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GPC based PID control of the roll Channel of flying vehicle with two actuators

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Abstract

A strategy to increase the control ability of aerodynamic control flying vehicles at the altitude higher than the standard range by using continuous time cold gas thruster is proposed in this paper. At the first an optimal PID (Proportional Integral Derivative) controller algorithm based on predictive control technique is defined, and a generalized predictive control (GPC) is used to derive the control law. Finally, based on the GPC control law PID control, parameters are calculated recursively. The result shows a proper response in the control of flying vehicle.

Keywords: Optimal PID Generalized predictive control, Flight control**.**

Nomenclature

PID	Proportional Integral Derivative	u_2, u_1	output controllers
GPC	Generalized Predictive Control	φ	roll angle
CSA	Canadian Space Agency	p	predictive horizon
δ_a	roll channel aileron	m	control horizon
θ	electro valve	M	moment
Q	error weighting	R	control weighting
y_{sp}	set point	y	controlled output
Δu	signal control variation	J	cost function
$F_{Truster}$	thruster force	I_{XX}	Inertia moment

1. Introduction

Control of flying vehicles commonly based on two types of controlling force is possible; Control by thrust force and the other is to control the aerodynamic force. Often, thrust force is used in systems with a long range to control the flight of the middle phase and control of low range systems is usually based on the aerodynamic forces. With range increasing, altitude increase so aerodynamics forces drastically decrease. Therefore control of system with disturbance and uncertainty is difficult, especially in roll channel. For this reason, in this paper, using of the clod gas thruster actuator for control roll channel of flying vehicles is presented. Although cold gas thruster actuator is used in Nano-satellite and spacecraft position's control. But in this paper at the same time are used from fin control actuator and cold gas thruster actuator continuous time.

J.K, Etal, [1] presented a hybrid cold gas micro-thruster system for spacecraft and Jeon in [2] presented a novel limit cycle analysis of creation for thruster control systems with time delay using a PWM-Based PD controller. Precision attitude stabilization with rise and fall's times incorporation in Gas-Based thrusters [3] and a neural network based modelling strategy to precisely identify the thrusts of cold-gas thrusters deployed in a Nano-satellite experimental test-bed developed at the Canadian Space Agency (CSA) [4] have been proposed. Attitude tracking control for variable structure near space vehicles has been presented based on switched nonlinear systems, that topology optimization of load-bearing structural components for reducing attitude control efforts of miniature space vehicles is investigated [5]. Delay and slowness in cold gas thruster actuator, is one of the challenges in roll channel system designing. Using the Predictive control can solve these challenges, on the other side PID controller can be used to improve stability of flying vehicles. For tunings PID controller parameters we use predictive control approaches and GPC to control law. Then based on the GPC control law, the parameters of PID controller recursively would be calculated. Finally PID controller design based on the application of GPC represented in a general linear industrial system model [6-8].

2. Modeling

The system dynamic model based on the fundamental laws of motion are derived.By linearization of flying vehicle dynamics around the set point, the roll channel equation can be considered as (1), [9].

$$\delta\ddot{\varphi}+C_{11}.\delta\dot{\varphi}=C_{12}\delta_a+M_{xdis} \tag{1}$$

The dynamic coefficients are

$$C_{11}=-\frac{\bar{q}sd^2.C_{l_\omega}}{2V.I_{XX}}=-A_2 \tag{2}$$

$$C_{12}=\frac{\bar{q}sd.C_{l_{\delta a}}}{I_{xx}}=A_1 \tag{3}$$

$$\dot{\varphi}=\omega_1 \tag{4}$$

Where $\bar{q}$,s, d, C_{l_ω} ,$C_{l_{\delta a}}$ and M_{dis} are dynamic pressure, flying vehicle cross sectional, diameter of the flying vehicle, is damping characteristic of flying vehicles in the roll channel, is roll angle variation due to fin angle variation and finally shows a moment of disturbance into roll channel which create from various sources such as no mass balance and fins installation error respectively.

