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Convenient Vector Superspaces Without Norm And Their Properties

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ABSTRACT

One of the most widespread methods of modeling nature through elementary particles is to use field theory. In this paper this leads to the study of super spaces and super manifolds based on topological spaces without norm. To generalize and simplify the model to be valid in algebraic setting one needs to express it in category theory. However this might take time to achieve. In this paper, we introduce bornology and use it instead of topology in the modeling of elementary particles. The set with bornology is called by Kriegl and Michor convenient vector space. When the superstructure is added to it, we get convenient vector superspace. This space is shown, following Kriegl and Michor to be a Cartesian closed category. This should show us the way of modeling of field theory using category theory. This generality however has not been done in this paper.

1.0 INTRODUCTION

The main part of the paper studies smooth mappings and their calculus. Let us now try to describe the basic ideas of smooth calculus. As Kriegl and Michor mentioned, one can say that it is a more or less unique consequences of taking variational calculus seriously. We start by looking at the space of smooth curves $C^{\infty}(IR, E)$ with values in a locally convex space E and note that it does not depend on the topology of E only on the underlying system of bounded sets. This is due to the fact, that for a smooth curve difference quotients converge to the derivative much better than arbitrary converging nets or filters, smooth curves have integral in E if and only if a weak completeness condition is satisfied [1]. It appeared as bornologically complete or locally complete we call it C^{∞}- complete. This is equivalent to the condition that scalarwise smooth curves are smooth. All calculus in this thesis will be done on convenient vector superspace. These are locally convex vector super spaces which are $C^{\infty}-$complete. Note that the locally convex topology on convenient vector super space can vary in some range only the system of bounded sets must remain the same.

A mapping between convenient vector super space is called smooth if it maps smooth curves to smooth curves, and everything else that is existence, smoothness, and linearity of derivatives, the chain rule, and also the most important feature, Cartesian closedness holds without any restriction, as pointed out [1] in some convenient vector spaces there are smooth functions which are not continuous in the locally convex topology.

2.0 LITERATURE REVIEW

2.1.1 CONVENIENT LINEAR SUPER SPACES AND THEIR MAPPINGS

2.1.2 DEFINITION (Convex Superset)

Let (x_1, y_1) and (x_2, y_2) be point in a superspace. Then if (x_1, y_1) + $(1-\lambda)$ (x_2, y_2), $0 \leq 1 \leq \lambda$, $\lambda \varepsilon IR$ also belong to the same subsuperset, then the subsuperset is a convex super set.

2.1.3 DEFINITION (Bounded Superset)

A neighbourhood U_{xo} is bounded if for every convex bounded neighbourhood of (0,0), U_o, there exist $\lambda \varepsilon IR : \lambda_{xo} \subset U_o$ The set of all bounded subsets form a bornology of the space.

2.1.4 DEFINITION (Locally Convex Superspace)

A locally convex superspace E is a vector space together with a Hausdorff topology such that addition $E + E \rightarrow E$ and scalar multiplication $IRxE \rightarrow E$ are continuous and have a basis of neighborhoods consisting of convex sets.

2.1.5 DEFINITION (Bornological Superspace)

A locally convex vector super space E is called bornological if and only if

For any locally convex vector super spaces E,F any bounded linear mapping $T : E \rightarrow F$ is continuous..

2.1.6 DEFINITION 3 (Convenient Superspace)[Compare with the earlier]

Let E be a locally convex vector super space. E is said to be c^{∞} complete or convenient if one of the following equivalent completeness conditions are satisfied:

1. Any Lipschitz curve in E is locally integrable.
2. For any $C_1 \varepsilon C^{\infty}(R, E)$, there is $C_2 \varepsilon C^{\infty}(R, E)$ with $C_2^1 = C_1$ (existence of an anti-derivative).

1. E is C^{∞} $-$closed in any locally convex super space.
2. If $C : IR \rightarrow E$ is a curve such that $Loc : IR \rightarrow IR$ is smooth for all $L \varepsilon E^*$ then c is smooth. (Existence of supersmooth maps.)
3. Any Mackey-Cauchy sequence E is mackey complete.
4. If β is bounded closed absolutely convex then E_b is locally convex topological space. This property is called locally complete.
5. Any continuous linear mapping from a space into E has a continuous extension to the completion of the space.

