 

# Analysis of Some Reliability Measures of a Deteriorating Reinforced Concrete Structure

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**Abstract**
In this paper, we study the reliability and availability characteristics of two-stage deterioration reinforce concrete structure .Failure and repair times are assumed exponential. The explicit expressions of reliability and availability characteristics such as mean time to system failure (MTSF), steady-state availability and busy period are derived using Kolmogorov forward equations method. Various cases are analyzed graphically to investigate the impact of system parameters on MTSF, availability and busy period

**Keywords:** Reliability, deterioration, reinforce concrete

## 1. Introduction

As the age of concrete structure increases, the structure slowly deteriorates correspondingly. Most of these structures are subjected to random deterioration which can results in unexpected failures and disastrous effect on safety and the economy it is therefore important to find a way to slow down the deterioration rate, and to prolong structure's service life. However in the real world some sections of a concrete structure are exposed to many of these environments hazard, once deterioration has begun it can occur in the embedded steel as well as the surface of the concrete. However, a concrete structure can be susceptible to a number of factors that can cause deterioration which could lead to a reduction in strength making for unsafe conditions. Therefore it is essential that a concrete structure is maintained correctly. The concept of deterioration and its impact on reliability measure of system effectiveness has been introduced by several authors (see for instance [3, 7, 8, 11]). Reliability is vital for proper utilization and maintenance of any reinforces concrete structure. It involves technique for increasing system effectiveness through reducing failure frequency and maintenance cost. For this reason, many researchers have studied reliability problem of different reinforce concrete structure (see, for instance, [1,5, 6, 9] and the references therein).

In this paper, we consider a three layer reinforce concrete structure exposed to slow and fast deterioration and derived its corresponding mathematical models using Kolmogorov's forward equation method. The contributions of this paper are twofold. First is to develop the explicit expressions for mean time to system failure (MTSF), steady-state availability and steady-state busy period. The second is to determine the impact of failure rate, repair rate, slow and fast deterioration rate, minor and major minimal maintenance rate on mean time to system failure, steady-state availability, and steady-state busy period.

The rest of the paper is organized as follows. Section 2 is the description and states of the reinforce concrete structure. Section 3 deals with models formulation. The results of our numerical simulations are presented and discussed in Section 4. The paper is concluded in Section 5.

## 2. **Description of the Reinforce Concrete Structure**

We consider three layers reinforce concrete structure with three modes: normal, deterioration and failure. The layers are coating, concrete and reinforcement. The deterioration mode consists of two consecutive stages: slow and fast. It is assumed that the structure transits from normal to slow and later to fast deterioration with rate $\delta_1$ and $\delta_2$ respectively. It is also assumed that the two upper layers (coating and concrete) never fail simultaneously. Whenever the coating layer deteriorates with rate $\delta_1$, minor minimal maintenance is invoke with rate $\mu_1$ to regain the structure to its early stage prior to deterioration or the deterioration will be faster with rate $\delta_2$ where major minimal maintenance will be done with rate $\mu_2$. Concrete repair involves cleaning and preparing the defective area and the exposed reinforcing steel, after which rebuilding up the defective area using a suitable repair mortar. Coating layer fails with rate $\beta_1$ and is repaired with rate $\alpha_1$. If coating is not repaired, concrete will be exposed which later fails with rate $\beta_2$ and repaired with rate $\alpha_2$. Once any concrete has been repaired it is very important to protect the structure from future deterioration and failure by applying a protection coat. The structure failed when two consecutive layers (coating and concrete) have failed.

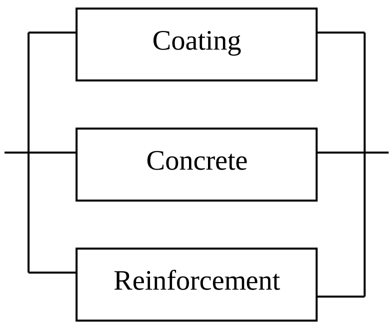


Fig. 1 Reliability block diagram of the reinforce concrete structure

 

Table 1 Transition rate table

| | $S_0$ | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ |
|---|---|---|---|---|---|---|
| $S_0$ | | $\delta_1$ | | $\beta_1$ | | |
| $S_1$ | $\mu_1$ | | $\delta_2$ | $\beta_1$ | | |
| $S_2$ | | $\mu_2$ | | | $\beta_1$ | |
| $S_3$ | $\alpha_1$ | $\alpha_1$ | | | $\delta_2$ | $\beta_2$ |
| $S_4$ | | | $\alpha_1$ | $\mu_2$ | | $\beta_2$ |
| $S_5$ | | | | $\alpha_2$ | $\alpha_2$ | |

2.1. **States of the System**

State $S_0$ : The coating, concrete and reinforcement are normal

State $S_1$ : The structure is under slow deterioration and is receiving minor minimal maintenance

State $S_2$ : The structure is under fast deterioration and is receiving major minimal maintenance

State $S_3$ : Coating layer failed and is under repair, the concrete structure is in slow deterioration

State $S_4$ : Coating layer failed and is under repair, the concrete structure is in fast deterioration

State $S_5$ : Concrete layer failed and is under repair, the reinforcement is exposed and the structure failed.

**3. Models Formulation**

Let $P(t)$ be the probability row vector at time $t$, then the initial conditions for this problem are as follows:

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0)] = [1,0,0,0,0,0]$$

we obtain the following system of differential equations:

$$P_0'(t) = -(\beta_1 + \delta_1)P_0(t) + \mu_1 P_1(t) + \alpha_1 P_3(t)$$

$$P_1'(t) = -(\beta_1 + \mu_1 + \delta_2)P_1(t) + \delta_1 P_0(t) + \mu_2 P_2(t) + \alpha_1 P_3(t)$$

$$P_2'(t) = -(\beta_1 + \mu_2)P_2(t) + \delta_2 P_1(t) + \alpha_1 P_4(t)$$

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$$P_3'(t) = -\left(\beta_2 + \delta_2 + 2\alpha_1\right)P_3(t) + \beta_1 P_0(t) + \beta_1 P_1(t) + \mu_2 P_4(t) + \alpha_2 P_5(t)$$

