# Investigating and Modeling Colors with Set theory
# A Case of Color Mixing

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**Abstract**
The paper introduced Colors in general and how they affect us and the environment. This work used set theory to explain the concept of Color mixing. The idea of set can be very useful to model colors; we here consider additive and Subtractive color mixing, for better understanding of the subject.
**Keyword:** Additive color mixing, Colors, Set theory, Subtractive color mixing, Intersection of set, Union of set, Universal set and Complement of set.

## 1.0 Introduction
### 1.1 Colors

In an experiment, first performed by Isaac Newton (1665), it was observed that when white light is passed through a prism, an elongated colored path of light is obtained on a screen placed behind the prism. The colored pattern is known as the spectrum of white light. The spectrum consists of the Red, Orange, Yellow, Green, Blue, indigo, violet (ROYGBIV), the colors were obtained in that form from apex side of the Prism,[1].

According to [ [2],[3] ], With colors one can set a mood, attract attention, or make a statement. The beauty of color cannot be overemphasis; in fact color can be used to energizes, or to cool down. By selecting the right color scheme, an ambiance of elegance warmth or tranquility can be created; you can convey an image of playful youthfulness. Color can be most powerful design element if you learn to use it effectively. Colors affect us in numerous ways, both mentally and physically. A strong red color has been shown to raise the blood pressure, while a blue color has a calming effect. Being able to use color consciously and harmoniously can help one create spectacular results.
There are two basic ways to mix colors to make other colors,[4].

1. One of these methods is accomplished by combining color illumination or light. Mixing color light is called "additive" color mixing.

2. The other method is accomplished by mixing together media such as paint, inks, dyes and other colorants. This is called "subtractive" color mixing.

### 1.2 Sets

A set is a word for a collecting of objects, numbers, ideas, items, etc. such that given an object , it is possible to determine whether that object belongs to the given collection or not.

The different objects, members etc. which form a set are called the members or elements of the set,

### 1.3 Elementary sets operations

We will like to use three operations for associating to any two or more given sets. These three operations namely, intersection, union and complementation of set are of great important to our study. Another set which needs to be introduced is the universal set, which is the entire set under consideration. For this study, we are considering additive and subtractive color mixing as the different sets under consideration.

The color of an object depends on the colors in the light incident or falling on it, and also on the absorption and reflection of this light by the object.

### 1.4 Additive mixing of colors

We can obtain a variety of colors by mixing the different colors of the spectrum. We cannot however, obtain Red, Green and Blue colors by mixing other colors. These three colors which are therefore used, to produce other colors is known as additive color mixing or additive combination of colors.

Secondary colors are the colors we obtain by mixing any two of the primary colors.

### 1.5 Subtractive color mixing

One can obtain other colors through mixing colors by subtracting other colors from the mixture

## 2.0 Methodology

### 2.1 COLOR MIXING

1. Additive color mixing RGB Model
   - Multiple light sources
   - Television, RGB displays, theatre lighting
   - Red + Green + Blue =White

   Red + Green = Yellow.
   Red + Blue= Magenta
   Green + Blue = Cyan

Yellow, Cyan and Magenta are the secondary colors or complementary colors. All the three primary colors combine to produce white.

Red + Green + Blue = White.

2. (a) Subtractive color mixing RYB Model
   - single light source
   - mixing paint
   - Red + Yellow + Blue = Black

   Red + Yellow = Orange
   Red + Blue = Purple
   Blue + Yellow = Green

   (b) Subtractive color mixing CMY Model

   In this CYM Model the primary color are Cyan, Magenta and Yellow

While the secondary colors are Green, Red and Blue

Cyan + Yellow = Green
Cyan + Magenta = Blue
Yellow + Magenta = Red
Cyan + Magenta + Yellow = Black

### 2.2 ADDITIVE COLOUR MIXING

- RGB are light primary colors which create colors by mixing light sources.

Complement color of the primary colors are at 180 degrees away from a primary color.

### 2.3 Physiological fact

A very wide range of intermediate colors can be produced by
1. Direct mixing beams of Red, Green and Blue light.
2. Indirect by using dyes or pigments.

When we apply Yellow it absorbs blue light, Magenta to absorb Green light and Cyan to absorb Red light.

