Some Fixed Point Theorem for Expansion onto Mappings on Cone metric Spaces

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Abstract

In this article, we prove some fixed point theorems in cone metric spaces by using expansion mapping.

Key Words

Cone metric space, fixed point , Common fixed point , expansion mapping.

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1 Introduction and Preliminaries

Let E be a real Banach space and P a subset of E. The subset P is called a cone if and only if :

(i) P is closed , non-empty and $P \neq \{0\}$
(ii) $ax + by \in P$ for all $x, y \in P$ and non-negative real numbers a, b.
(iii) $P \cap (-P) = \{0\}$

Given a cone $P \subset E$, we define a partial ordering $\leq$ on E with respect to P by $x \leq y$ if and only if $y - x \in P$. We shall write $x < y$ if $x \leq y$ and $x \neq y$; we shall write $x \ll y$ if $y - x \in intP$, where $intP$ denotes the interior of P .The cone P is called normal if there is a number $K > 0$ such that for all $x, y \in E$,

$0 \leq x \leq y$ implies $\|x\| \leq K\|y\|$.

The least positive number satisfying the above inequality is called the normal constant of P [1].

Definition1.1:[1]

Let X be a non-empty set. Suppose that mapping $d: X \times X \to E$ satisfies the following :

(d_1) $0 \leq d(x, y)$ for all $x, y \in X$ and $d(x, y) = 0$ if and only if $x = y$,

(d_2) $d(x, y) = d(y, x)$ for all $x, y \in X$,

(d_3) $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in X$.

Then d is called a cone metric space on X and (X, d) is called a cone metric space .

Definition1.2:[1]

Let (X, d) be a cone metric space, $x \in X$ and $\{x_n\}_{n\geq1}$ a sequence in X .Then we say that

(i) $\{x_n\}_{n\geq1}$ converges to x whenever for every $c \in E$ with $0 \ll c$, there is a natural number N such that $d(x_n, x) \leq c$ for all $n \geq N$. We denote this by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$.
(ii) $\{x_n\}_{n\geq1}$is a Cauchy sequence whenever for every $c \in E$ with $0 \ll c$ there is a natural number N such that $d(x_n, x_m) \ll c$ for all $n, m \geq N$.
(iii) (X, d) is a complete cone metric space if every Cauchy sequence is convergent .

2 Main Results

Theorem 2.1 Let (X,d) be a complete cone metric space and the mapping $T: X \to X$ is onto and satisfies the contractive condition

$$d(Tx,Ty) \geq K \min\left\{\frac{d(x,Tx).d(y,Ty)+d(x,Ty).d(y,Tx)}{d(x,y)}, d(x,y), d(x,Tx), d(y,Ty)\right\}$$

For all $x,y \in X$, where $K > 1$ is a constant. Then T has a unique fixed point in X.

Proof : For each $x_0 \in X$, since T is onto , there exist $x_1 \in X$ such that $Tx_1 = x_0$ similarly ,we can write

$$x_n = Tx_{n+1} \quad \text{for each} \quad n \geq 1$$

If $x_{n-1} = x_n$ then x_n is a fixed point of T.

Now suppose that $x_{n-1} \neq x_n$ for all $n \geq 1$. Then

$$d(x_n, x_{n-1}) = d(Tx_{n+1}, Tx_n)$$

$$\geq K \min\left\{\begin{matrix}\frac{d(x_{n+1},Tx_{n+1}).d(x_n,Tx_n)+d(x_{n+1},Tx_n).d(x_n,Tx_{n+1})}{d(x_{n+1},x_n)}, \\ d(x_{n+1},x_n), d(x_{n+1},Tx_{n+1}), d(x_n,Tx_n)\end{matrix}\right\}$$