$$P_4'(t) = -\left(\beta_2 + \alpha_1 + \mu_2\right)P_4(t) + \beta_1 P_2(t) + \delta_2 P_3(t) + \alpha_2 P_5(t)$$

$$P_5'(t) = -2\alpha_2 P_5(t) + \beta_2 P_3(t) + \beta_2 P_4(t) \tag{1}$$

The differential equations in (1) above is transformed into matrix as $P' = TP$ (2)

where

$$T = \begin{bmatrix} -\left(\beta_1+\delta_1\right) & \mu_1 & 0 & \alpha_1 & 0 & 0 \\ \delta_1 & -\left(\beta_1+\mu_1+\delta_2\right) & \mu_2 & \alpha_1 & 0 & 0 \\ 0 & \delta_2 & -\left(\beta_1+\mu_2\right) & 0 & \alpha_1 & 0 \\ \beta_1 & \beta_1 & 0 & -\left(\beta_2+\delta_2+2\alpha_1\right) & \mu_2 & \alpha_2 \\ 0 & 0 & \beta_1 & \delta_2 & -\left(\beta_2+\alpha_1+\mu_2\right) & \alpha_2 \\ 0 & 0 & 0 & \beta_2 & \beta_2 & -2\alpha_2 \end{bmatrix}$$

**3.1 Mean Time to System Failure Analysis**

It is difficult to evaluate the transient solutions, hence we follow [2,4,10], the procedure to develop the explicit expression for $MTSF$ is to delete the sixth row and column of matrix $T$ and take the transpose to produce a new matrix, say $Q$ . The expected time to reach an absorbing state is obtained from

$$E\left[T_{P(0)\to P(absorbing)}\right] = MTSF = P(0)(-Q^{-1})\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix} = \frac{N_1}{D_1} \tag{3}$$

Where

$$Q = \begin{bmatrix} -\left(\beta_1+\delta_1\right) & \delta_1 & 0 & \beta_1 & 0 \\ \mu_1 & -\left(\beta_1+\mu_1+\delta_2\right) & \delta_2 & \beta_1 & 0 \\ 0 & \mu_2 & -\left(\beta_1+\mu_2\right) & 0 & \beta_1 \\ \alpha_1 & \alpha_1 & 0 & -\left(\beta_2+\delta_2+2\alpha_1\right) & \delta_2 \\ 0 & 0 & \alpha_1 & \mu_2 & -\left(\beta_2+\mu_2+\alpha_1\right) \end{bmatrix}$$

$$N_1 = (\beta_1\beta_2^2\delta_2 + 2\beta_1\beta_2\mu_2\delta_2 + \alpha_1\beta_1\mu_2\delta_2 + \beta_2\mu_1\mu_2\delta_2 + \alpha_1\mu_1\mu_2\delta_2 + \beta_1\beta_2\mu_1\delta_2 + \beta_1\beta_2\delta_2^2 + \beta_1^2\beta_2\delta_2 + 2\alpha_1^2\mu_1\mu_2 + \beta_2^2\mu_1\mu_2 +$$

$$\beta_2\mu_1\mu_2^2 + 2\alpha_1\mu_1\mu_2^2 + \beta_1^2\beta_2\mu_2 + \alpha_1\beta_1^2\beta_2 + \alpha_1\beta_1^2\mu_2 + \beta_1\beta_2\mu_2^2 + \beta_1\beta_2^2\mu_2 + \alpha_1\beta_1\mu_2^2 + \beta_1^2\beta_2^2 + 3\alpha_1\beta_2\mu_1\mu_2 + \beta_1\beta_2\mu_1\mu_2 +$$

$$2\alpha_1\beta_1\beta_2\mu_2 + 2\alpha_1\beta_1\mu_1\mu_2 + 2\alpha_1\beta_1\beta_2\delta_2 + 2\alpha_1\beta_1\beta_2\mu_1 + \alpha_1^2\beta_1\mu_2 + \beta_1\beta_2^2\mu_1) + (2\alpha_1^2\mu_2\delta_1 + \alpha_1^2\beta_1\mu_2 + 2\alpha_1\mu_2^2\delta_1 + \alpha_1\beta_1\mu_2^2 +$$

$$2\alpha_1\beta_1\mu_2\delta_1 + \alpha_1\beta_1^2\mu_2 + \alpha_1\beta_1\mu_2\delta_2 + 3\alpha_1\beta_2\mu_2\delta_1 + \alpha_1\mu_2\delta_1\delta_2 + \alpha_1\beta_1\beta_2\mu_2 + \alpha_1\beta_1^2\beta_2 + 2\alpha_1\beta_1\beta_2\delta_1 + \beta_2\mu_2^2\delta_1 + \beta_2\mu_2\delta_1\delta_2 +$$

$$\beta_1\beta_2\mu_2\delta_1 + \beta_2^2\mu_2\delta_1 + \beta_1\beta_2\delta_1\delta_2 + \beta_1\beta_2^2\delta_1) + \delta_2(\alpha_1\beta_1\mu_1 + 2\alpha_1^2\delta_1 + \alpha_1^2\beta_1 + 2\alpha_1\mu_2\delta_1 + \alpha_1\beta_1\mu_2 + \alpha_1\beta_1\delta_2 + 3\alpha_1\beta_2\delta_1 + \alpha_1\beta_1^2 +$$

$$\alpha_1\delta_1\delta_2 + \alpha_1\beta_1\beta_2 + \alpha_1\beta_1\delta_1 + \beta_2\mu_2\delta_1 + \beta_2^2\delta_1 + \beta_2\delta_1\delta_2) + \beta_1(\mu_1\mu_2^2 + \beta_2\mu_1\mu_2 + \alpha_1\mu_1\mu_2 + \beta_1\mu_1\mu_2 + \beta_1\beta_2\mu_1 + \alpha_1\beta_1\mu_2 +$$

$$\beta_1\mu_2^2 + \mu_2^2\delta_1 + \beta_1\beta_2\mu_2 + \beta_1^2\mu_2 + \beta_2\mu_2\delta_1 + \beta_1\mu_2\delta_2 + \beta_1\mu_2\delta_1 + \mu_2\delta_1\delta_2 + \beta_1\beta_2\delta_1 + \beta_1^2\beta_2 + \beta_1\beta_2\delta_2) + \beta_1\delta_2(\mu_1\mu_2 + \beta_1\mu_1 +$$