### 2.4 Complementary color

Each of these complements has an equal amount of the primaries on either side of it and none of the opposite it.

| PRIMARY | COMPLEMENT |
|---|---|
| Red | Cyan |
| Green | Magenta |
| Blue | Yellow |

Complementary colors are also called Secondary Colors.

Print primary colors refer to the facts that complementary light primaries are the pigments/dyes used to mix all possible print colors. Two basic methods of mixture:

- Direct-------------Additive
- Indirect----------Subtractive

We shall use the following Letters to represent the different colors: R (Red), G (Green), B (Blue), K (Black), W (White), C (Cyan), M (Magenta), Y (Yellow), P (Purple), O (Orange) for easy modeling of the colors.

## 3.0 Result

### 3.1 Additive color process (RGB) Model

This begins with black, or the absence of light and therefore no color

Additive color mixture: light from two or more forces is combining before reaching the eye-

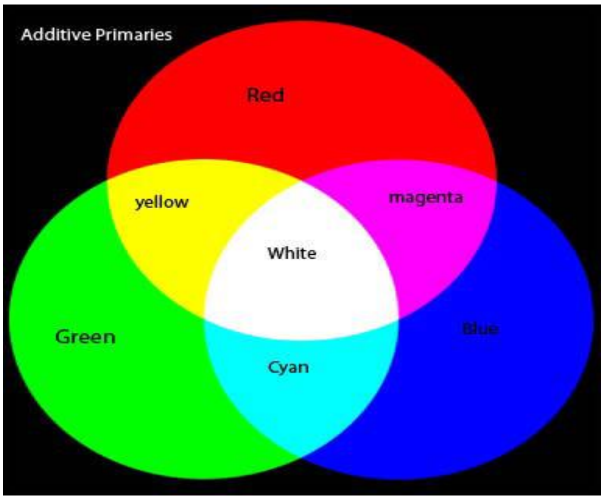
Adding and mixing the three primary wavelengths of light Red, Green and Blue in different combination process, form a full spectrum of colors.

(i) Adding and combining the blue and red wavelengths produce the sensation we call "Magenta"
(ii) Adding and combing the blue wavelengths produces the sensation called "cyan"
(iii) Adding and combining the red and green wavelengths process the sensation of "yellow"

### 3.2 Intersection of Sets as related to Colors

We shall now use the relationship of intersection of sets to explain the concept of additive and subtractive color mixing.

A Venn Diagram of Additive color mixing for the RGB Model




The Mathematical expression of additive color mixing which explain the Venn Diagram above with the universal set $\mu = \{B, G, R, Y\}$ is in equation (1)

$$\left.\begin{array}{r} B \cap R = M \\ B \cap G = C \\ R \cap G = Y \\ R \cap G \cap B = W \end{array}\right\} \qquad (1)$$

### 3.3 MORE ENERGY

The energy of the combination is equal to the sum of two initial beams. The result enhanced brightness when the beams are superimposed as in (1).

- White appears where Blue, Green and Red overlap.
- Mixing any two of the additive primary colors produces another color. Secondary colors in additive process: Cyan, Magenta and Yellow are same as the primary colors of the subtractive process. Examples of additive colors mixing:
  – projection system,
  - stage lighting

### 3.4 SUBTRACTIVE COLOR PROCESS (CMY) MODEL

This process starts with light already present and reflect from an object.

LIGHT AFFECTED BY PIGMENTS OR DYES

- This light absorbs or subtracts certain wavelengths.
- They allowing others to be reflected.

- They can be combined to form Red Green and Blue as secondary colors. - Combining the ideal subtractive primaries is equal to the amount that produces black. When all the colors superimposed there final effect is black as in (2). The primary subtractive colors:

- Cyan
- magenta
- Yellow.

