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Analysis of DO sag for Multiple Point Sources

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Abstract

The classical model of Streeter and Phelps for the Dissolved Oxygen (DO) is of limited practical importance as it is unable to address the situation when partially treated/untreated waste is discharged in a river through multiple points scattered along the river. The Streeter and Phelps model is modified to take into account additional factors like settleable Biochemical Oxygen Demand (BOD) and Dispersion. The presented model addresses a practical situation where the waste is discharged in a river through multiple point sources that increases the complexity of computation in predicting DO conditions in river. The presented model is applied on a real field data collected for the river "Yamuna" along Delhi (Capital of India).
Keywords: Mathematical Model, BOD, Water Pollution, MATLAB programming.

1. Introduction

The fresh water present on the surface of the earth is required for many purposes including domestic and municipal uses, agricultural irrigation, industries and recreation. The water gets polluted with organic and inorganic chemicals while passing through the urban settlements, agricultural lands and industries. It has been a common practice throughout the world to discharge waste material into streams/rivers. Organic matter discharged into the river utilizes the dissolved oxygen of water through its aerobic decomposition. The decreasing DO level may interfere with uses of water for varied purposes. Mathematical models are accepted as an efficient tool to predict levels of DO in a water body after discharge of organic waste.

The works of Streeter and Phelps as early as in 1925 included the mathematical formulations of the major processes associated with DO balance in a river. The model for DO sag parameters proposed by them had been in use since then for DO related stream studies. The models includes the effect of advection, BOD reaction (first order decay of BOD) and reaeration on stream's DO. The resulting mathematical model which is a system of coupled linear differential equations represent the transport of BOD exerting organic matter and its impact on the river's DO. The well known DO sag curve resulting from deoxygenation and reoxygenation was modelled several decades ago but it is still used in understanding the basic phenomenon related to BOD-DO relationships in river system . The one dimensional model represented by Streeter and Phelps is valid only after the mixing length is over. With the advancement of computational powers as well as the knowledge in field of modelling, the model is found to have some limitations in it's applicability. Firstly, the model includes only the dissolved part of BOD and does not account for the settleable part of BOD that is often present in a partially treated/untreated waste. Secondly, it did not take into account of dispersive effects. The effect of settleable part of BOD is incorporated by various authors in different ways. Thomas(1940) introduced the lag time concept in his model. Orford and Ingram(1953) introduced a coefficient of retardation in their model. Velz and Gannon (1962) took account of the BOD removals through formation and settling of biological flocks by adding a factor α. Bhargava(1983, 1986 a & b), however, considered that the total BOD of partially treated or untreated waste entering into the river consists of the settleable part and dissolved part. Tyagi et al(1999) presented a one dimensional advection dispersion model that incorporated both types of BOD. The model predicts the DO conditions in rivers when partially treated/untreated waste is discharged into the river through a single point source. In this paper, a one dimensional steady state model is developed that takes into account, both type of BOD, dissolved as well as settleable. The DO sag is analysed for some hypothetical situations. The model is then used for the real data obtained for the river Yamuna along Delhi. A fair agreement of predicted and observed values establishes the robustness of the model.

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2. Development of model

Consider an unpolluted river that receives waste water through multiple discharge points situated along it's bank. A one dimensional mathematical model with the following assumptions is used to predict the DO conditions in river.

- The waste is being discharged from five outfalls scattered on the bank of river in the considered reach.
- Reaeration is modelled according to Henry's law.
- Only advection and dispersion are considered as relevant transport mechanism.
- The BOD present in waste consists of two parts namely Dissolved (denoted by B_d) and Settleable (denoted by B_s). The dissolved part is decaying according to first order kinetics while settleable part is being removed by a linear law. It is assumed that the settleable part of BOD gets removed at a distance x_1 from the concerned outfall and once this part is settled at bottom, it does not put any demand from river's DO.