By definition $\delta_{\dot{\varphi}}=\dot{\varphi}\,,\delta_{\ddot{\varphi}}=\ddot{\varphi}$ we have:

$$\ddot{\varphi}=A_2.\dot{\varphi}+A_1\delta_a+M_{dis} \tag{5}$$

From aerodynamic coefficients and (5), we can be defined as:

$$\ddot{\varphi}=\frac{\bar{q}sd^2C_{l_\omega}}{2VI_{XX}}.\dot{\varphi}+\frac{\bar{q}sd}{I_{XX}}.C_{l_{\delta a}}\delta_a+M_{dis} \tag{6}$$

Finally the transfer function roll channel is defined as:

$$\varphi(s)=\frac{A_1}{s(s+A_2)}\delta_a+\frac{1}{s(s+A_2)}M_{dis} \tag{7}$$

2.1. Composition roll channel transfer function with cold gas thruster actuator

According to Newton's law for rotational motion in an inertial space ($\Sigma\vec{M}=\frac{d}{dt}(\overrightarrow{H})$, (where $\vec{H}=I\times\vec{\omega}$)), total moment on the roll channel is defined as:

$$\sum M=I_{XX}\ddot{\varphi} \tag{8}$$

After a cold gas thruster actuator adding we have:

$$I_{XX}\ddot{\varphi}=\frac{\bar{q}sd^2.C_{l_\omega}}{2V}.\dot{\varphi}+\bar{q}sd.C_{l_{\delta a}}\delta_a+F_{Thruster}.R \tag{9}$$

Where *R*, is the distance of the thruster to flying vehicle's axis (roughly radial) and $F_{Truster}$ is the two thruster force. Finally the total Composition of roll channel transfer function with cold gas thruster actuator can be defined as:

$$\varphi(s)=\frac{A_1}{s(s+A_2)}\delta_a+\frac{KA_3}{s(s+A_2)}.F \tag{10}$$

A_1, A_2 Can be assumed as (7) and $A_3 = \frac{R}{I_{XX}}$

According this fact that Equation (10) is considering without the cold gas thruster and fin aerodynamic actuators. If actuators dynamics are considered as well as the following second-order :

$$W_{DEF}(s) = \frac{\delta_a}{u_1} = \frac{\omega_1^2}{s^2 + 2\eta_1\omega_1 s + \omega_1^2} e^{-T_{d1}s} \quad (11)$$

$$W_{IMM}(s) = \frac{F_{Thruster}}{u_2} = \frac{\omega_2^2}{s^2 + 2\eta_2\omega_2 s + \omega_2^2} e^{-T_{d2}s} \quad (12)$$

Where $u_2 = \theta$, so the system transfer function with actuators and disturbance moment can be obtained as:

$$\varphi = \left(\frac{A_1}{s(s+A_2)} \cdot \frac{\omega_1^2}{(s^2 + 2\eta_1\omega_1 s + \omega_1^2)} e^{-T_{d1}s}\right) u_1 + \left(\frac{KA_3}{s(s+A_2)} \cdot \frac{\omega_2^2}{(s^2 + 2\eta_2\omega_2 s + \omega_2^2)} e^{-T_{d2}s}\right) u_2 + \frac{1}{s(s+A_2)} M_{dis} \quad (13)$$

Where u_2, u_1 are output controllers. Parameter value of the system transfer function in relation (13) has been shown in Table 2. Figure 1 shows that system has two inputs (u_2, u_1) and one output φ (roll angle) is.

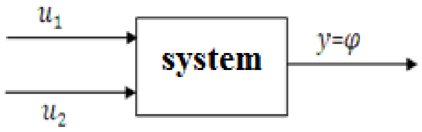

Figure1. MISO system
(Inputs are without considering the actuators dynamic)

For independently control of yaw and pitch channel consider $\varphi = 0$ (roll angle is zero).

3. Optimal PID control design based on predictive method

According to the desired general form of system transfer function

$$y = g_1 u_1 + g_2 u_2 \quad (14)$$

In equation (14), each g_i $(where \quad i = 1,2)$ contains a subsystem which , a and b, represented as [8],[10],[11]:

$$g_i = \frac{d_i s + c_i}{(s + a_i)(s + b_i)} e^{-sL_i} \quad (15)$$

Since each of g_i is forth-order, must be decreased to second-order to design of mentioned controller according to equation (15). That is done according to criteria [12]. The equation (15) in discrete-time transfer function can be represented as

$$g_i' = \frac{b_{1i}' z + b_{2i}'}{z^2 + a_{1i}' z + a_{2i}'} z^{-h_i} \quad (16)$$

Consider error described by $e = y_{sp} - y$, where y_{sp} is the set point and y is the controlled output, then in terms of error, with $y_{sp} = 0$, equivalent equation can be represented as, $(where \quad i = 1,2)$

$$e_i(k+1) = -a_{1i}' e_i(k) - a_{2i}' e_i(k-1) - b_{1i}' \tilde{u}_i(k - h_i) \quad (17)$$