$$\geq K \min\left\{\begin{matrix}\frac{d(x_{n+1},x_n).d(x_n,x_{n-1})+d(x_{n+1},x_{n-1}).d(x_n,x_n)}{d(x_{n+1},x_n)}, \\ d(x_{n+1},x_n), d(x_{n+1},x_n), d(x_n,x_{n-1})\end{matrix}\right\}$$

$$\geq K \min\{d(x_n, x_{n-1}), d(x_{n+1},x_n), d(x_{n+1},x_n), d(x_n,x_{n-1})\}$$

$$\geq K \min\{d(x_{n+1},x_n), d(x_n,x_{n-1})\}$$

Case I

$$d(x_n,x_{n-1}) \geq K d(x_n,x_{n-1})$$
$$\Longrightarrow 1 \geq K$$

Which is contradiction.

Case II

$$d(x_n,x_{n-1}) \geq K d(x_{n+1},x_n)$$
$$d(x_{n+1},x_n) \leq \frac{1}{K} d(x_n,x_{n-1})$$

$$d(x_{n+1},x_n) \leq h\, d(x_n,x_{n-1}) \qquad \text{Where } h = \frac{1}{K} < 1 \ (As\ K > 1)$$

From this we get $d(x_n,x_{n+1}) \leq h^n\, d(x_0,x_1)$

Now for $n < m$ we have

$$d(x_n,x_m) \leq d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \cdots \ldots\ldots + d(x_{m-1}, x_m)$$

$$\leq (h^n + h^{n+1} + \cdots \ldots + h^{m-1})\, d(x_0,x_1)$$

$$\leq \frac{h^n}{1-h} d(x_0,x_1)$$

Let $0 \leq c$ be given. Choose a natural number N_1 such that $\frac{h^n}{1-h} d(x_0,x_1) \leq c$ for all $n \geq N_1$.Thus, $d(x_n,x_m) \leq c$, for $n < m$. Therefore, $\{x_n\}_{n\geq 1}$ is a Cauchy sequence in (X,d). Since (X,d) is a complete cone metric space, there exists $x^* \in X$ such that $x_n \to x$ as $n \to \infty$.

If T is continuous, then $d(Tx^*,x^*) \leq d(Tx_n,Tx^*) + d(Tx_n,x^*) \to 0$ as $n \to \infty$. Therefore, $d(Tx^*,x^*) = 0$ and so $Tx^* = x^*$. Thus T has a fixed point in X.

Theorem 2.2 Let (X,d) be a complete cone metric space and the mapping $T: X \rightarrow X$ is onto and satisfies the contractive condition

$$d(Tx,Ty) \geq K\{d(x,Tx).d(y,Ty) + d(x,Ty).d(y,Tx)\}^{\frac{1}{2}}$$

For all $x, y \in X$, where $K > 1$ is a constant. Then T has a unique fixed point in X.

Proof : For each $x_0 \in X$, since T is onto , there exist $x_1 \in X$ such that $Tx_1 = x_0$ similarly ,we can write

$$x_n = Tx_{n+1} \quad \text{for each} \quad n \geq 1$$

If $x_{n-1} = x_n$ then x_n is a fixed point of T.

Now suppose that $x_{n-1} \neq x_n$ for all $n \geq 1$. Then

$$d(x_n, x_{n-1}) = d(Tx_{n+1}, Tx_n)$$

$$\geq K\{d(x_{n+1}, Tx_{n+1}).d(x_n, Tx_n) + d(x_{n+1}, Tx_n).d(x_n, Tx_{n+1})\}^{\frac{1}{2}}$$

$$\geq K\{d(x_{n+1}, x_n).d(x_n, x_{n-1}) + d(x_{n+1}, x_{n-1}).d(x_n, x_n)\}^{\frac{1}{2}}$$

$$\geq K\{d(x_{n+1}, x_n).d(x_n, x_{n-1})\}^{\frac{1}{2}}$$

$$\{d(x_n, x_{n-1})\}^2 \geq K^2 d(x_{n+1}, x_n).d(x_n, x_{n-1})$$

$$d(x_n, x_{n-1}) \geq K^2 d(x_{n+1}, x_n)$$

$$d(x_n, x_{n+1}) \leq \frac{1}{K^2} d(x_{n-1}, x_n)$$

$$d(x_n, x_{n+1}) \leq h\, d(x_{n-1}, x_n) \qquad \text{Where } h = \frac{1}{K^2} < 1 \ (As\ K > 1)$$