$$\alpha_1\beta_1 + 2\alpha_1\delta_1 + \beta_1\mu_2 + \mu_2\delta_1 + \delta_1\delta_2 + \beta_1\delta_1 + \beta_1^2 + \beta_1\delta_2 + \beta_2\delta_1)$$

$$D_1 = \alpha_1\beta_1\mu_2 + \beta_1\beta_2\mu_2 + \beta_1\beta_2\mu_1 + \beta_1\mu_1\mu_2 + 2\beta_1\mu_2\delta_2 + \beta_1\beta_2\delta_2 + \beta_2\mu_1\mu_2 + \beta_1\mu_1\delta_2 + 2\mu_2\delta_1\delta_2 + \beta_1\delta_2^2 + \alpha_1\beta_1\delta_2 + \beta_1^2\delta_2 +$$

$$\mu_1\mu_2\delta_2 + \alpha_1\mu_1\mu_2 + \beta_2\mu_2\delta_1 + \beta_1\mu_2\delta_1 + \beta_1\beta_2\delta_1 + \beta_2\delta_1\delta_2 + \beta_1\delta_1\delta_2 + \delta_1\delta_2^2 + \beta_1\mu_2^2 + \beta_1^2\mu_2 + \beta_1^2\beta_2 + \mu_1\mu_2^2 + \mu_2^2\delta_1 +$$

$$2\alpha_1\delta_1\delta_2 + \alpha_1\mu_2\delta_1$$

**3.2 System Availability Analysis**

For the availability case of table 1 using the initial condition in section 3 for this system, $P(0) = [P_1(0), P_2(0), P_3(0), P_4(0), P_5(0)] = [1,0,0,0,0,0]$

The system of differential equations in (1) for the system above can be expressed in matrix form as:

$$\begin{bmatrix} P_0' \\ P_1' \\ P_2' \\ P_3' \\ P_4' \\ P_5' \end{bmatrix} = \begin{bmatrix} -(\beta_1+\delta_1) & \mu_1 & 0 & \alpha_1 & 0 & 0 \\ \delta_1 & -(\beta_1+\mu_1+\delta_2) & \mu_2 & \alpha_1 & 0 & 0 \\ 0 & \delta_2 & -(\beta_1+\mu_2) & 0 & \alpha_1 & 0 \\ \beta_1 & \beta_1 & 0 & -(\beta_2+\delta_2+2\alpha_1) & \mu_2 & \alpha_2 \\ 0 & 0 & \beta_1 & \delta_2 & -(\beta_2+\alpha_1+\mu_2) & \alpha_2 \\ 0 & 0 & 0 & \beta_2 & \beta_2 & -2\alpha_2 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix}$$

Let $V$ be the time to failure of the system. Following [2,4,10], the steady-state availability is given by

$$A_V = P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) \quad (4)$$

In steady state, the derivatives of state probabilities become zero, thus (2) becomes

$$TP(\infty) = 0 \quad (5)$$

which in matrix form is

 

$$\begin{bmatrix} -(\beta_1+\delta_1) & \mu_1 & 0 & \alpha_1 & 0 & 0 \\ \delta_1 & -(\beta_1+\mu_1+\delta_2) & \mu_2 & \alpha_1 & 0 & 0 \\ 0 & \delta_2 & -(\beta_1+\mu_2) & 0 & \alpha_1 & 0 \\ \beta_1 & \beta_1 & 0 & -(\beta_2+\delta_2+2\alpha_1) & \mu_2 & \alpha_2 \\ 0 & 0 & \beta_1 & \delta_2 & -(\beta_2+\alpha_1+\mu_2) & \alpha_2 \\ 0 & 0 & 0 & \beta_2 & \beta_2 & -2\alpha_2 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

using the normalizing condition

$$P_0(\infty)+P_1(\infty)+P_2(\infty)+P_3(\infty)+P_4(\infty)+P_5(\infty)=1 \qquad (6)$$

we substitute (6) in the last row of (5) following [2,3,5]. The resulting matrix is

$$\begin{bmatrix} -(\beta_1+\delta_1) & \mu_1 & 0 & \alpha_1 & 0 & 0 \\ \delta_1 & -(\beta_1+\mu_1+\delta_2) & \mu_2 & \alpha_1 & 0 & 0 \\ 0 & \delta_2 & -(\beta_1+\mu_2) & 0 & \alpha_1 & 0 \\ \beta_1 & \beta_1 & 0 & -(\beta_2+\delta_2+2\alpha_1) & \mu_2 & \alpha_2 \\ 0 & 0 & \beta_1 & \delta_2 & -(\beta_2+\alpha_1+\mu_2) & \alpha_2 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Expression for $A_V$ thus is:

$$A_V = \frac{N_2}{D_2}$$

$$N_2 = \alpha_1\alpha_2(4\alpha_1\mu_1\mu_2 + 4\mu_1\mu_2^2 + 4\beta_1\mu_1\mu_2 + 3\beta_2\mu_1\mu_2 + 2\mu_1\mu_2\delta_2 + 2\beta_1\beta_2\mu_1 + 2\alpha_1\beta_1\mu_2 + 2\beta_1\mu_2^2 + 2\beta_1\mu_2\delta_2 + \beta_1\beta_2\mu_2 +$$

$$2\beta_1^2\mu_2 + \beta_1\beta_2\delta_2 + \beta_1^2\beta_2) + \alpha_1\alpha_2(4\alpha_1\mu_2\delta_1 + 2\alpha_1\beta_1\mu_2 + 4\mu_2^2\delta_1 + 2\beta_1\mu_2^2 + 4\beta_1\mu_2\delta_1 + 2\beta_1^2\mu_2 + 2\mu_2\delta_1\delta_2 + 2\beta_1\beta_2\mu_2 +$$

$$3\beta_2\mu_2\delta_1 + 2\beta_1\mu_2\delta_2 + 2\beta_1\beta_2\delta_1 + \beta_1^2\beta_2) + \alpha_1\alpha_2(\beta_1\beta_2\mu_1 + 2\beta_1\mu_1\delta_2 + 2\alpha_1\beta_1\delta_2 + 4\alpha_1\delta_1\delta_2 + 2\beta_1\mu_2\delta_2 + 4\mu_2\delta_1\delta_2 + 2\beta_1^2\delta_2 +$$