Where, $\tilde{u}_i(k-h_i) = u_i(k-h_i) + \frac{b_{2i}'}{b_{1i}'} u_i(k - h_i - 1)$, $h_i = \frac{L_i}{T_s}$ and $t = 0$ is the sampling time. Equation (17) can be changed to state space form for the design of two controllers:

$$X_i(k+1) = F_i X_i(k) + B_i \tilde{u}_i(k - h_i) \quad (18)$$

Where

$$F_i = \begin{pmatrix} 0 & 1 & 0 \\ -a_{2i}^{'} & -a_{1i}^{'} & 0 \\ 0 & 1 & 1 \end{pmatrix}, B_i = \begin{pmatrix} 0 \\ -b_{1i}^{'} \\ 0 \end{pmatrix}, X_i(k) = \begin{pmatrix} e_i(k) \\ e_i(k+1) \\ \theta_i(k+1) \end{pmatrix}$$

And $\theta_i = \sum_{j=1}^{k} e_i(j)$ is the integral error. Using p and m prediction and control horizons respectively. The predicted error in compact form can be represented as[8], [10], [11]. $(where \quad i = 1,2)$

$$\tilde{X}_i = G_i F_i X_i(k) + A_i \tilde{U}_i \tag{19}$$

Where

$$\tilde{X}_i = (X_i^T(k+1) \quad X_i^T(k+2) \quad \dots \quad X_i^T(k+p))^T$$

$$G_i = \begin{pmatrix} I \\ F_i \\ \vdots \\ F_i^{p-1} \end{pmatrix}, A_i = \begin{pmatrix} B_i & 0 & \cdots & 0 \\ F_i B_i & B_i & \cdots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ F_i^{p-1} B_i & F_i^{p-2} B_i & \cdots & F_i^{p-m} B_i \end{pmatrix}$$

$$\tilde{U}_i = (\tilde{u}_i(k-h_i) \quad \tilde{u}_i(k+1-h_i) \quad \tilde{u}_i(k+m-1-h_i))^T$$

Since GPC is an optimal control strategy, therefore a performance index or cost function must be minimized in order to obtain an optimal control signal. Considering the following cost function, $(where \quad i = 1,2)$

$$J_i = \sum_{l=1}^{p} \left\| x_i(k+l) \right\|_{Q(l)}^2 + \sum_{j=1}^{m} \left\| u_i(k+j-1) \right\|_{R(j)}^2 \tag{20}$$

Where Q and R are the error and control weighting matrices respectively. Substitution of the prediction equation (19) in the cost function (20) i.e., an optimization step, resulted an optimal control sequence, like[8],[10],[11], $(where \quad i = 1,2)$

$$\tilde{U}_i = -[A_i^T Q A_i + R]^{-1} [A_i^T Q G_i F_i] X_i(k) \tag{21}$$

Under the receding horizon principle, only the first value of the optimal control sequence is applied at each sampling time while the rest are discarded. Therefore $(where \quad i = 1,2)$

$$\begin{aligned} \tilde{u}_i(k-h_i) &= -H[A_i^T Q A_i + R]^{-1} [A_i^T Q G_i F_i] X_i(k) \\ &= -D_i X_i(k) \end{aligned} \tag{22}$$

Where $D_i = H[A_i^T Q A_i + R]^{-1}[A_i^T Q G_i F_i]$ and $H = [I \quad 0 \quad \cdots \quad 0]$

From equation (22), it follows that $\tilde{u}_i = -D_i X_i(k+h_i)$, which means that the control law given above includes the time delay information and the current control u(k) is actually a feedback of the future state at time k + h. This implies that the controller has a prediction capability in both the output and the control signal. And which means that current control value depends on the future predicted state. In case of significant time delay, this problem would be Solved in two different range i.e., $0 \le k < h_i$ and $k \ge h_i$. However, in the absence of time delay (i.e., $h_i = 0$) then control law of plant would simply be

$$\tilde{u}_i = -D_i X_i(k) \tag{23}$$

Then based on this GPC control law, the equivalent set of PID control parameters will be back calculated. The state feedback of $D_i X_i(k)$ is simply PID control, where $D_i = [K_{Pi} \quad K_{Ii} \quad K_{Di}]$.

So PID Controllers tuning parameters would only consider for $k \ge h_i$, $(where \quad i = 1,2)$

$$\begin{cases} K_{Pi} = -(K_{1i}(h_i) + K_{2i}(h_i)) & \\ K_{Ii} = -K_{3i}(h_i) & k \ge h_i \\ K_{Di} = K_{1i}(h_i) & \end{cases}$$

4. Simulation Results

In this section, Optimal PID control design based on predictive method is described and compared by analysis method based on important system's signals such as outputs and inputs with defined criterion. Figure 2 shows the simulation Process in the MATLAB software.