2.1.7 DEFINITION (Super Matrix)

The Matrix

$$\left(\begin{array}{c|c} \textit{Commuting}\text{ elements} & \text{anticommuting elements} \\ \hline \textit{anticommuting}\text{ elements} & \text{commuting elements} \end{array} \right)$$

Where multiplication and addition are performed like in ordinary matrices is called a supermatrix.

2.1.8 DEFINITION (SUPERGEOMETRY)

A vector superspace with matrices as its endomorphism is called a vector super-geometry
Example in E^2

2.1.9 DEFINITION [SUPER SMOOTH MAPPING]

A mapping between convenient vector super spaces is called smooth if it maps smooth curves to smooth curves. A curve is supersmooth if it is differentiable at every point.

2.1.10 DE FINITION [Superderivative in a Bornological space]

Let X and Y be locally convex topological super space, $\beta\delta$ a system of bounded convex balanced and closed sets in X and Y respectively, B and C with dashes or not are sets from $\beta\delta$ a system of bounded convex, balanced and closed sets in X and Y respectively, B and C with dashes or not are sets from β and δ respectively. W is an open set in X. Then map $f : w \to Y$ is $D_{\beta\beta}$ superdifferentiable at the point $X_o \varepsilon W$ if there exist an $A\varepsilon L(X,Y)$ such that

$$o(h) = f(a+R) - f(a) - Ah$$

$\exists C \forall B \exists U : (h\varepsilon B + U, th\varepsilon u, x - a\varepsilon U) \Rightarrow o(th)\varepsilon tc$. We shall denote A by $f'(a)$.

2.1.11 DEFINITION (Synthetic Derivative)

Let X, Y be rings, $f : x \to y$ be a homomorphism between rings, then if $\forall x, b \in x \exists! d \in x \therefore$

$$f(x+d) = f(x) + db$$

where d is such that $d^2 = 0$

Theorem 1:

In a field, the superderivative is a synthetic derivative.

Proof.

In synthetic setting, $d^2 = 0$ and the ring structure is used in the products ab and d^2. In Bornological spaces, b is bounded and hence $\frac{o(h)}{d} \underset{h \to \partial}{\to} 0$, then 0(h)=0 in synthetic setting. This then implies that $f(a+d) = f(a) + bd$ where d belongs to the bornological space X.

Theorem 2 [Compare with 1]

Let $U_i \to (U_i + U_i) \subseteq E_{ii})$ be C^∞ – open subsets in locally convex super spaces, which need not be C^∞ – complete. Then a super mapping $f : U_i x U_2 \to F$ is smooth if and only if the canonically associated super mapping $f^v : U_i \to C^\infty(U_2, F)$ exist and is supersmooth.

Proof:
As in [1] but the rules of superstructures must be follows. From the above we have the following implication.

$f^{v}:U_1 \to C^{\infty}(U_2, F)$ is supersmooth.

$\Leftrightarrow foc:\Re \to C^{\infty}(U_2, F)$ is smooth for all smooth curves C_1 in U_1
$\Leftrightarrow C_2of^{v}oc: IR \to C^{\infty}(IR, F)$ is smooth for all smooth curves C_iU_i
$\Leftrightarrow fo(C_1xC_2) = (C_x^{*}of_o^{v}C_1):\Re^2 \to F$ is smooth for all smooth curves C_1 in U_i
$\Leftrightarrow f: U_1xU_2 \to P$ is smooth

Here the last equivalent is seen as follows: Each curve into U_1xU_2 is of the form $C_1, C_2) = (C_1xC_2)o\Delta$ where Δ is the diagonal mapping.

Conversely $fo(c_1 \text{ x } c_2): IR^2 \to F$ is smooth for all smooth curves $c_i \, in \, U_i$ since the product and all the composite of smooth mapping is supersmooth..

Note: that the derivative as in 3.1.9 has been put into consideration and the superspaces may be without norm.