From this we get $d(x_n, x_{n+1}) \leq h^n\, d(x_0, x_1)$

Now for $n < m$ we have

$$d(x_n, x_m) \leq d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \cdots \ldots .. + d(x_{m-1}, x_m)$$

$$\leq (h^n + h^{n+1} + \cdots \ldots + h^{m-1})\, d(x_0, x_1)$$

$$\leq \frac{h^n}{1-h} d(x_0, x_1)$$

Let $0 \leq c$ be given. Choose a natural number N_1 such that $\frac{h^n}{1-h} d(x_0, x_1) \leq c$ for all $n \geq N_1$.Thus, $d(x_n, x_m) \leq c$, for $n < m$. Therefore, $\{x_n\}_{n \geq 1}$ is a Cauchy sequence in (X, d). Since (X, d) is a complete cone metric space, there exists $x^* \in X$ such that $x_n \rightarrow x$ as $n \rightarrow \infty$.

If T is continuous, then $d(Tx^*, x^*) \leq d(Tx_n, Tx^*) + d(Tx_n, x^*) \rightarrow 0$ as $n \rightarrow \infty$. Therefore, $d(Tx^*, x^*) = 0$ and so $Tx^* = x^*$. Thus T has a fixed point in X.

Theorem 2.3 Let (X,d) be a complete cone metric space and the mapping $T: X \rightarrow X$ is onto and satisfies the contractive condition

$$d(Tx,Ty) \geq \frac{K}{2} \frac{[d(x,Tx) + d(y,Ty)]^2}{d(x,Ty) + d(y,Tx)}$$

For all $x, y \in X$, where $1 < K \leq 2$ is a constant. Then T has a unique fixed point in X.

Proof : For each $x_0 \in X$, since T is onto , there exist $x_1 \in X$ such that $Tx_1 = x_0$ similarly ,we can write

$$x_n = Tx_{n+1} \quad \text{for each} \quad n \geq 1$$

If $x_{n-1} = x_n$ then x_n is a fixed point of T.

Now suppose that $x_{n-1} \neq x_n$ for all $n \geq 1$. Then

$$d(x_n, x_{n-1}) = d(Tx_{n+1}, Tx_n)$$

$$\geq \frac{K}{2} \frac{[d(x_{n+1}, Tx_{n+1}) + d(x_n, Tx_n)]^2}{d(x_{n+1}, Tx_n) + d(x_n, Tx_{n+1})}$$

$$\geq \frac{K}{2} \frac{[d(x_{n+1}, x_n) + d(x_n, x_{n-1})]^2}{d(x_{n+1}, x_{n-1}) + d(x_n, x_n)}$$

$$d(x_n, x_{n-1}).\, d(x_{n+1}, x_{n-1}) \geq \frac{K}{2}[d(x_{n+1}, x_n) + d(x_n, x_{n-1})]^2$$

$$d(x_n, x_{n-1}).\, [d(x_{n-1}, x_n) + d(x_n, x_{n+1})] \geq \frac{K}{2}[d(x_{n+1}, x_n) + d(x_n, x_{n-1})]^2$$

$$d(x_n, x_{n-1}) \geq \frac{K}{2}[d(x_{n+1}, x_n) + d(x_n, x_{n-1})]$$

$$Kd(x_{n+1}, x_n) \leq (2 - K)d(x_n, x_{n-1})$$

$$d(x_{n+1}, x_n) \leq \frac{2-K}{K} d(x_n, x_{n-1})$$

$$d(x_{n+1}, x_n) \leq h\, d(x_n, x_{n-1}) \qquad \text{Where } h = \frac{2-K}{K} < 1\ (As 1 < K \leq 2)$$