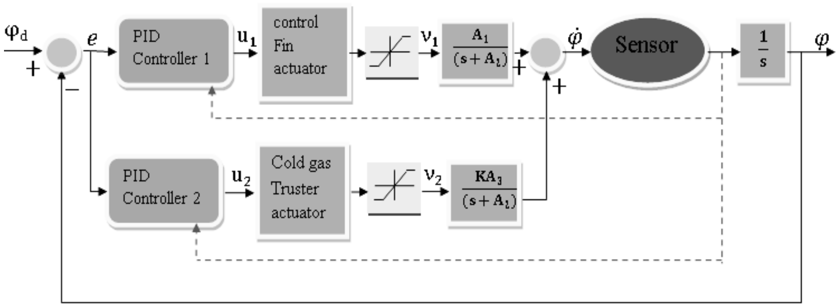

Figure 2. Simulation Process

Here the second-order is used as sensor transfer function, which is as follows:

$$\frac{\hat{\omega}}{\omega} = \frac{k\,\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \qquad (24)$$

In the simulation, the following specifications are used:

Table 1. Parameters of simulation

Sampling time	$T_s = 0.01$
Predictive horizon	$p = 80$
Control horizon	$m = 4$
Constant	$k = 1$
Natural frequency	$\omega_n = 2\pi \times 100$
Damping ratio	$\xi = 0.7$

Also output signal's of cold gas thruster actuator continuous time must be in the interval degree (-3, +3) that is near angle of electro valve. Also for the output signal of control fin must be in the interval degree (-10, +10) that is near roll channel aileron. In this simulation, due to noise $\dot{\varphi}$, before pre-designed controller, $\dot{\varphi}$ must be filtered as below: $(where \quad i = 1,2)$

$$u_i = k_{Pi}\,e + k_{Ii}\,.\int e + \frac{k_{Di}}{\tau_f\, s + 1}.\dot{\varphi} \qquad \tau_f = 0.01\sec \qquad (25)$$

Parameter values of equation (13) are shown in Table 2.

Table 2. The parameters value of the system transfer function in relation (13)

Parameters	Units	Values	Parameters	Units	Values
ω_2	rad/s	70	A_1	_	1
A_2	_	0.1	η_1	_	0.6
R	m	0.35	η_2	_	0.75
I_{XX}	$N.m/(rad/s^2)$	10	K	N	10
A_3	_	0.035	T_{d_1}	sec	0.01
ω_1	rad/s	60	T_{d_2}	sec	0.01

In this section presented simulation result in three set points of the system, first, when flying vehicles are placed at high altitudes, the parameters of the system transfer function change as follows: $A_1 = 0.02$

$$\varphi = \left(\frac{0.02}{s(s+0.1)}.\frac{3600}{(s^2+72s+3600)}e^{-0.01s}\right)u_1 + \left(\frac{0.35}{s(s+0.1)}.\frac{4900}{(s^2+105s+4900)}e^{-0.01s}\right)u_2 \qquad (26)$$

When flying vehicles is placed at middle altitudes, the parameters of the system transfer function change as follows: $A_1 = 1 \quad , A_2 = 0.1$

$$\varphi = \left(\frac{1}{s(s+0.1)} \cdot \frac{3600}{(s^2+72s+3600)} e^{-0.01s} \right) u_1 + \left(\frac{0.35}{s(s+0.1)} \cdot \frac{4900}{(s^2+105s+4900)} e^{-0.01s} \right) u_2 \tag{27}$$

And finally when flying vehicles are placed at low altitudes, the parameters of the system transfer function change as follows: $A_1 = 20 \quad , A_2 = 1$

$$\varphi = \left(\frac{20}{s(s+1)} \cdot \frac{3600}{(s^2+72s+3600)} e^{-0.01s} \right) u_1 + \left(\frac{0.35}{s(s+1)} \cdot \frac{4900}{(s^2+105s+4900)} e^{-0.01s} \right) u_2 \tag{28}$$

Figure 3 shows the reference input (set point) tracking system without thruster actuator in high altitude. Figure 4 shows the system control signal without the thruster actuator in high altitude.

