

Periodic Solutions of Certain Fifth Order Differential Equations

Melike Karta

Department of Mathematics, Faculty of Science and Arts,
Agri Ibrahim Cecen University, Agri-Turkey
E-mail: melike_karta2010@hotmail.com

Abstract

In this study, it is investigated that no periodic solution other than the trivial solution $X = 0$ to the nonlinear fifth order differential equation

$$X^5 + AX^4 + B\ddot{X} + G(X)\dot{X} + \Theta(X, \dot{X}, \ddot{X}, \ddot{X}, X^{(4)})\dot{X} + F(X, Y, Z, W, U)X = 0$$

by Lyapunov function.

Keywords: Nonlinear vector differential equation of fifth, Lyapunov function, Periodic solutions

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1. Introduction

Periodic solutions of high order scalar and vector differential equations have appeared in the sciences as some practical mechanical problems, physics, chemistry, biology, economics, and control theory. According to studies available in the literature, the problems related to the periodic behavior of solutions of a higher order non-linear scalar and vector differential equation have been analyzed by many authors. Bereketoglu [2-3] have obtained periodic solutions of certain class of seventh-order and eighth-order differential equations. Ezeilo [4-6] have proposed periodic solutions of a certain fourth and fifth order differential equations. Tejumola [8] have considered instability and periodic solutions of certain nonlinear seventh-order and sixth-order ordinary differential equations. Tiriyaki [9] have analyzed periodic solutions of a certain fourth and fifth order differential equations. Tunç [10] have presented periodic solutions of a certain fourth and fifth order vector differential equations. In all of the papers mentioned above, authors used Lyapunov's second (or direct) method [11].

Lǐ and Yu [1] investigated the instability of trivial solution to fifth-order nonlinear scalar differential equation

$$x^{(5)} + ax^{(4)} + b\ddot{x} + \varphi(x, \dot{x}, \ddot{x}, \ddot{x}, x^{(4)})\dot{x} + g(x)\dot{x} + f(x) = 0$$

by introducing a Lyapunov function, where a and b are some positive constants.

Karta [7] examined the instability of trivial solution of the fifth order non-linear vector differential equation

$$X^{(5)} + AX^{(4)} + B\ddot{X} + \Theta(X, \dot{X}, \ddot{X}, \ddot{X}, X^{(4)})\dot{X} + G(X)\dot{X} + F(X, Y, Z, W, U)X = 0 \quad (1.1)$$

In this study, we are concerned with periodic solution of the solution $X = 0$ of the fifth order nonlinear vector differential equation described by (1.1). In the real Euclidean space R^n , where $X \in R^n$; A and B are constant $n \times n$ -symmetric matrices; Ψ , G and F are $n \times n$ -symmetric continuous matrix function. We use,

instead of (1.1) the equivalent differential system

$$\dot{X} = Y, \dot{Y} = Z, \dot{Z} = W, \dot{W} = U \tag{1.2}$$

$$\dot{U} = -AU - BW - \Psi(X, Y, Z, W, U)Z - G(X)Y - F(X, Y, Z, W, U)X$$

which was obtained as usual by setting

$$\dot{X} = Y, \ddot{X} = Z, \dddot{X} = W, X^{(4)} = U$$

from (1.1). Let $J_G(X)$ displays the Jacobian matrix.

$$J_G(X) = \left(\frac{\partial g_i}{\partial x_j} \right) \quad (i, j = 1, 2, \dots, n)$$

in which (x_1, x_2, \dots, x_n) and (g_1, g_2, \dots, g_n) are components of X and G , respectively.

2. Main results

Our main result is the following the theorem.

Theorem 2.1.

In addition to the basic assumptions imposed on A, B, Ψ, G and F appearing in (1.1), we assume there are constants a, b and a positive constant k such that the following conditions hold

- (i) $\lambda_i(A) \geq a, \lambda_i(A) \geq b, bsgna > 0$
- (ii) $\lambda_i(F(X))sgna - (1/4|a|) \lambda_i(\Psi(X, Y, Z, W, U))^2 \geq k \quad (i = 1, 2, \dots, n)$

Then equation (1.1) has no periodic solution other than $X=0$.