From this we get $d(x_n, x_{n+1}) \leq h^n\, d(x_0, x_1)$

Now for $n < m$ we have

$$d(x_n, x_m) \leq d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \cdots \ldots + d(x_{m-1}, x_m)$$

$$\leq (h^n + h^{n+1} + \cdots \ldots + h^{m-1})\, d(x_0, x_1)$$

$$\leq \frac{h^n}{1-h} d(x_0, x_1)$$

Let $0 \leq c$ be given. Choose a natural number N_1 such that $\frac{h^n}{1-h} d(x_0, x_1) \leq c$ for all $n \geq N_1$.Thus, $d(x_n, x_m) \leq c$, for $n < m$. Therefore, $\{x_n\}_{n\geq 1}$ is a Cauchy sequence in (X, d). Since (X, d) is a complete cone metric space, there exists $x^* \in X$ such that $x_n \to x$ as $n \to \infty$.

If T is continuous, then $d(Tx^*, x^*) \leq d(Tx_n, Tx^*) + d(Tx_n, x^*) \to 0$ as $n \to \infty$. Therefore, $d(Tx^*, x^*) = 0$ and so $Tx^* = x^*$. Thus T has a fixed point in X

Theorem 2.4 Let (X, d) be a complete cone metric space and the mapping $T: X \to X$ is onto and satisfies the contractive condition

$$d(Tx, Ty) \geq \alpha \left[\frac{d^2(x, Tx) + d^2(y, Ty)}{d(x, Tx) - d(y, Ty)}\right] + \beta \left[\frac{d^2(x, Ty) + d^2(y, Tx)}{d(x, Ty) - d(y, Tx)}\right]$$

$$+\gamma[d(x, Tx) + d(y, Ty)] + \delta d(x, y)$$

For all $x, y \in X, \alpha > 1, \beta > 1, \gamma > 1, \delta > 1$ *and* $2\gamma + \delta > 1$.

Then T has a unique fixed point in X.

Proof : For each $x_0 \in X$, since T is onto , there exist $x_1 \in X$ such that $Tx_1 = x_0$ similarly ,we can write

$$x_n = Tx_{n+1} \qquad \text{for each } n \geq 1$$

If $x_{n-1} = x_n$ then x_n is a fixed point of T.

Now suppose that $x_{n-1} \neq x_n$ for all $n \geq 1$. Then

$$d(x_n, x_{n-1}) = d(Tx_{n+1}, Tx_n)$$

$$\geq \alpha \left[\frac{d^2(x_{n+1},Tx_{n+1})+d^2(x_n,Tx_n)}{d(x_{n+1},Tx_{n+1})-d(x_n,Tx_n)}\right] + \beta \left[\frac{d^2(x_{n+1},Tx_n)+d^2(x_n,Tx_{n+1})}{d(x_{n+1},Tx_n)-d(x_n,Tx_{n+1})}\right]$$

$$+\gamma[d(x_{n+1}, Tx_{n+1}) + d(x_n, Tx_n)] + \delta d(x_{n+1}, x_n)$$

$$\geq \alpha \left[\frac{d^2(x_{n+1},x_n)+d^2(x_n,x_{n-1})}{d(x_{n+1},x_n)-d(x_n,x_{n-1})}\right] + \beta \left[\frac{d^2(x_{n+1},x_{n-1})+d^2(x_n,x_n)}{d(x_{n+1},x_{n-1})-d(x_n,x_n)}\right]$$