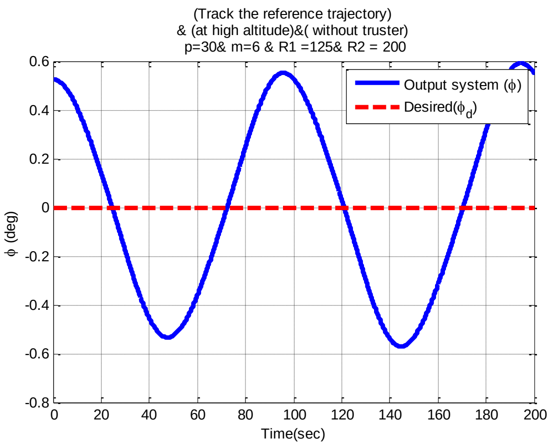

Figure 3. Study of the system tracking the reference Without thruster actuator in high altitude. $p = 30 \,\&\, m = 6$

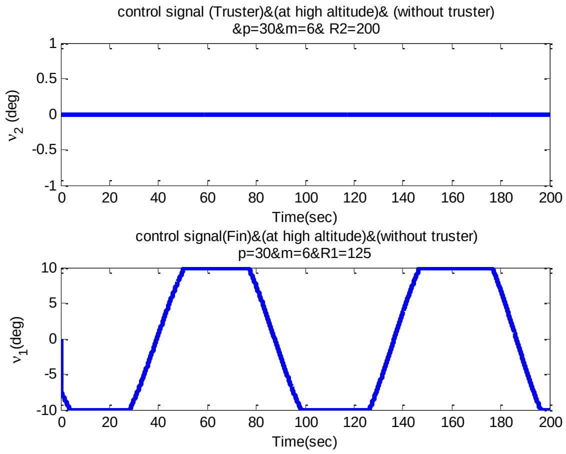

Figure 4. Study of system control signal without thruster actuator in high altitude. $p = 30 \,\&\, m = 6$

Figure 5 shows the system tracking reference with the thruster actuator in high altitude. Figure 6 shows the System control signal with the thruster actuator in high altitude**.**

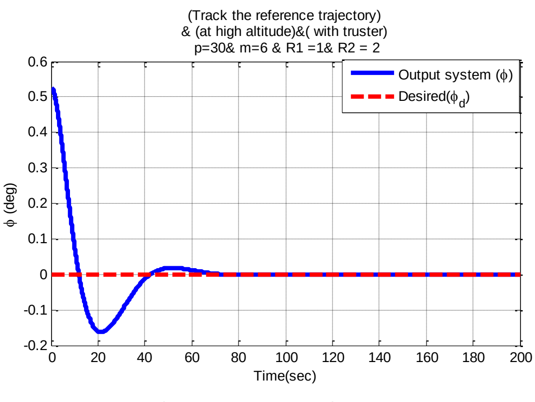

Figure 5. Study of tracking the reference with thruster actuator in high altitude $p = 30 \,\&\, m = 6$

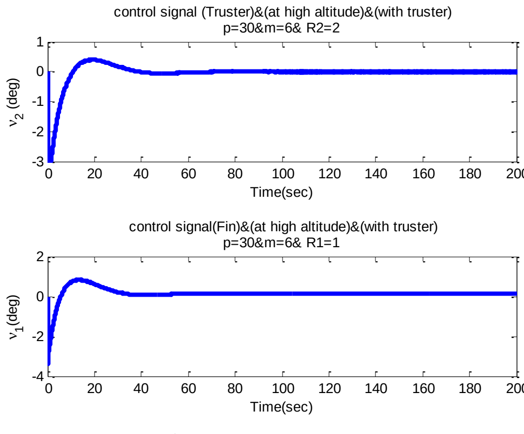

Figure 6. Study of system control signal with Thruster actuator in high altitude. $p = 30 \,\&\, m = 6$

Signal control thruster energy in high altitude = 1.0145
Signal control fin energy in high altitude = 0.8171

Simulation result comparison between tracks the reference system and system control signal (with and without thruster actuator) shows that the system ($p = 30 \,\&\, m = 6$) is unstable in high altitude when we don't have any thruster actuator and fin control signal is saturated. So suggested for better controlling of roll channel flying vehicles, actuator of cold gas thruster continuous time should be used. By proper initialization to the weighting matrix R and according to energy of cold gas thruster continuous time control signal and energy of control fin control signal, we can find this fact that in high altitude, energy of signal control in thruster actuator is more than energy of control fin control signal. On the other hand cold gas thruster continuous time in flying vehicles controlling is more effective in high altitude. Table 3 and 4 Show relation between energy values of control signal with the weight matrix R.