$$2\beta_1\beta_2\delta_2 + 2\delta_1\delta_2^2 + 2\beta_1\delta_2^2 + 3\beta_2\delta_1\delta_2 + 2\beta_1\delta_1\delta_2 + \beta_1^2\beta_2 + \beta_1\beta_2\delta_1) + \alpha_2\beta_1(2\alpha_1\mu_1\mu_2 + 2\mu_1\mu_2^2 + \beta_2\mu_1\mu_2 + 2\beta_1\mu_1\mu_2 + \beta_1\beta_2\mu_1 +$$

$$2\alpha_1\mu_2\delta_1 + 2\alpha_1\beta_1\mu_2 + 2\mu_2^2\delta_1 + 2\beta_1\mu_2^2 + 2\beta_1\mu_2\delta_1 + 2\beta_1^2\mu_2 + 2\beta_1\mu_2\delta_2 + 2\mu_2\delta_1\delta_2 + \beta_2\mu_2\delta_1 + \beta_1\beta_2\mu_2 + \beta_1\beta_2\delta_1 + \beta_1\beta_2\delta_2 +$$

$$\beta_2\delta_1\delta_2 + \beta_1^2\beta_2) + \alpha_2\beta_1(\beta_2\mu_1\mu_2 + 2\mu_1\mu_2\delta_2 + 2\beta_1\mu_1\delta_2 + \beta_1\beta_2\mu_1 + 2\alpha_1\beta_1\delta_2 + 4\alpha_1\delta_1\delta_2 + 2\beta_1\mu_2\delta_2 + \beta_2\mu_2\delta_1 + 2\mu_2\delta_1\delta_2 +$$

$$\beta_1\beta_2\mu_2 + 2\beta_1^2\delta_2 + \beta_2\delta_1\delta_2 + 2\beta_1\delta_1\delta_2 + \beta_1\beta_2\delta_2 + 2\beta_1\delta_2^2 + \beta_1^2\beta_2 + 2\delta_1\delta_2^2 + \beta_1\beta_2\delta_1)$$

$$D_2 = 2\beta_1^2\beta_2\mu_2\delta_2 + \alpha_1\beta_1^2\beta_2\delta_2 + 2\alpha_2\beta_1^2\beta_2\delta_2 + 2\alpha_1\alpha_2\beta_1\delta_2^2 + 4\alpha_1\alpha_2\beta_1^2\delta_2 + 4\alpha_2\beta_1^2\mu_2\delta_2 + \beta_1^2\beta_2\delta_2^2 + \beta_1^2\beta_2^2\delta_2 + \beta_1^3\beta_2\delta_2 +$$

$$2\alpha_2\beta_1^2\delta_2^2 + 2\alpha_2\beta_1^3\delta_2 + 6\alpha_1\alpha_2\beta_1\mu_2\delta_2 + 2\alpha_1\alpha_2\mu_1\mu_2\delta_2 + \beta_1\beta_2\delta_1\delta_2^2 + \beta_1\beta_2\mu_1\mu_2\delta_2 + 2\beta_1\beta_2\mu_2\delta_1\delta_2 + 2\alpha_2\beta_1^2\mu_1\delta_2 +$$

$$2\alpha_2\beta_1\mu_1\mu_2\delta_2 + 2\alpha_2\beta_1^2\delta_1\delta_2 + 2\alpha_2\beta_1\delta_1\delta_2^2 + 2\alpha_2\beta_1\beta_2\delta_1\delta_2 + 4\alpha_2\beta_1\mu_2\delta_1\delta_2 + \beta_1\beta_2^2\delta_1\delta_2 + \beta_1^2\beta_2\mu_1\mu_2 + \beta_1^2\beta_2\mu_2\delta_1 + 2\alpha_2\beta_1^2\mu_2\delta_1 +$$

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$$2\alpha_2\beta_1^2\beta_2\delta_1 + 2\alpha_2\beta_1^2\beta_2\mu_1 + 2\alpha_2\beta_1^2\mu_1\mu_2 + 2\alpha_2\beta_1^2\beta_2\mu_2 + 2\alpha_2\beta_1\mu_1\mu_2^2 + \beta_1\beta_2^2\mu_2\delta_1 + 2\alpha_2\beta_1\mu_2^2\delta_1 + \beta_1\beta_2\mu_1\mu_2^2 + \beta_1\beta_2\mu_2^2\delta_1 +$$

$$\beta_1\beta_2^2\mu_1\mu_2 + \beta_1^3\beta_2^2 + \beta_1^3\beta_2\mu_2 + 2\alpha_2\beta_1^3\beta_2 + 2\alpha_2\beta_1^3\mu_2 + 2\alpha_2\beta_1^2\mu_2^2 + \beta_1^2\beta_2^2\mu_2 + \beta_1^2\beta_2\mu_2^2 + \beta_1^2\beta_2^2\delta_1 + \beta_1^2\beta_2^2\mu_1 +$$

$$2\alpha_2\beta_1\beta_2\mu_2\delta_1 + 2\alpha_2\beta_1\beta_2\mu_1\mu_2 + \alpha_1\beta_1^2\beta_2\mu_2 + 6\alpha_1\alpha_2\beta_1^2\mu_2 + 3\alpha_1\alpha_2\beta_1^2\beta_2 + 4\alpha_1\alpha_2\mu_1\mu_2^2 + 4\alpha_1\alpha_2\beta_1\mu_2^2 + 4\alpha_1\alpha_2\mu_2^2\delta_1 +$$

$$2\alpha_1^2\alpha_2\beta_1\delta_2 + 4\alpha_1^2\alpha_2\beta_1\mu_2 + 4\alpha_1^2\alpha_2\delta_1\delta_2 + 4\alpha_1^2\alpha_2\mu_1\mu_2 + 4\alpha_1^2\alpha_2\mu_2\delta_1 + 2\alpha_1\beta_1\beta_2\delta_1\delta_2 + 2\alpha_1\alpha_2\beta_1\mu_1\delta_2 + 6\alpha_1\alpha_2\beta_1\delta_1\delta_2 +$$

$$\alpha_1\beta_1\beta_2\mu_1\mu_2 + \alpha_1\beta_1\beta_2\mu_2\delta_1 + 3\alpha_1\alpha_2\beta_2\delta_1\delta_2 + 3\alpha_1\alpha_2\beta_2\mu_2\delta_1 + \alpha_1\alpha_2\beta_2\mu_1\mu_2 + 3\alpha_1\alpha_2\beta_1\beta_2\delta_2 + 3\alpha_1\alpha_2\beta_1\beta_2\mu_1 + 6\alpha_1\alpha_2\beta_1\mu_1\mu_2 +$$

$$3\alpha_1\alpha_2\beta_1\beta_2\delta_1 + 3\alpha_1\alpha_2\beta_1\beta_2\mu_2 + 6\alpha_1\alpha_2\beta_1\mu_2\delta_1$$