$$+\gamma[d(x_{n+1}, x_n) + d(x_n, x_{n-1})] + \delta d(x_{n+1}, x_n)$$

$$\geq \alpha \left[\frac{\{d(x_{n+1},x_n)-d(x_n,x_{n-1})\}^2+2d(x_{n+1},x_n).d(x_n,x_{n-1})}{d(x_{n+1},x_n)-d(x_n,x_{n-1})}\right] + \beta \left[\frac{d^2(x_{n+1},x_{n-1})}{d(x_{n+1},x_{n-1})}\right]$$

$$+\gamma[d(x_{n+1}, x_n) + d(x_n, x_{n-1})] + \delta d(x_{n+1}, x_n)$$

$$\geq \alpha \left[\frac{\{d(x_{n+1},x_n)-d(x_n,x_{n-1})\}^2}{d(x_{n+1},x_n)-d(x_n,x_{n-1})}\right] + \beta[d(x_{n+1}, x_{n-1})]$$

$$+\gamma[d(x_{n+1}, x_n) + d(x_n, x_{n-1})] + \delta d(x_{n+1}, x_n)$$

$$\geq \alpha[d(x_{n+1}, x_n) - d(x_n, x_{n-1})] + \beta[d(x_{n+1}, x_n) - d(x_{n-1}, x_n)]$$

$$+\gamma[d(x_{n+1}, x_n) + d(x_n, x_{n-1})] + \delta d(x_{n+1}, x_n)$$

$$\geq \alpha d(x_{n+1}, x_n) - \alpha d(x_n, x_{n-1}) + \beta d(x_{n+1}, x_n) - \beta d(x_{n-1}, x_n)$$

$$+\gamma d(x_{n+1}, x_n) + \gamma d(x_n, x_{n-1}) + \delta d(x_{n+1}, x_n)$$

$$(1 + \alpha + \beta - \gamma) d(x_n, x_{n-1}) \geq (\alpha + \beta + \gamma + \delta) d(x_{n+1}, x_n)$$

$$(\alpha + \beta + \gamma + \delta) d(x_{n+1}, x_n) \leq (1 + \alpha + \beta - \gamma) d(x_n, x_{n-1})$$

$$d(x_{n+1}, x_n) \leq \frac{1+\alpha+\beta-\gamma}{(\alpha+\beta+\gamma+\delta)} d(x_n, x_{n-1})$$

$d(x_{n+1}, x_n) \leq h\, d(x_n, x_{n-1})$ Where $h = \frac{1+\alpha+\beta-\gamma}{(\alpha+\beta+\gamma+\delta)} < 1$

From this we get $d(x_n, x_{n+1}) \leq h^n\, d(x_0, x_1)$

Now for $n < m$ we have

$$d(x_n, x_m) \leq d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \cdots \ldots + d(x_{m-1}, x_m)$$

$$\leq (h^n + h^{n+1} + \cdots \ldots + h^{m-1})\, d(x_0, x_1)$$

$$\leq \frac{h^n}{1-h} d(x_0, x_1)$$

Let $0 \leq c$ be given. Choose a natural number N_1 such that $\frac{h^n}{1-h} d(x_0, x_1) \leq c$ for all $n \geq N_1$.Thus, $d(x_n, x_m) \leq c$, for $n < m$. Therefore, $\{x_n\}_{n\geq 1}$ is a Cauchy sequence in (X, d). Since (X, d) is a complete cone metric space, there exists $x^* \in X$ such that $x_n \to x$ as $n \to \infty$.

If T is continuous, then $d(Tx^*, x^*) \leq d(Tx_n, Tx^*) + d(Tx_n, x^*) \to 0$ as $n \to \infty$. Therefore, $d(Tx^*, x^*) = 0$ and so $Tx^* = x^*$. Thus T has a fixed point in X.

Theorem 2.5 Let (X, d) be a complete cone metric space and the mapping $T: X \to X$ is onto and satisfies the contractive condition

$d(Tx,Ty) \geq \alpha \frac{d(x,Ty)[d(x,y)+d(x,Tx)+d(y,Tx)]}{d(x,Ty)+d(y,Tx)} + \beta \frac{d(x,Tx)[d(x,y)+d(y,Tx)+d(y,Ty)]}{d(x,Ty)+d(y,Tx)}$

For all $x, y \in X, \alpha > 1, \beta > 1$ and $2\alpha + \beta > 1$.