Table3. Relation between energy values of control signal with the weight matrix in high altitude

Energy values of control signal	Values of weight matrix R	In high altitude
0.8315	250	Cold gas thruster actuator continuous time
0.7702	125	Fin control actuator

Table4. The relation between energy values of control signal with the weight matrix in low altitude

Energy values of control signal	Values of weight matrix R	In low altitude
0.4231	10000	Cold gas thruster actuator continuous time
0.4693	1000	Fin control actuator

The simulation result show ability of disturbance rejection and noise measurement reducing based on tracking error variance criteria are evaluated. Figure7 show Comparison of tracking the reference with PID controller based on predictive control method for tracking the reference with PID controller by Ziegler & Nichols method. Figure8. show Comparison of PID controller based on predictive control method to the PID controller by Ziegler & a Nichols method with disturbance in 100 sec, where the amplitude of input disturbance, applied to the fin control actuator (one grade) and amplitude of input disturbance, applied to the cold gas thruster actuator continuous time (0.3 degree) have been chosen. Simulation result in figure 8., show ability of PID controller disturbance rejection based on predictive control method, is better than a PID controller on Ziegler & Nichols method. Figure9. show Comparative is tracking the reference of PID controller based on predictive control method to PID controller by Ziegler & the Nichols method by applying noise (variant 0.09). As shown in figure10. Tracking error variance of PID controller based on predictive control method has been compared with tracking error variance Ziegler & Nichols method. So PID controller based on predictive control method operation is better than Ziegler & Nichols method to decrease the noise. In this section, the system robust against uncertainty has been studied based on physical uncertainty definition. Some phenomenon makes an error in aerodynamic coefficient calculation in wide tunnel. Thus aerodynamic coefficients were studied in roll channel transfer function as system physical uncertainty. Aerodynamic coefficient $C_{l_{\omega 1}}$ is damping characteristics of flying vehicles in rolling channel, and aerodynamic coefficient $C_{l_{\delta a}}$ is rolling angle variation due to fin angle changes. By considering relations (2), (3), (5) and (6) in system transfer function A_1, A_2 must be studied as possible physical uncertainty of the system. In this section despite 30 percent variation of A_1, A_2 we pore over system robust. Figure 11 shows the system robustness by 30 percent variation of A_1. Also, Figure 12 shows the system robustness by 30 percent variation of A_2, figure13 and 14. Show PID controller with 30 percent variation of A_1, A_2 values by Ziegler & Nichols method.

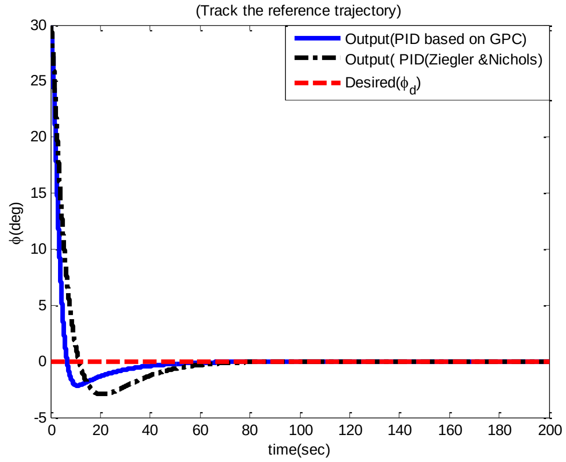

Figure 7. Show Comparison of tracking Reference with PID controller based on Predictive control method by Ziegler & Nichols method

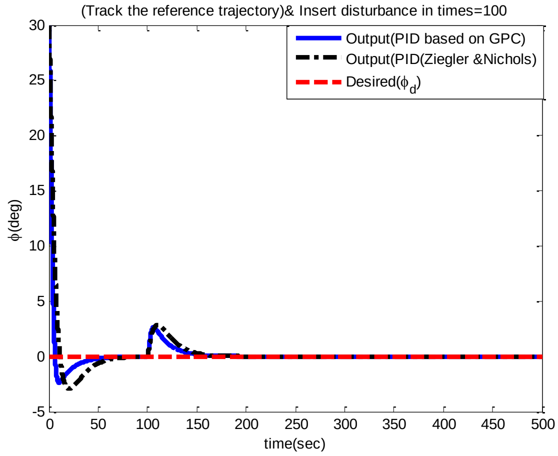

Figure 8. The Comparison of PID controller based on predictive control method to PID.Controller by Ziegler & Nichols method With disturbance in 100 Sec.