### 3.3 Busy Period Analysis

Using the same initial condition in section 3 above as for the reliability case

$P(0) = [P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0)] = [1, 0, 0, 0, 0, 0, 0]$ and (5) and (6) the busy period is obtained as follows:

In the steady state, the derivatives of the state probabilities become zero and this will enable us to compute steady state busy period :

The system of differential equations in (1) for the system above can be expressed in matrix form as:

$$\begin{bmatrix} P_0' \\ P_1' \\ P_2' \\ P_3' \\ P_4' \\ P_5' \end{bmatrix} = \begin{bmatrix} -(\beta_1+\delta_1) & \mu_1 & 0 & \alpha_1 & 0 & 0 \\ \delta_1 & -(\beta_1+\mu_1+\delta_2) & \mu_2 & \alpha_1 & 0 & 0 \\ 0 & \delta_2 & -(\beta_1+\mu_2) & 0 & \alpha_1 & 0 \\ \beta_1 & \beta_1 & 0 & -(\beta_2+\delta_2+2\alpha_1) & \mu_2 & \alpha_2 \\ 0 & 0 & \beta_1 & \delta_2 & -(\beta_2+\alpha_1+\mu_2) & \alpha_2 \\ 0 & 0 & 0 & \beta_2 & \beta_2 & -2\alpha_2 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix}$$

Let $V$ be the time to failure of the system. The steady-state busy period is given by

$$B_V = P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) + P_5(t) \qquad (4)$$

In steady state, the derivatives of state probabilities become zero, thus (2) becomes

$$TP(\infty) = 0 \qquad (5)$$

which in matrix form is

$$\begin{bmatrix} -(\beta_1+\delta_1) & \mu_1 & 0 & \alpha_1 & 0 & 0 \\ \delta_1 & -(\beta_1+\mu_1+\delta_2) & \mu_2 & \alpha_1 & 0 & 0 \\ 0 & \delta_2 & -(\beta_1+\mu_2) & 0 & \alpha_1 & 0 \\ \beta_1 & \beta_1 & 0 & -(\beta_2+\delta_2+2\alpha_1) & \mu_2 & \alpha_2 \\ 0 & 0 & \beta_1 & \delta_2 & -(\beta_2+\alpha_1+\mu_2) & \alpha_2 \\ 0 & 0 & 0 & \beta_2 & \beta_2 & -2\alpha_2 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

using the normalizing condition

$$P_0(\infty)+P_1(\infty)+P_2(\infty)+P_3(\infty)+P_4(\infty)+P_5(\infty)=1 \qquad (6)$$

we substitute (6) in the last row of (5) following [2,3,5]. The resulting matrix is

$$\begin{bmatrix} -(\beta_1+\delta_1) & \mu_1 & 0 & \alpha_1 & 0 & 0 \\ \delta_1 & -(\beta_1+\mu_1+\delta_2) & \mu_2 & \alpha_1 & 0 & 0 \\ 0 & \delta_2 & -(\beta_1+\mu_2) & 0 & \alpha_1 & 0 \\ \beta_1 & \beta_1 & 0 & -(\beta_2+\delta_2+2\alpha_1) & \mu_2 & \alpha_2 \\ 0 & 0 & \beta_1 & \delta_2 & -(\beta_2+\alpha_1+\mu_2) & \alpha_2 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

In the steady state, the derivatives of the state probabilities become zero and this will enable us to compute steady state busy period :

$$B_V(\infty)=1-P_0(\infty) \qquad (7)$$

The steady state busy period $B_V(\infty)$ is therefore:

$$B_V(\infty)=\frac{N_3}{D_2}$$

$$N_3=\alpha_1\alpha_2(4\alpha_1\mu_2\delta_1+2\alpha_1\beta_1\mu_2+4\mu_2^2\delta_1+2\beta_1\mu_2^2+4\beta_1\mu_2\delta_1+2\beta_1^2\mu_2+2\mu_2\delta_1\delta_2+2\beta_1\beta_2\mu_2+$$

$$3\beta_2\mu_2\delta_1+2\beta_1\mu_2\delta_2+2\beta_1\beta_2\delta_1+\beta_1^2\beta_2)+\alpha_1\alpha_2(\beta_1\beta_2\mu_1+2\beta_1\mu_1\delta_2+2\alpha_1\beta_1\delta_2+4\alpha_1\delta_1\delta_2+2\beta_1\mu_2\delta_2+4\mu_2\delta_1\delta_2+2\beta_1^2\delta_2+$$

$$2\beta_1\beta_2\delta_2+2\delta_1\delta_2^2+2\beta_1\delta_2^2+3\beta_2\delta_1\delta_2+2\beta_1\delta_1\delta_2+\beta_1^2\beta_2+\beta_1\beta_2\delta_1)+\alpha_2\beta_1(2\alpha_1\mu_1\mu_2+2\mu_1\mu_2^2+\beta_2\mu_1\mu_2+2\beta_1\mu_1\mu_2+\beta_1\beta_2\mu_1+$$

$$2\alpha_1\mu_2\delta_1+2\alpha_1\beta_1\mu_2+2\mu_2^2\delta_1+2\beta_1\mu_2^2+2\beta_1\mu_2\delta_1+2\beta_1^2\mu_2+2\beta_1\mu_2\delta_2+2\mu_2\delta_1\delta_2+\beta_2\mu_2\delta_1+\beta_1\beta_2\mu_2+\beta_1\beta_2\delta_1+\beta_1\beta_2\delta_2+$$