The theorem above shows that:

Collorary I

A convenient vector superspace is a Cartesian closed category.

Proof:

(a) $f^{v}:U_i \to C^{\infty}(U_2, F)$ is a terminal object in $C^{\infty}(U_2, F)$. Thus property (a) of Cartesian closedness is fulfilled.

(b) For any two objects U_1 and U_2 in $E\varepsilon C^{\infty}$ then $U_1xU_2\varepsilon \text{ E x E} \subseteq C^{\infty}$. Thus property (b) of definition of Cartesian closedness is fulfilled.
(c) Every C, has a right adjoint C_x in $C,^{\infty}$ thus property (c) of definition of Cartesian closedness is fulfilled.

Corollary (2)

Let E, F, G etc be locally convex spaces, and let U, V be $C^{\infty}-$open subsets of such. Then the following canonical super mapping are supersmooth.

1. $Ev.C^{\infty}(U, F) \text{ x U} \to \text{F(f, x)} \to \text{f(x)}$
2. Ins. $E \to C^{\infty}(F, E \text{ x F}), \text{x} \to (\text{Y} \to (\text{x, y})$
3. $(\ )^{\wedge} :: C^{\infty}(U, C^{\infty}(V, G)) \to C^{\infty}(U \text{ x V, G})$
4. $(\ )^{v}: C^{\infty}(U \text{ x V, G}) \to C^{\infty}(V, G))$
5. $Comp: C^{\infty}(F, G)xC^{\infty}(U, F) \to C^{\infty}(U, G), (f, g) \to fog,$
6. $C^{\infty}(\ )C^{\infty}(E_D, E)xC^{\infty}(F_1, F_2) \to$
7. $\to C^{\infty}(C^{\infty}(E, F), C^{\infty}(E_2F_2)), (f, g) \to h \to gohof)$

8. $\pi : \pi C^{\infty}(E_I, F_I) \to C^{\infty}(\pi Ei, \pi Fi)$ for any index set.

Proof:

1. The mapping associated to $ev.v_{ia}$ Cartesian closedness in the identity on $C^{\infty}(U, F)$, which is C^{∞}, thus ev is also C^{∞}.
2. The mapping associated to $ins.via$ Cartesian closedness is the identity on E x F_1 hence *ins* is C^{∞}
3. The mapping associated to $(\ )^{\wedge}$ via Cartesian closedness is a smooth composition of evaluation $evo(ev\,\mathrm{xid}) : (\mathrm{f}, \mathrm{x}, \mathrm{y}) \to \mathrm{f(x)(y)}$
4. We apply Cartesian closedness twice to get the associated mapping $(f(x, y) \to f(x, y),$ which is just a smooth evaluation mapping.
5. The mapping associated to *comp.via* Cartesian closedness is $(f, g, x) \to f(g(x))$ which is the smooth mapping *ev.o(id x ev)*.
6. The mapping associated to the one in question by applying Cartesian closed twice is $(f, g, h, x) \to g(h(f(x))$, which is the C^{∞} mapping *evo(vd x ev) o (id x id x ev)*.
7. Up to a flip of factors the mapping associated via Cartesian closedness is the product of the evaluation mapping $C^{\infty}(E_i, F_i)\,\mathrm{x}\,\mathrm{E_i} \to F_i$.

2.1.12 LEMMA

A curve into C^{∞} closed subspace of a space is supersmooth if and only if it is supersmooth into the total super space.

Proof

Since the derivative of a supersmooth curve is mackey limit of the difference quotient, the C^{∞} closedness implies that this limit belong to the super subspace. Thus we deduce inductively that all super derivatives belong to the super subspace and hence the curve is supersmooth into the super subspace.

Corollary:

The supersmooth mapping on open subset of $X_o{}^n\ x\ X_1^m$ in the sense of definition are exactly the usual supersmooth mappings.

Proof

Both conditions are of local nature, so we may assume that the open subset of $X_o{}^n\ x\ X_1^m$ is an open box and in turn even $X_o{}^n\ x\ X_1^m$ itself.