Based on the above stated assumptions, the following one dimensional model representing concentration of BOD and DO in river is presented:

$$B_s = B_{0-s}\left(1-\frac{v}{d}.\frac{x}{u}\right), \qquad x \le x_1$$

$$= 0 \qquad x \ge x_1 \qquad (1)$$

$$-u\frac{dB_d}{dx} - D_L\frac{d^2B_d}{dx^2} - k_1B_d = 0 \qquad (2)$$

$$-u\frac{dC}{dx} - D_l\frac{d^2C}{dx^2} - k_1B_d - mB_s + K_r(C_s - C) = 0 \qquad (3)$$

Where B_d = dissolved part of BOD (mg/L), Bo-s is the initial settleable BOD (mg/L), B_s = settleable part of BOD (mg/L), x_1 is the distance at which settling is over (m). K_r = coefficient of reaeration (s-^{1}); v = settling velocity of particles (m/s); d = Depth of the river (m); u = mean cross-sectional flow velocity (m/s); D_L = Co efficient of Dispersion (m^2/s); D = DO deficit (mg/L); C is the concentration of DO in (mg/L) ; k_1 is the decay rate of dissolved part of BOD in (s^{-1}); m is the removal rate of the settleable part of BOD (s^{-1}) and C_s is the concentration of DO at saturation level (mg/L).

3. Method of solution

The values of B_s and B_d as obtained from Eq.1 and Eq.2 are substituted in Eq.3 to get the value of c for two different distances as follows:

$$c = C_s - C_2 e^{\gamma x} - \frac{k_1 B_{0-d} e^{\beta x}}{D_L \beta^2 - u\beta - k_r} - \frac{m B_{0-s}}{k_r} - \frac{m B_{0-s} v}{k_r u d}\left(x - \frac{u}{k_r}\right); \quad x < x_1 \text{…………….} \quad (4)$$

$$c = C_s - C_2 e^{\gamma x} - \frac{k_1 B_{0-d} e^{\beta x}}{D_L \beta^2 - u\beta - k_r}; \quad x \ge x_1 \qquad \text{…………} \qquad \text{…}(5)$$

Where, $\beta = \dfrac{u - \sqrt{u^2 + 4k_1 D_L}}{2D_L}$

$$\gamma = \frac{u - \sqrt{u^2 + 4k_r D_L}}{2D_L}$$

4.Analysis of DO sag for a hypothetical multi outfall system

Consider a hypothetical case where three outfalls are located along the river bank . Let the contribution of settleable and dissolved BOD from each outfall (say 1, 2 and 3) be denoted by B_{si} , B_{di} ;$i \quad 1, 2 , 3$respectively, where

$B_{d1} \quad 9.6\ mg/l$, $B_{s1} \quad 6.4\ mg/l$

$B_{d2} \quad 6.0\ mg/l$, $B_{s2} \quad 4.0\ mg/l$

$B_{d3} \quad 3.6\ mg/l$, $B_{s3} \quad 2.4\ mg/l$

Let the distance x_1 at which the settleable part from each outfall is settled be 562m. The concentration of DO is calculated using the presented model for four different situations represented by cases 4.1-4.4 and the predicted values of DO are plotted against the distances in Figs. 1-4.

4.1 Non-interfering settling fields

Let the second outfall be located at x= 600m (A point whose distance from first outfall is more than 562 meters) and third outfall is located at x= 1200m (A point whose distance from first outfall is more than 1162m). This case represents a situation where the second waste is discharged when the settling field of the first outfall is over and third waste is discharged when the settling field of second waste is over.

4.2 Partial interfering settling fields

Let the second outfall be located at x=600m (A point whose distance from first outfall is more than 562 meters) and third outfall is located at x=1000m (A point whose distance from first outfall is less than 1162 meters). This case represents a situation where the second waste is discharged when the settling field of the first outfall is over and third waste is discharged when the settling field of second waste is not completed.