Then T has a unique fixed point in X.

Proof : For each $x_0 \in X$, since T is onto , there exist $x_1 \in X$ such that $Tx_1 = x_0$ similarly ,we can write

$$x_n = Tx_{n+1} \quad \text{for each} \quad n \geq 1$$

If $x_{n-1} = x_n$ then x_n is a fixed point of T.

Now suppose that $x_{n-1} \neq x_n$ for all $n \geq 1$. Then

$$d(x_n, x_{n-1}) = d(Tx_{n+1}, Tx_n)$$

$\geq \alpha \frac{d(x_{n+1},Tx_n)[d(x_{n+1},x_n)+d(x_{n+1},Tx_{n+1})+d(x_n,Tx_{n+1})]}{d(x_{n+1},Tx_n)+d(x_n,Tx_{n+1})}$

$+\beta \frac{d(x_{n+1},Tx_{n+1})[d(x_{n+1},x_n)+d(x_n,Tx_{n+1})+d(x_n,Tx_n)]}{d(x_{n+1},Tx_n)+d(x_n,Tx_{n+1})}$

$\geq \alpha \frac{d(x_{n+1},x_{n-1})[d(x_{n+1},x_n)+d(x_{n+1},x_n)+d(x_n,x_n)]}{d(x_{n+1},x_{n-1})+d(x_n,x_n)}$

$+\beta \frac{d(x_{n+1},x_n)[d(x_{n+1},x_n)+d(x_n,x_n)+d(x_n,x_{n-1})]}{d(x_{n+1},x_{n-1})+d(x_n,x_n)}$

$\geq 2\alpha[d(x_{n+1}, x_n)] + \beta \frac{d(x_{n+1},x_n)[d(x_{n+1},x_n)+d(x_n,x_{n-1})]}{d(x_{n+1},x_{n-1})}$

$\geq 2\alpha[d(x_{n+1}, x_n)] + \beta d(x_{n+1}, x_n)$

$\geq (2\alpha + \beta)[d(x_{n+1}, x_n)]$

$d(x_{n+1}, x_n) \leq \frac{1}{2\alpha+\beta} d(x_n, x_{n-1})$

$d(x_{n+1}, x_n) \leq h\, d(x_n, x_{n-1})$ Where $h = \frac{1}{2\alpha+\beta} < 1$

From this we get $d(x_n, x_{n+1}) \leq h^n\, d(x_0, x_1)$

Now for $n < m$ we have

$d(x_n, x_m) \leq d(x_n\, , x_{n+1}) + d(x_{n+1}\, , x_{n+2}) + \cdots \ldots .. + d(x_{m-1}\, , x_m)$

$\leq (h^n + h^{n+1} + \cdots \ldots + h^{m-1})\, d(x_0, x_1)$

$\leq \frac{h^n}{1-h} d(x_0, x_1)$

Let $0 \leq c$ be given. Choose a natural number N_1 such that $\frac{h^n}{1-h} d(x_0, x_1) \leq c$ for all $n \geq N_1$.Thus, $d(x_n, x_m) \leq c$, for $n < m$. Therefore, $\{x_n\}_{n\geq 1}$ is a Cauchy sequence in (X, d). Since (X, d) is a complete cone metric space, there exists $x^* \in X$ such that $x_n \to x$ as $n \to \infty$.

If T is continuous, then $d(Tx^*, x^*) \leq d(Tx_n, Tx^*) + d(Tx_n, x^*) \to 0$ as $n \to \infty$. Therefore, $d(Tx^*, x^*) = 0$ and so $Tx^* = x^*$. Thus T has a fixed point in X.

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