastic stability of thin- walled cylindrical and conical shells under axial compression. AIAA J 1965;3:500-05.

[2] Singer J. Buckling of circular conical shells under uniform axial compression. AIAA J 1965;3:985-87.

[3] Weingarten VI, Morgan EJ, Seide P. Elastic stability of thin-walled cylindrical and conical shells under combined internal pressure and axial compression. AIAA J 1965;3:1118-25.

[4] Tani J, Yamaki N. Buckling of Truncated Conical Shells under Axial Compression. AIAA J 1970;8:568-71.

[5] Tong L, Tabarrok B, Wang TK. Simple solutions for buckling of orthotropic conical shells. International Journal of Solids Structures. 1992;29:933-46.

[6] Pariatmono N, Chryssanthopoulos MK. Asymmetric elastic buckling of axially compressed conical shells with various end conditions. AIAA J 1995;33:2218-27.

[7] Tavares SA. Thin conical shells with constant thickness and under axisymmetric load. Computer & Structures 1996;60:895-921.

[8] Teng JG, Barbagallo M. Shell restraint to ring buckling at cone-cylinder intersections. Engineering Structures 1997;19:425-31.

[9] Chryssanthopoulos MK, Spagnoli A. The influence of radial edge constraint on the stability of stiffened conical shells in compression. Thin-Walled Structures 1997;27:147-63.

[10] Spagnoli A, Chryssanthopoulos MK. Elastic buckling and postbuckling behaviour of widely-stiffened conical shells under axial compression. Engineering Structures 1999;21:845-55.

[11] Chryssanthopoulos MK, Poggi C. Collapse strength of unstiffened conical shells under axial compression. Journal of Constructional Steel Research 2001;57:165-84.

[12] Xue P, Yu TX, Tao XM. Flat-topped conical shell under axial compression. International Journal of Mechanical Sciences 2001;43:2125-45.

[13] Spagnoli A. Different buckling modes in axially stiffened conical shells. Engineering Structures 2001;23:957-65.

[14] Spagnoli A. Koiter circles in the buckling of axially compressed conical shells. International Journal of Solids and Structures 2003;40:6095-109.

[15] Thinvongpituk C, El-Sobky H. The effect of end conditions on the buckling load characteristic of conical shells subjected to axial loading. ABAQUS Users' Conference, Munich, 4-6 June 2003.

[16] Thinvongpituk C, El-Sobky H. Buckling load characteristic of conical shells under various end conditions. The 17th Annual Conference of Mechanical Engineering Network of Thailand, Prachinburi, 15-17 October 2003.

[17] Ifayefunmi O. Plastic buckling of axially compressed thick unstiffened steel cones. Ocean Engineering 2015;103:1-9.

[18] FEM Computer program COSMOS/M GeoStar 2010, SRAC (Structural Research and Analysis Corporation), Santa Monica, California.

[19] COSMOS/M User's Guide. Structural Research and Analysis Corporation. 2001.

[20] Chung Y. Buckling of composite conical shells under combined axial compression, external pressure, and bending. Ph.D. Thesis, New Jersey Institute of Technology, Newark, NJ, 2001.

[21] Abdulkadir Yasar, Durmus Ali Bircan, Design, Analysis and Optimization of Heavy Vehicle Chassis Using Finite Element Analysis, International Journal of Scientific and Technological Research, Vol 1, No.6, 2015.