$$\beta_2\delta_1\delta_2+\beta_1^2\beta_2)+\alpha_2\beta_1(\beta_2\mu_1\mu_2+2\mu_1\mu_2\delta_2+2\beta_1\mu_1\delta_2+\beta_1\beta_2\mu_1+2\alpha_1\beta_1\delta_2+4\alpha_1\delta_1\delta_2+2\beta_1\mu_2\delta_2+\beta_2\mu_2\delta_1+2\mu_2\delta_1\delta_2+$$

$$\beta_1\beta_2\mu_2+2\beta_1^2\delta_2+\beta_2\delta_1\delta_2+2\beta_1\delta_1\delta_2+\beta_1\beta_2\delta_2+2\beta_1\delta_2^2+\beta_1^2\beta_2+2\delta_1\delta_2^2+\beta_1\beta_2\delta_1)+\beta_1\beta_2(\alpha_1\beta_1\mu_2+\beta_1\beta_2\mu_2+\beta_1\beta_2\mu_1+$$

$$\beta_1\mu_1\mu_2+2\beta_1\mu_2\delta_2+\beta_1\beta_2\delta_2+\beta_2\mu_1\mu_2+\beta_1\mu_1\delta_2+2\mu_2\delta_1\delta_2+\beta_1\delta_2^2+\alpha_1\beta_1\delta_2+\beta_1^2\delta_2+\mu_1\mu_2\delta_2+\alpha_1\mu_1\mu_2+\beta_2\mu_2\delta_1+$$

$$\beta_1\mu_2\delta_1+\beta_1\beta_2\delta_1+\beta_2\delta_1\delta_2+\beta_1\delta_1\delta_2+\delta_1\delta_2^2+\beta_1\mu_2^2+\beta_1^2\mu_2+\beta_1^2\beta_2+\mu_1\mu_2^2+\mu_2^2\delta_1+2\alpha_1\delta_1\delta_2+\alpha_1\mu_2\delta_1)$$

## 4. Results and Discussions

In this section, we numerically obtained the results for mean time to system failure, system availability, busy period and

profit function for all the developed models. For the model analysis, the following set of parameters values are fixed

throughout the simulations for consistency:

$$\beta_1 = 0.1, \beta_2 = 0.2, \alpha_1 = 0.4, \alpha_2 = 0.1, \delta_1 = 0.1, \delta_2 = 0.1, \mu_1 = 0.3, \mu_2 = 0.4$$