Proof. Let $(\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5) = (\varphi_1(t), \varphi_2(t), \varphi_3(t), \varphi_4(t), \varphi_5(t))$

be an arbitrary α -periodic solution of (1.1), that is

$$\begin{aligned} &(\varphi_1(t), \varphi_2(t), \varphi_3(t), \varphi_4(t), \varphi_5(t)) \\ &= (\varphi_1(t + \alpha), \varphi_2(t + \alpha), \varphi_3(t + \alpha), \varphi_4(t + \alpha), \varphi_5(t + \alpha)) \end{aligned} \tag{2.1}$$

For some $\alpha > 0$. It will be shown that, under the conditions in Theorem (2.1),

$$\varphi_1 = \varphi_2 = \varphi_3 = \varphi_4 = \varphi_5 = 0.$$

As basic tool the proof of Theorem (2.1), we will use Lyapunov function

$$V = V_0(X, Y, Z, W, U)sgna$$

given as

$$V_0 = \langle Y, W \rangle + \langle Y, AZ \rangle - \langle X, U \rangle - \langle X, AW \rangle - \langle X, BZ \rangle - \frac{1}{2} \langle Z, Z \rangle + \frac{1}{2} \langle BY, Y \rangle - \int_0^1 \langle \sigma G(\sigma X) X, X \rangle d\sigma$$

$$\theta(t) = V_0(\varphi_1(t), \varphi_2(t), \varphi_3(t), \varphi_4(t), \varphi_5(t)) \text{sgna.}$$

Since $\theta(t)$ is continuous and $\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5$ are periodic in t , $\theta(t)$ is clearly bounded. An elementary differentiation will Show that

$$\dot{V}_0 = \frac{d}{dt} V_0(X, Y, Z, W, U) = \langle AZ, Z \rangle + \langle F(X) X, X \rangle + \langle X, \Psi(X, Y, Z, W, U) Z \rangle$$

and it is clear $\dot{V} \geq k \|X\|^2 \geq 0$ that, under the conditions in Theorem (2.1) .

Hence $\theta'(t) \geq 0$; so that $\theta(t)$ is monotone in t , and therefore, being bounded, tends to a limit , θ_0 say, as $t \rightarrow +\infty$. That is $\lim_{t \rightarrow +\infty} \theta(t) = \theta_0$. It is readily showed that

$$\theta(t) = \theta_0 \text{ for all } t. \tag{2.2}$$

from by (2.1) ,

$$\theta(t) = \theta(t + m\alpha)$$

For any arbitrary fixed t an for arbitrary integer m , and then for $t \rightarrow +\infty$, it implies that

$$\lim_{t \rightarrow +\infty} \theta(t) = \lim_{t \rightarrow +\infty} \theta(t + m\alpha) = \theta_0.$$

Due to $\lim_{t \rightarrow +\infty} \theta(t) = \theta_0$ and $\theta(t) = \theta(t + m\alpha)$ for all t , it is $\theta(t) = \theta_0$.thus for all t $\dot{\theta}(t) = 0$.

Furthermore $\dot{\theta}(t) = 0$ necessarily implies that $\varphi_1=0$ for all t . That is , it is showed that

$$\varphi_1 = \varphi_2 = \varphi_3 = \varphi_4 = \varphi_5 = 0.$$

Because $\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5$ are solutions of (1.2) system, it is obtained

$$(\varphi_1(t), \varphi_2(t), \varphi_3(t), \varphi_4(t), \varphi_5(t)) = (X(t), Y(t), Z(t), W(t), U(t)) = (0,0,0,0,0).$$

This completes the proof of Theorem (2.1).

3. Conclusion

The proof of the Theorem 2.1 showed that the equation (1.1) have no periodic solution other than the trivial solution $X = 0$. So, this has helped us to find a new result.

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