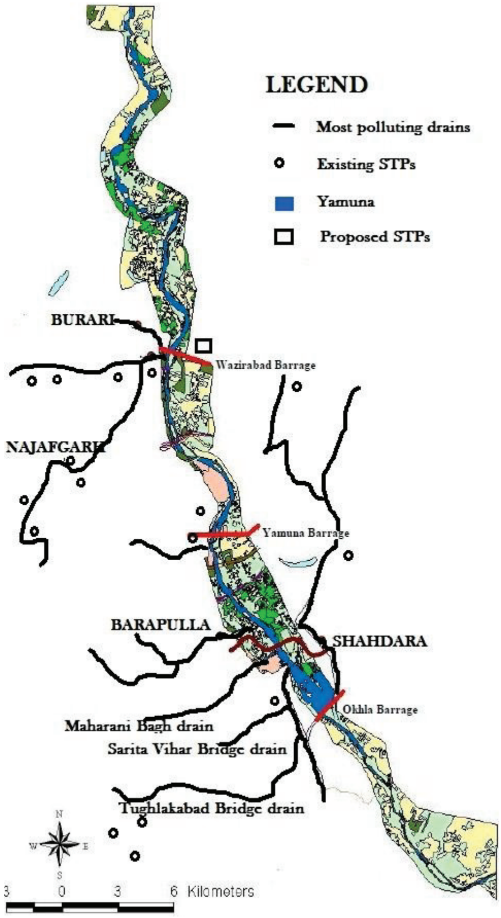
4.3 Single Interfering settling fields

Let the second outfall be located at x = 400m (A point whose distance from first outfall is less than 562 meters) and third outfall is located at x = 800m (A point whose distance from first outfall is less than 1162 meters). This case represents a situation where the second waste is discharged when the settling field of the first outfall is not completed and third waste is discharged when the settling field of second waste is not completed.

4.4 Multiple interfering settling fields

Let the second outfall be located at x=300m and third outfall is located at x=500m. This case represents a situation where represents a situation where the second and third wastes are discharged before the settling field of the first outfall is not completed. The settleable part of second and third wastes are settled at x=862m and x= 1062m respectively.

The data presented in Table-1 is used for the computation of DO values given by Eq.3, at various distances downstream of the three outfalls, in all the situations. Fig. 1-4 represents the predicted values of DO for cases 4.1-4.4 respectively



5. Analysis of DO sag for stretch of River Yamuna in Delhi.

5.1 About the River

The Yamuna is the largest tributary river of the Ganga in northern India. Originating from the Yamunotri Glacier at a height of 6,387 metres on the south western slopes of Banderpooch peaks. Its coordinates are 25°30′N 81°53′E . The Yamuna river passing through 22 km in Delhi has become one of the dirtiest rivers in the country. it travels a total length of 1,376 kilometres. The water of Yamuna is of "reasonably good quality" through its length from Yamunotri in the Himalayas to Wazirabad in Delhi, about 375 km, where the discharge of waste water through 15 drains between Wazirabad barrage and Okhla barrage renders the river severely polluted after Wazirabad in Delhi. Delhi generates about 3,267 million litres per day (ml/d) of sewage while the city's installed waste water treatment capacity is only 2,330 ml/d. More than 937 ml/d of waste is not treated. Out of Delhi's 2,330 ml/d treatment capacity, 37 per cent is under-utilised and 1,270 ml/d of sewage is untreated and allowed to enter the river every day. The Najafgarh drain contributes to 60% of the total wastewater and 45 per cent of the total BOD load being discharged from Delhi into the Yamuna. At downstream Okhla, the DO level declined to 1.3 mg/l with the BOD at 16 mg/l, indicating considerable deterioration in water quality due to discharge of sewage and industrial effluents. In Delhi stretch of Yamuna river, 16 drains outfall into the river from Wazirabad to Okhla. The data of these drains as well as the individual contribution of the BOD's by each drain in the river, **a**re shown in table 2(ref 5).