$\Rightarrow if\ \mathrm{f} : X_o{}^n\ x\ X_1^m \to F$ is supersmooth then by Cartesian closedness, for each coordinate the respective associated mapping

$f : X_o^{\;n}\; x\, X_1^{m^{n-1}} \to C^{\infty}(X_o^{\;n}\; x\, X_1^{m}, F)$ is supersmooth, so again by Cartesian closedness we have $\partial if = (\partial * f^{\,vi})^{\Delta}$, so all first partial derivatives exist and are super smooth, inductively, all iterated partial derivatives exist and are supersmooth, thus continuous, so *f* is supersmooth in the usual sense.

2.1.13 DEFINITION [Second Super Derivative]

A C^2 mapping $U \to A$ is called G^2 at $a\varepsilon U$ if the second derivative $f''(a)$, is a symmetric bilinear map $AxA \to A$ such that $f''(a).(h,k)$ is a quadratic form with coefficients in A.

Theorem

A G^2 and G^1 mapping $U \to A$ is also G^2 at $a\varepsilon U$ if $f''(a)(h-,k-) = 0$

Proof

Since *f* is *G'* we have $f'(a) - h = h + U(a) + h - V(a)$

Thus

$$f''(a)(h,k) = h + (U'(a),k) + h - (V'(a),k)..........(1)$$

replacing *k* by $K_+ \oplus K-$ and using the symmetry in *h* and *k* we obtain,

$$h + (U'(a).k+) = K_+(U'(a).h+)..................(2)$$ and

$$h + (U'(a)).k-) + h - (v'(a) - k+) = K + (U'(a)h-) + K - (V'(a).h+)...........(3)$$

taking $K_+ = e$ in (2) gives

$$U'(a).h_+ h_+ U'(a)$$

$K_+ = e$ and $h+ = 0$ in (3) gives

$$U'(a)h.- = h - (v'(a).e)$$

Therefore if,

$$f''(a)(h-,k-) \cong h - (v'(a),k-) = 0$$

then,

$f''(a)(h,k)$ is given by a quadratic form with coefficient in A.

$$\partial + + f(a) = U'(a), e, \;\; \partial + - f(a) = \partial - + f(a) - V'(a).e$$

2.1.13 PROPOSITION

To an entire function $IR \to IR$ given by $f : x \to \sum_{n=0}^{\infty} c_n X^n$.........(1) corresponds a C^{∞} mapping $A \to A$ given by $f : a \to \sum_{n=0}^{\infty} c_n a^n$

Proof

(i) The mapping is defined from A into A: a term $f_i, \ldots\ldots\ldots ip \quad \varepsilon_i^{l_i \ldots\ldots \varepsilon_i^{ip}}$ in f(a) is well defined because it contains only finite number of terms of the formal series which defines a, and the series (1) is absolutely convergent for all x.

(ii) f is c^{∞} in the locally convex topology and G^{∞} indeed

$$f(a) = \sum_{n=0}^{\infty} c_n (a_+)^n + c_n (a_+)^{n-1} a-)$$

$$f'(a)h = h_+ \sum c_n (a_+^{n-1} + n(n-1)a^{n-2} a-) + h - \sum C_n a +^{n-1}$$ A G^k mapping, with K large enough for the application at hand, is also called super smooth.

2.1.14 REMARKS

The definition of supersmoothness extends in a standard way to mappings from A^n into A^p or more generally from $A_+ m \times A_- q$ into $A_+ m' \times A_- q$ such a mapping is graded or super, if the derivative is given by an $(n+q)x(m' \times q')$ matrix with elements in A_1 that is there exist elements of $A_+ m' \times A_- q'$ denoted $\partial_{+c} f$, $\partial - j \ f$ such that

$$f'(a).h = \sum_{i=1}^{in} \partial + ifh_+^i + \sum_{j=1}^{q} \partial - fh_i^j$$

$$h+ = (h_+^i) \varepsilon A_+ m, \quad (h_-^j) = h - \varepsilon A q_-$$

2.1.15 DEFINITION [Super polynomial]

A supersmooth mapping $f : E \to F[F_+ + F-]$ is called a superpolynomial if some derivative $d^n f$ vanishes on $E[E_+ + E_-]$. The largest ρ such that $d^p f \neq 0$ is called the degree of the polynomial. The mapping *f* is called a monomial of degree ρ if it is of the form $f(x) = f(x, \ldots. x)$ for some $f \varepsilon \, L^{o} \varepsilon sym(E, F)$.