A Venn diagram showing the Subtractive color mixing (CMY) Model

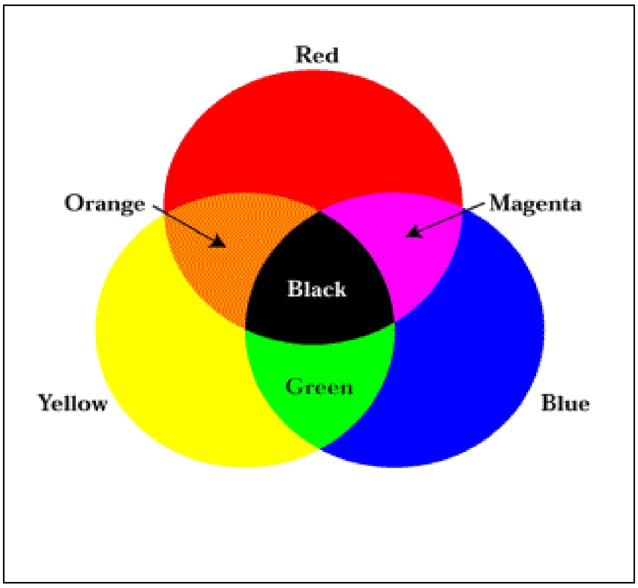

The Mathematical expression of Subtractive color mixing which explain the Venn Diagram above with the universal set $\mu = \{C, M, Y, K\}$ is in equation (2)

$$\left.\begin{array}{r} C \cap M = B \\ C \cap Y = G \\ M \cap Y = R \\ C \cap M \cap Y = K \end{array}\right\} \qquad (2)$$

## 3.5 Selective absorption

Each filter selectively absorbs light in its own way the emerging beam is composed of wavelengths that escape the absorption process. Subtractive Color Mixing is demonstrated by passing white light beam through filters.

A Venn diagram showing the Subtractive color mixing (RYB) Model



The Mathematical expression of Subtractive color mixing which explain the Venn Diagram above with the universal set $\mu = \{R, Y, B, K\}$ is in equation (3)

$$\left.\begin{aligned} \mathbf{R \cap Y = O} \\ \mathbf{R \cap B = P} \\ \mathbf{B \cap Y = G} \\ \mathbf{R \cap Y \cap B = K} \end{aligned}\right\} \qquad (3)$$

In the RYB (or subtractive) color model, the **primary colors** are red, yellow and blue. The three **secondary colors** (green, orange and purple) are created by mixing two primary colors. When the three primary colors superimposed they absorbed all colors to give black as in (3).

### 3.6 Less Energy

Because energy is removed from the beam by absorption, the brightness of the emerging beams is always diminished. The same process occurs when paints or ink are mixed. Examples of subtractive process:

- Color transparencies (Slides)
- Color Photographic prints (darkroom)
- Subtractive Color process allowed us to perceive color in environment.

### 3.7 Union of Sets as related to Colors

We here used the relationship of Union of sets to explain subtractive color mixing.

**From (2) above CMY Model**

$$\left.\begin{aligned} M \cup Y \cup G^1 \cup B^1 = R \\ M \cup C \cup G^1 \cup R^1 = B \\ C \cup Y \cup B^1 \cup R^1 = G \\ C \cup M \cup Y = K \end{aligned}\right\} \qquad (4)$$

Green and Blue are absorbed when combine the mixture of Magenta and Yellow to give Red color.
It is also seen that Magenta and Cyan when combined, it result to Blue color while Green and Red are absorbed. When combine the mixture of Cyan and Yellow color then absorbed both Blue and Red Color, only Green color is seen. The mixture of Cyan, Magenta and Yellow Color gives Black which means Green, Blue and Red colors were all absorbed as in (4).

**From (3) the RYB Model**

$$\left.\begin{aligned} R \cup Y \cup B^1 = O \\ R \cup B \cup Y^1 = P \\ B \cup Y \cup R^1 = G \\ R \cup Y \cup B = K \end{aligned}\right\} \qquad (5)$$

The mixture of Red and Yellow colors removes Blue color to form Orange color.
Purple color is seen when Yellow color is subtracted from the mixture of Red and Blue colors. The combination of Blue and Yellow colors absorbs Red color and only green color was seen. Finally when Red, Yellow and Blue colors were combined all colors were removed leaving only black as in (5).

## 4.0 Conclusion

From equation (1) – (5) of three models of color mixture, shows that Set theory can be used on the three models of color mixture, the RGB, CMY and RYB Models. We can therefore claim here that set theory can be used to explain different ideas in many other fields as well as the concept of color mixing, just as observed from this paper additive and Subtractive color mixing was considered and modeled with Sets, which give a better understanding of the subject and made it more interested.

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