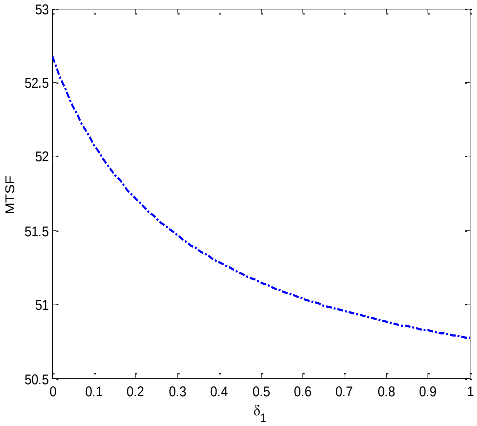


Fig. 2 Effect of $\delta_1$ on MTSF

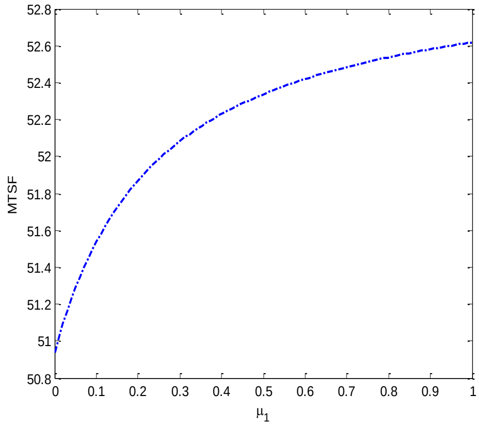


Fig. 3 Effect of $\mu_1$ on MTSF

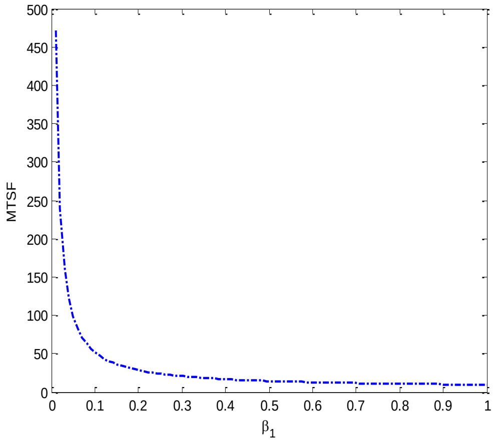


Fig. 4 Effect of $\beta_1$ on MTSF

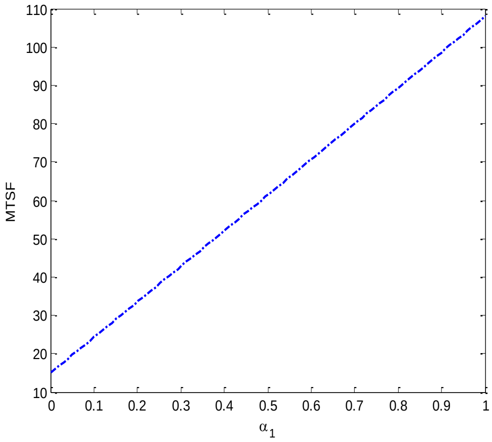


Fig. 5 Effect of $\alpha_1$ on MTSF

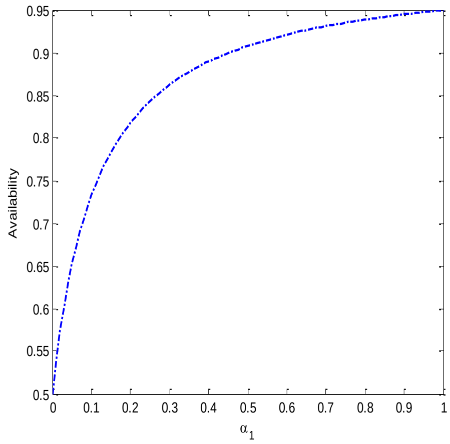


Fig. 6 Effect of $\alpha_1$ on Availability

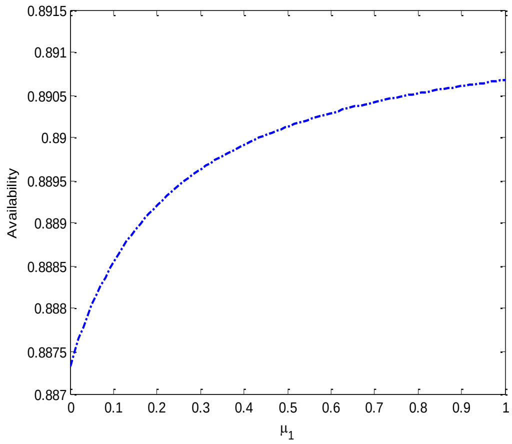


Fig. 7 Effect of $\mu_1$ on Availability

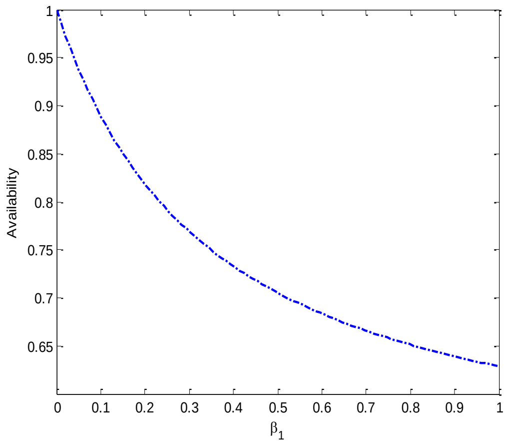


Fig. 8 Effect of $\beta_1$ on Availability

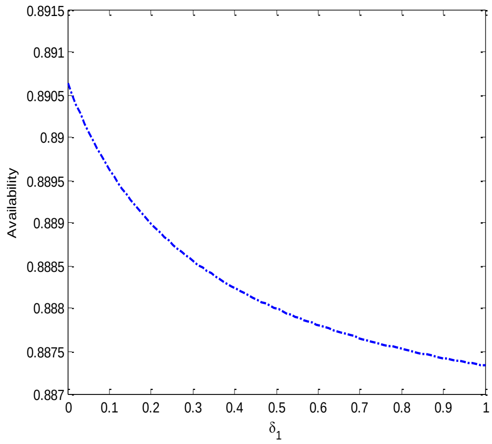


Fig. 9 Effect of $\delta_1$ on Availability

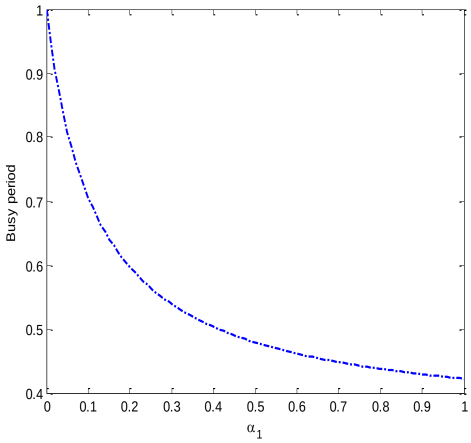


Fig. 10 Effect of $\alpha_1$ on Busy period

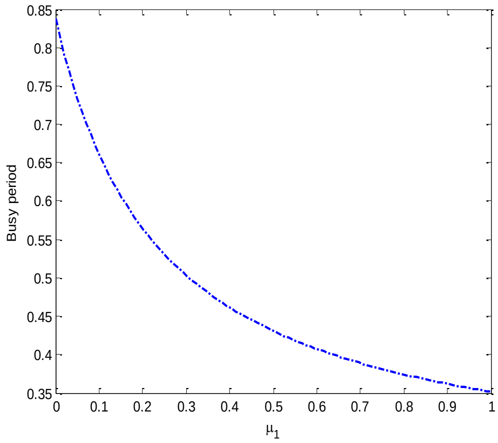


Fig. 11 Effect of $\mu_1$ on Busy period

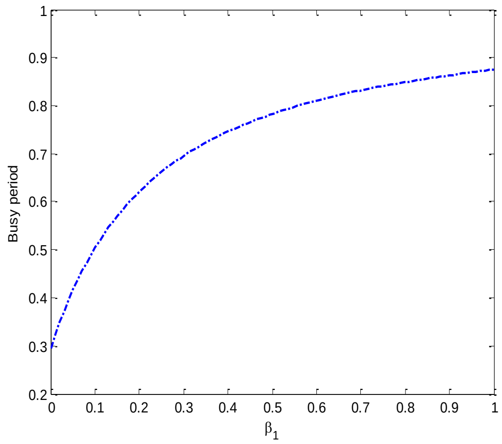


Fig. 12 Effect of $\beta_1$ on Busy period

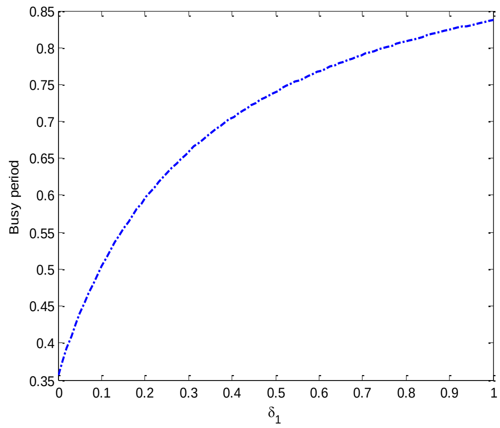


Fig. 13 Effect of $\delta_1$ on Busy period

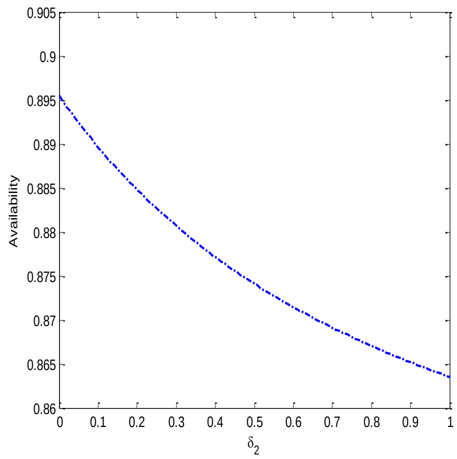


Fig. 14 Effect of $\delta_2$ on availability

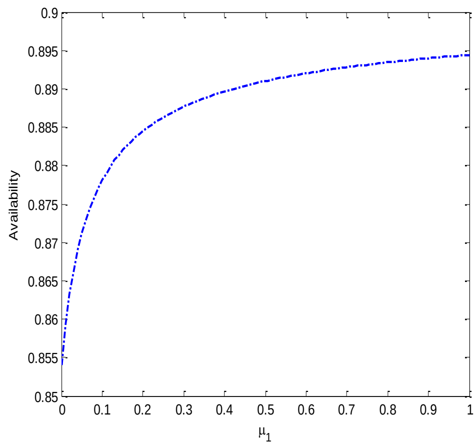


Fig. 15 Effect of $\mu_2$ on availability

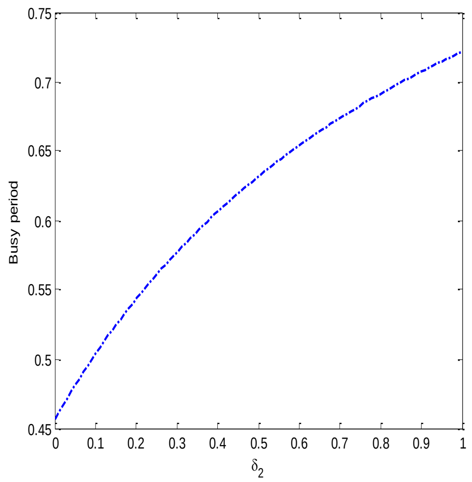


Fig. 16 Effect of $\delta_2$ on Busy period

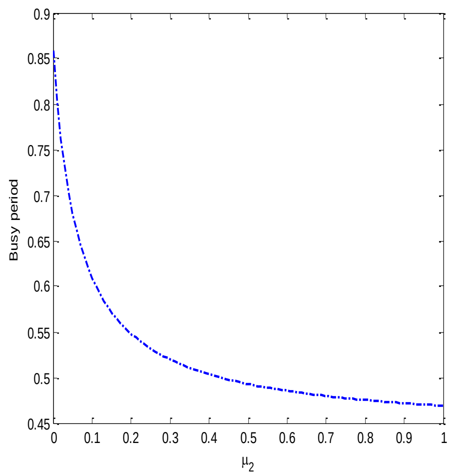


Fig. 17 Effect of $\mu_2$ on Busy period

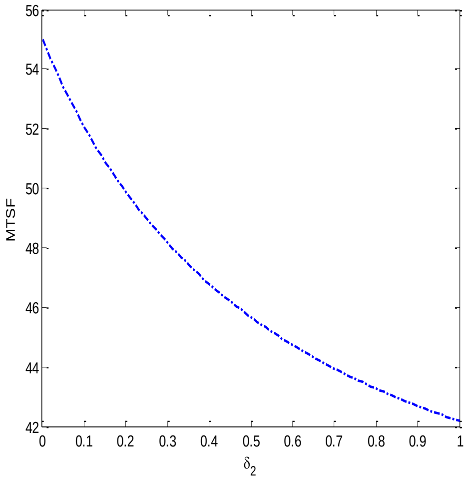


Fig. 18 Effect of $\delta_2$ on MTSF

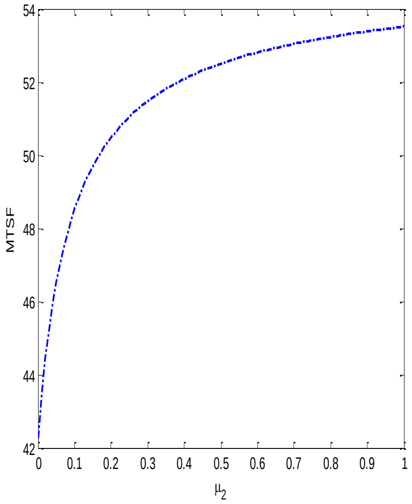


Fig. 19 Effect of $\mu_2$ on MTSF

The impact of $\delta_1$ on MTSF, steady-state availability and busy period can be observed in Figures 2,9 and 13. From Figures 2, and 9, it is evident that the MTSF and steady-state availability decreases as $\delta_1$ increases while in Fig. 13, busy period increases with increase in $\delta_1$. Similar results can be observed in Figures 14,16 and 18 on steady-state availability, busy period and MTSF with respect to $\delta_2$. From Figures 14 and 18 steady-state availability and MTSF decreases as $\delta_2$ increases and the busy period increases with increase in $\delta_2$ from Fig. 16. Results of MTSF, steady-state availability and busy period with respect to $\beta_1$ are given in Figures 4,8 and 12. It is evident from Figures 4,8 and 12 that as $\beta_1$ increases, the MTSF, steady-state availability and busy period decreases while from Figures 5,6 and 10, the MTSF, steady-state availability and busy period increases with increase in $\alpha_1$. Furthermore, the impact of $\mu_1$ on MTSF, steady-state availability and busy period can be observed in Figures 3,7 and 11. From Figures 3 and 7, it is evident that MTSF and steady-state availability increases with increase in $\mu_1$ while busy period decreases as $\mu_1$ increase from Fig. 11. Similarly, the effect of $\mu_2$ on steady-state availability, busy period and MTSF can be seen in Figures 15,17 and 19. In Figures 15 and 19, the steady-state availability and MTSF increase as $\mu_2$ increases and the busy period decreases with increase in $\mu_2$ from Fig. 17.

## 5. Conclusion

In this paper, we constructed a two-stage deteriorating three layers reinforce concrete structure to study the effectiveness of the structure. Explicit expressions MTSF, steady-state availability and busy period were derived. We performed numerical investigation to see the effect of slow deterioration rate, fast deterioration rate, minor minimal maintenance, major minimal maintenance, failure and repair rates on MTSF, steady-state availability and busy period of repairman. It is evident from the results that repair rate, minor minimal maintenance rate and major minimal maintenance rates prolong the life span of the reinforce concrete structure while slow deterioration, fast deterioration and failure rate decrease the life span of the structure.

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