Hydraulic parameters

Table 1

Parameter	Value	Units
U	2808	m/s
D_L	12675	m^2/s
B_{do}	12	mg/L
B_{s0}	10	mg/L
D_0	0.0	mg/L
k_1	3	s^{-1}
k_r	5	s^{-1}
m	5	s^{-1}

5.2 Results and Discussion

In the present case study, the proposed model is applied on the river Yamuna along Delhi.
The contribution from the drains mentioned at serial numbers 1,8,11,12 and 14 in Table-2 are considered significant and therefore included in the present study. The following values of initial BOD and DO are used (reference 3)
B_0=2.0 mg/l, $D_{0-1} = 0.67$ mg/l, p=0.6, k=1.5/day, $k_r = 5.0\ day^{-1}$, m=$5.0\ day^{-1}$.
The hydraulic parameters given in table-1 are used in preparing the DO sag curve using the presented model. A DO sag curve is also obtained using the model presented by Bhargava (ref 5). A comparison of DO sag curve obtained from both the models i.e Bhargava's model and presented model is presented with the observed values in fig-5.

Table 2

S.No.	Discharge	Distance(km)		BOD(mg/l)
1	Najafgarh	0.0		36.5
2	Magazine road	1.1		0.05
3	Sweeper colony	1.8		0.02
4	Khyber pass	2.5		0.01
5	Metcalf House	3.8		0.15
6	Kudsia	5.4		0.40
7	Moat	6.5		0.03
8	Trans-Yamuna MCD	7.4		3.50
9	Mori Gate	8.7		0.08
10	Civil Mill	9.6		0.17
11	Power House	11.9		0.50
12	Sen Nursing Home	14.4		0.45
13	Drain number 14	15.5		0.015
14	Barapulla	19.8		6.00
15	Maharani Bagh	21.6		0.25
16	Kalka ji	25.0		0.05

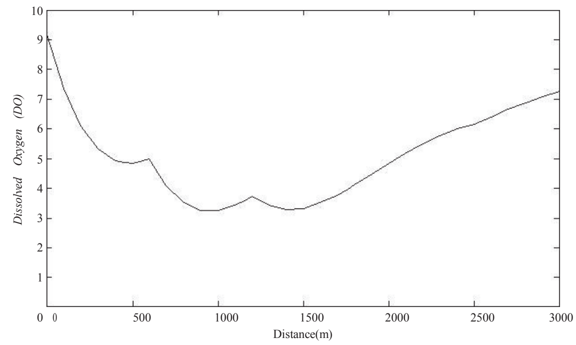


Fig.1 Concentration of DO for non interfering settling field

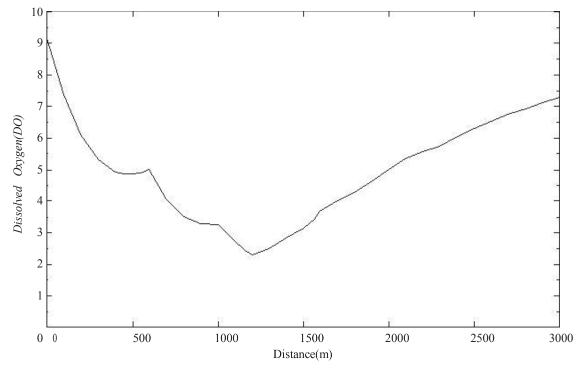


Fig. 2 Concentration of DO for partial interfering settling field.

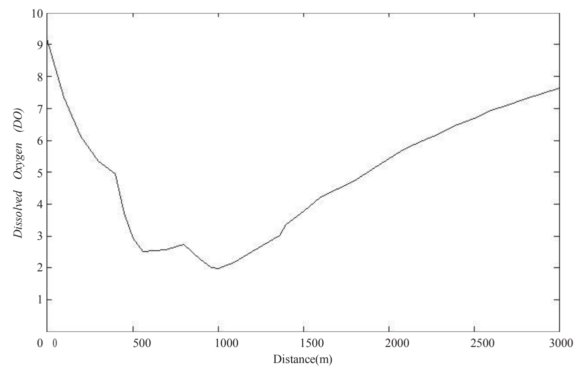


Fig. 3 Concentration of DO for single interfering settling field.