2.1.16 DEFINITION [Approximation Property in vector Superspace]

Another important addition property of convenient vector superspace is the approximation property i.e. the denseness of $E' \oplus E$ in $L(E, E)$. A convenient vector super space E is said to have the bornological approximation property if $E \otimes E$ in L(E,E) with respect to the bornological topology. It is said to have the C^{∞} $-$approximation property if this is true with respect to the $C^{\infty}(-L[E, E])$.

2.1.17 LEMMA: (Curves into Limits)

A curve into a C^{∞} closed subspace of a space is supersmooth if and only if it is supersmooth into total space. In particulars a curve is supersmooth into a projective limit if and only if all its components are supersmooth.

Proof

Since the derivative of supersmooth curve is the mackey limit of the difference quotient, the C^{∞} closedness implies that this limit belongs to the subspace. Thus we deduce inductively that all derivatives belong to the subspace, and hence the curve is supersmooth into the subspace. The result on projective limit now follows, since obviously a curve is supersmooth into a product, if all its components are supersmooth.

Theorem (Simplest Case of Exponential Law)(Kriegl)

Let $f : IR^2 \to IR$ be an arbitrary mapping. Then all iterated partial derivatives exist and are locally bounded if and only if the associated mapping $f^{v} : IR \to C^{\infty}(IR, IR)$ exist as a super curve where $C^{\infty}(IR, IR)$ is considered as the Frechet space with $C^{\infty}(IR, IR)$, is considered as the Frechet space with the topology of uniform convergency of each derivative on compact sets. Furthermore, we have $(\partial, f)^{v} = d(f^{v})$ and $(\partial_2 f)^{v} = dof^{v} = d*(f^{v})$.

2.1.18 CALCULUS OF MAPPINGS:

The concept of a supersmooth curve with values in a locally convex vector super space is easy and without problems. Let E be a locally convex vector space.

2.1.19 DEFINITION [Super Differentiable Curves]

A curve $C : IR \to E$ is called differentiable if the derivative $c'(t) = \lim_{s \to o}(c\frac{t+s)-c(t)}{s})$ at t exist for all t. A curve $C'' IR \to E$ is called super smooth c^{∞} is all iterated derivatives exist. It is called C^{n} for some finite *n* if its iterated derivatives up to order *n* exist and are continuous.

Collary [Smoothness of the difference quotient]

For a supersmooth curve $C : IR \to E$ the different quotient

$$(t, s) \to \begin{cases} \frac{c(t)-c(s)}{t-s} & \text{for } t \neq s \\ c'(t) & \text{for } t = s \end{cases}$$

is a smooth mapping $IR^2 \to E$

Proof

We have $f : t, s) \to \frac{c(t)-c(s)}{t-s} = \int_{o}^{'} c'(s + r(t-s)dr$ and it is smooth $IR^2 \to E^{\Delta}$. The left hand side has values in E, and for $t \neq s$ this is also true for all iterated directional derivatives. It remains to consider the derivatives for *t*=*s*. The iterated directional derivatives are given by,

$$d^{p} vf(t, s) = dv^{p} \int_{0}^{11} c'(s + r(t-s)dr$$

$$= \int_0^1 dv^p c'(s + r(t - s))dr$$

where dv act on the *(t,s)* variables. The later integrand is for *t=s* just a linear combination of derivatives of *c* which are independent of *r*, hence $d^p vf(t,s)\varepsilon E$. The mapping *f* is supersmooth into *E.*

Likewise, a mapping $f : IR^n \to E[E + +E-]$ is called supersmooth if all iterated partial derivaties $\partial i_1 ....ipf = \frac{\partial}{\partial oi^i} .... \frac{\partial}{\partial x} i_p$ exist.