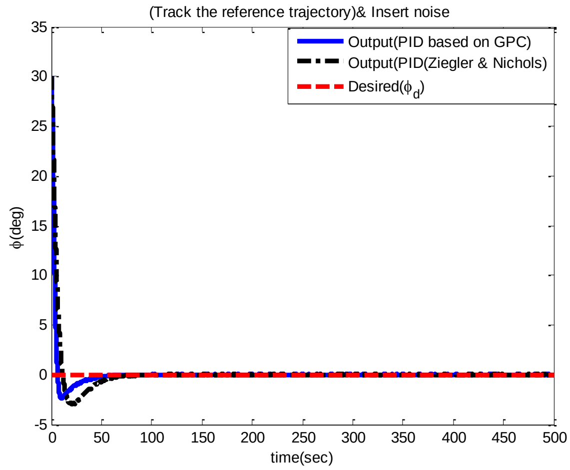

Figure 9. Show comparison of PID the tracking reference controller based on predictive control method to PID controller the tracking Reference by Ziegler & Nichols Method (with applying noise).

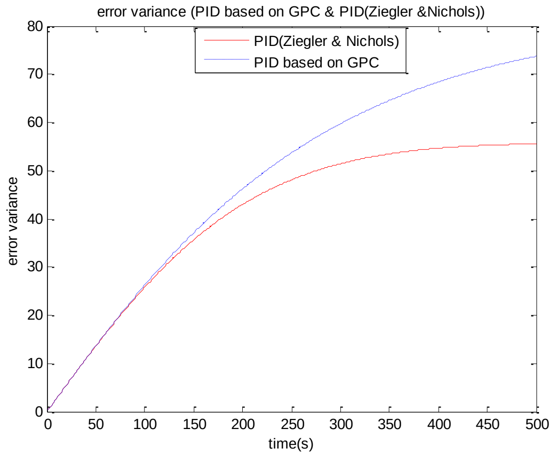

Figure 10. The Comparison of tracking error variance of PID controller based on predictive control method for tracking error variance of PID controller by Ziegler & Nichols method.

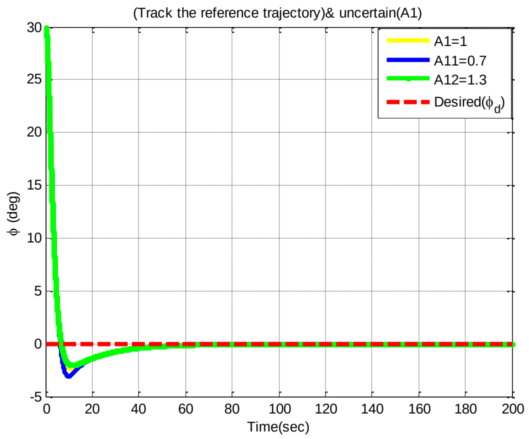

Figure 11. The tracking reference By studying of system robust with $A_1 = (1 \pm \%30)$ In PID controller based on predictive method.

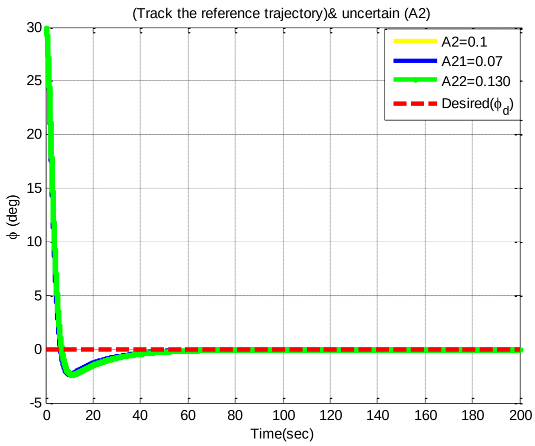

Figure 12. System the tracking reference by studying of system robust with $A_2 = 0.1(1 \pm \%30)$ in PID controller based on predictive method

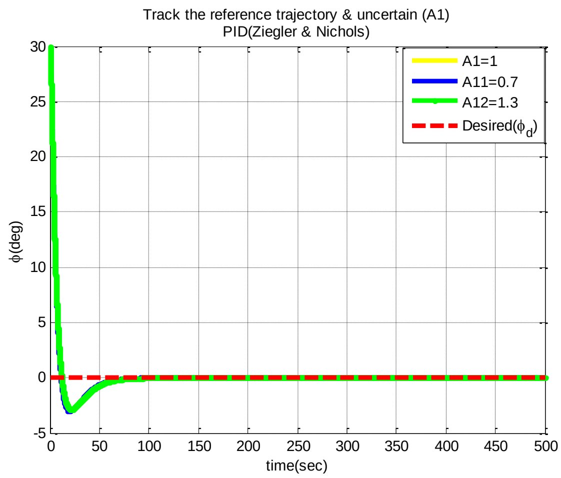

Figure 13. System the tracking reference by Studying of system robust with $A_1 = (1 \pm \%30)$ In PID controller by Ziegler & Nichols method