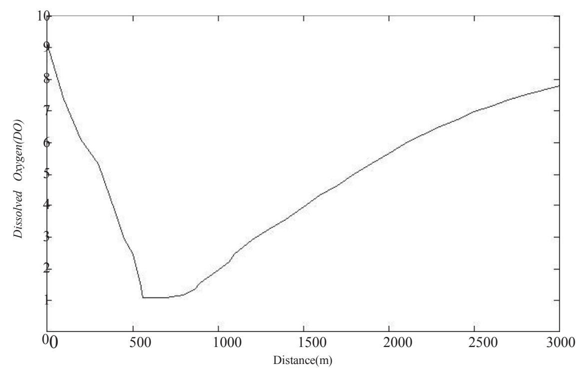


Fig. 4 Concentration of DO for multiple interfering settling fields.

The DO sag curve shown with dashed line in Fig-5 is obtained using presented model while the curve with broken line represents the DO sag curve obtained by Bhargava's model for the same data. The observed points using reference-10 are plotted and represented with triangular points while some observed points using reference-1 are plotted and represented by cross points in Fig-5. A fait agreement of observed values of DO with those obtained by the presented model shows the robustness of the presented model.

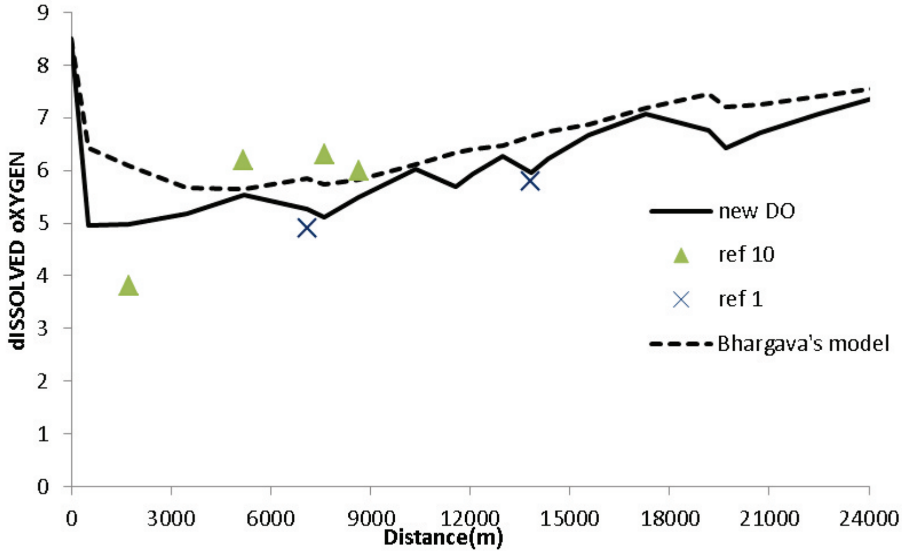


Fig. 5 Comparison of DO sag curve with Bhargava's model.

Conclusion

A mathematical model has been presented to analyse the DO sag in a situation when partially treated /untreated waste is discharged into the river through multiple point sources situated along the river bank. The model is analysed for different hypothetical situations. The model is applied for a practical situation where the stretch of holy river Yamuna along Delhi receives discharge from multiple sources situated along it's bank. For the present study, the sources with significant contributions of BOD are considered.. A fair agreement of DO values obtained from presented model with the observed values shows the robustness of model. The DO sag curve obtained from presented model is also compared with the DO sag curves obtained by Bhargava's model. The closeness of observed points with the presented DO sag curve shows the importance of including dispersion in the model. The presented model can be used in a practical situation when partially treated/untreated waste is discharged into the river from several point sources.

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