For all $i_1 .... ip\varepsilon\{\text{i........n}\}$

A curve $c : R \to E$ is called locally Lipschitzian if every point $r\varepsilon IR$ has a neighbourhood *U* such that the Lipschitz condition is satisfied on *U*, i.e. the set.

$\left\{\frac{1}{t-s}(c(t) - c(s) : t \pm s; t, s\varepsilon U\right\}$ is bounded.

Note that this implies that the curve satisfies the Lipschitz condition on each bounded interval, since for *(t_I)* increasing

$$\frac{c(t_n) - c(t_o)}{t_n - t_o} = \sum \frac{t_i + 1 - t_i}{t_n - to} \frac{c(ti + 1 - c(ti)}{ti + 1 - ti}$$

is in the absolutely convex hull of a finite union of bounded sets. A curve $c : IR \to E$ is called Lip^k or $C^{(k+1)-}$ if all derivatives up to order K exist and are locally Lipschitzian. For those properties we have the following implications.

$C^{m+1} \Rightarrow L_1 p^n \Rightarrow c^n$

differentiable $\Rightarrow c$,

In fact, continuity of the derivatives implies locally its boundness.

Lemma [continuous Linear Mapping are supersmooth]

A continuous linear mapping $L : E \to F$ between locally convex vector superspaces maps $Lip^k -$curves in *E* to $L_i p^k -$curves in *F*, for all $0 \le k \le \infty$, and for $k \succ 0$ one has $(Loe)'(t) = L(c')(t).$

Proof.

As a linear map L commutes with difference quotients, hence the image of Lipschitz curve is Lipschitz since L is bounded. As a continuous map it commutes with the formation at the respective limits. Hence $(LoC)'(t) = L(c')(t)).$

Note that a differentiable curve is continuous and that a continuously differentiable curve is locally Lipschitz: For $L\varepsilon E*$ [The space of all continuous linear functionals on E] we have

$$L\left(\frac{c(t) - c(s)}{t - s}\right) = \frac{((Loc)(t) - (Loc)(s)}{t - s}$$

$= \int_0^1 (Loc)'(s+(t-s)r)dr,$ which is bounded,

Since $(Loc)'$ is locally bounded

Proposition: Integral of Lipschitz Curves:

Let $c:[0,1] \to E[E_+ + E_-]$ be a Lipschitz curve into a Mackey complete space. Then the Riemann integral exist in *E* as (Mackey) limit of the Riemann sums.

2.1.20 DEFINITION: [THE INTEGRAL](Kriegl)

For continuous curves $C: IR \to E$ the definite integral is given by $\int_a^b e = \int (c)(b) - f(c)(a)$

2.1.21 COROLLARY: [BASICS ON THE INTEGRAL]

For a continuous curve $c: IR \to E$ we have

1. $L(\int_a^b c) = \int_a^o (Loc)$ for all $L \varepsilon E$
2. $\int_a^b c + \int_b^d c = \int_a^d c$
3. $\int_a^b (ao\psi)\psi' = \int_{\rho(a)}^{\rho(b)} c$ for $\varphi \varepsilon c'(R, IR)$
4. $\int_a^b c$ lies in the closed convex hull in $\hat{E}$ of the set $((b-a)c(t): a < t < b)$ in *E*
5. $\int_a^b :: c(IR, E) \to \hat{E}$ is linear
6. (Fundamental theorem of calculus). For each c'= curve c: $IR \to E$ we have $c(s) - c(t) = \int_t^s c'$

   All the above curves are on superstructures.

2.1.22 LEMMA [Integral of Continuous Curves]

Let $C: IR \to E$ be a continuous curve in a locally convex vector super space. Then there is a unique differentiable curve $\int c: IR \to \hat{E}$ in the completion $\hat{E}$ of E such that $(fc)(o) = 0$ and $(fc)' = oC$

Proof

We show uniqueness first. Let $C_1: IR \to \hat{E}$ be a curve with derivative C and $C_1(0) = 0$. For every $L \varepsilon E*$ The composite LoC, is an anti-derivative of Loc with initial value O, so it is uniquely determined, and since $E*$ separates point C, is also uniquely determined.