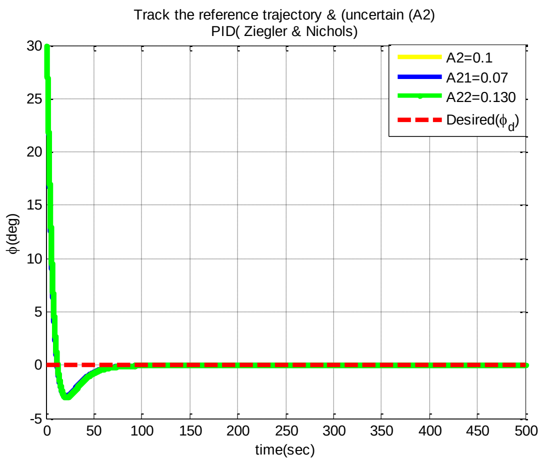

Figure 14. System the tracking reference by studying of system robust with $A_2 = 0.1(1 \pm \%30)$ in PID controller Ziegler & Nichols method

5. Conclusion

An optimal PID controller based on the GPC method for roll channel of flying vehicles has been designed in this paper. Setting up rolling angle of zero to control of the yaw and pitch channels independently, and control of flying vehicles roll channel increasing when the aerodynamic force is small, that Result by adding secondary cold gas thruster actuator continuous time, are two Targets in this paper. The simulation results have shown that the PID controller based on predictive control remain stable when disturbance rejection and noise reducing. Although simulation results have shown that Ability of disturbance rejection of PID controller based on predictive control is better than Ziegler & Nichols method but both controllers are robustness. Therefore, it is suggested for future designs that the predictive control criterion function (based on signal control variation (Δu)) is used. An on/off the cold gas thruster should be used instead of cold gas thruster continuous time. In order to use the system without degree reduce in controller algorithm, Generalized Increasing Predictive Control is suggested.

Reference

J.K., et al. (2002), " A hybrid cold gas microthruster system for spacecraft Sensors and Actuators", **A**(97-98): pp.587-598.

Jeon, S.-W. & S. Jung,. (2009), " Novel Limit Cycle Analysis of the Thruster Control System with Time Delay using a PWM-Based PD controller", IEEE International Symposiumon Industrial Electronics, pp.1245-1250.

Sukumar, S. & M.R. Akella. (2011), "Precision Attitude Stabilization: Incorporating Rise and Fall Times in Gas-Based Thrusters", Journal of Guidance, Control, and Dynamics, Vol.34, pp. 317-323.

Chaoui, H., et al., (2012), "Neural Network Modeling of Cold-Gas Thrusters for a Spacecraft Formation Flying Test-bed" , IEEE, pp.2619-2621.

Yufei, W., J. Changsheng, & W. Qingxian, (2013), "Attitude tracking control for variable structure near space vehicles based on switched nonlinear systems", Chinese Journal of Aeronautics, 26(1): pp.186-193.

Tan, K. & T.H. Lee, (1998), "Automatic tuning of PID cascade controllers for servo systems", Intelligent Automation and Soft Computing, A(A): pp.325-340.

Tan, K., et al., (1999), " Adaptive- predictive PI control of a class of SISO system", Proceedings of the American Control Conference, pp.3848-3852.

Tan, K., S.N.Huang, & T.H.Lee, (2000), " Development of a GPC-based PID controller for unstable systems with dead time", ISA Transaction, 39, pp. 57-70.

J. M. Blake Lock, J. Wiley, & S. Inc, (1992), "Automatic Control of Aircraft and Missiles",pp. 183-235.

Huang , S., Tan K. K., & Lee T.H. (2002), " Optimal PID Control Based on a Predictive Control Approach", Applied Predictive Control, chapter 5, pp.135-152.

K. K. Tan, S.N.H. T. H. Lee, & F.M. Leu., (2002), " PID Control Design Based on a GPC Approach", Ind. Eng. Chem. Res, 41, pp.2013-2022.

C.Kuo, B., (1991), " Automatic Control Systems", pp.357-359.