Now we show the existence, we have that $\hat{E}$ is (isomorphic t_o) the closure of E is the obviously complete space $L[E*_{equip}, R]$. We define $(fc)(t): E^* \to IR$ by $L \to \int_o^t (loc)(s)ds.$ It is a bounded linear functional on $E^{*}equip$ since for an equicontinuous subset $\varepsilon \subseteq E^*$ the set $((Loc)(s): L\varepsilon\varepsilon,\ s\varepsilon[o,t]$ is bounded so $\int c: IR \to L(E^* euip; IR)$.

Now we show that $\int c$ is differentiable with derivative δoc.

$$\left(\frac{\int c(t+r) - (\int c)((r)}{t} - (\delta oc)(r)\right) l$$

$$= \frac{i}{t}\left[\int_o^{\frac{t}{t+r}} (loc)(s)ds - \int_o^r \ (lov)(s) - t(loc)(r)\right]$$

$$= \frac{i}{t}\int_r^{r+t} (loc)(s) - (loc)(r)ds = \int_o^i \ L(c(r+ts) - c(r)ds$$

Let $\sum \subseteq E^*$ be eqicontinuous and let $\varepsilon > o$. Then there exist neighbourhood U of o such that $\lfloor L(U) \rfloor L\varepsilon$ for all $lo\varepsilon$. For sufficiently small t, all $SI[0,1]$ and fixed r we have $c(r+ts) - c(r)\varepsilon U$. So $\int_o^1 L(c(r+ts) - c(r))ds / < \varepsilon$. This shows that the difference quotient of $\int c$ at r converges to $\partial(c(r))$ uniformly on equicontinuous subset.

It remains to show that $(\int c)(t)\varepsilon\hat{E}$. By the mean value theorem the difference quotient.

$\frac{1}{t}(\int c)(t) - \left(\int c\right)(o)$ is contained in the closed convex hull in $L(E^* equi, IR)$ of the subset. $\langle c(s): o < s < t \rangle$ of E. so it lies in $\hat{E}$.

2.1.22 DIFFERENTIATION OF AN INTEGRAL

We return to the question of differentiation an integral, so let $f: ExIR \to F$ be supersmooth, and let $\hat{F}$ be the completion of the locally convex space F. Then we may form the function $f_o: E \to \hat{F}$ defined $x \to \int_o^i f(x,t)dt$. We claim that it is super smooth, and that, directional derivative is given $dvf_o(x) = \int_o^i dvf(x,t)dt.$ By Cartesian closedness the associated mapping $f^v: E \to C^\infty(IR, F)$ is super smooth, so the mapping $\int_o^1 \ of^v: E \to \hat{F}$ is super smooth since integration is a bounded linear operator and

$$dvf_o(x) = \frac{\partial}{2s}\bigg|_{s=} fo(x+sv) = \frac{\partial}{\partial s}\bigg|_{s=o} \int_o^1 \ f(x+sv)t)dt$$

$$= \int_o^1 \frac{\partial}{\partial s}\Big|_{s=o} f(x+sv,t)dt = \int_o^1 dv\, \mathrm{f(x+sv)(x)dt}$$

3.0 CONCLUSION

In this paper the theorem on the Cartesian closedness of convenient vector spaces, provided by Kriegl Michor has been shown to be valid in bornological topological spaces without norm. The super structures have been studied in a convenient vector space without norm. Mapping and smooth mappings have been derived; their properties have been studied with their calculus.

Super derivatives have been introduced in this paper. Here Super derivatives in convenient vector space without norm have been introduced for the first time. The properties of these derivatives have been worked in super structures. It has been shown for the first time that a super derivative can be approximated to a synthetic derivative in IR.

Approximation property and super polynomial vector spaces have been defined proved in convenient vector space without norm

4.0 APPLICATIONS

In conclusion this summary has shown that the three model studies, i.e., super mathematics due to its unification of laws in physics, synthetic mathematics and analysis in bornological spaces due to their generality in mathematics can be combined to one model through category theory. The main result in this paper is that it has shown that the three models are valid in sequential complete topological spaces without norm.

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