Cross-Diffusion Effect on Mixed Convective Fluid Flow through Horizontal Annulus

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Abstract

The unsteady mixed convective fluid flow through an annulus filled with a fluid-saturated porous medium is numerically investigated in this study in the presence of Cross-Diffusion effect and constant Heat Source. The flow configuration and coordinate system for an annulus which is horizontal position have been considered

Details of the effect of several parameters controlling the velocity, temperature and concentration profiles are shown graphically and the observations are discussed. These parameters include the Non-Darcy parameter, Pressure gradient, Soret effect, Schmidt Number, Dufour effect, Eckert number and Prandtl Number. The effect of these dimensionless parameters mentioned above is observed either to enhance, to decrease or have no effect on the velocity, temperature and the concentration profiles.

Keywords: Unsteady Flow, Cross-Diffusion effect, Constant heat source Effect.

Nomenclature

Symbol ***Quantity***

Roman Symbols

C	Dimensionless Concentration
C_p	Specific heat at constant pressure,
C_s	Concentration susceptibility
C_i', C_o'	Concentration of the fluid at the inner and outer pipes respectively
ΔC	Characteristic concentration difference
Δr	Space marching step
Δt	Time marching step
D^{-1}	Non-Darcy parameter
D_m	Mass diffusivity
D_u	Dufour number
e	Kinetic energy
E	Total energy
F	Body force
F_e	Electromagnetic force
g	Acceleration due to gravity
h	Convective heat transfer coefficient
i, j, k	Unit vectors in the r', θ', and z' directions respectively
J	Electric current density
K	Porous medium permeability
k_1	Coefficient of thermal conductivity
K_T	Thermal diffusion ratio
P	Pressure of the fluid.
P_r	Prandtl number
q	Velocity vector of the fluid
r	Radius of the Cylinder
R_e	Reynolds number
S	Dimensionless suction velocity
S_c	Schmidt number
S_r	Soret number

t	Representative time
T	Temperature
ΔT	Characteristic temperature difference
U	Uniform velocity
u, v, w	Velocity components
r, θ, z	Polar coordinates of the annulus

Greek Symbols

λ	Thermal diffusivity
β	Thermal coefficient
β'	Coefficient of expansion due to concentration gradients
σ	Fluid electrically conductivity,
$\sigma_{i,j}$	Strain tensor
τ	Turbulence time scale
$\varepsilon_{i,j}$	Rate of strain tensor
ρ	Fluid Density
ρ_e	Excess electrical charge
ρ_∞	Free stream fluid density
ν	Kinematic viscosity
θ	Dimensionless fluid temperature
μ	The coefficient of viscosity
μ_e	The magnetic permeability
ϕ	Viscous dissipation function
α	Heat source parameter

Abbreviations

FDM	Finite difference method
PDE	Partial differential equation

1. Introduction

Fluid dynamics is a sub-discipline of fluid mechanics that deals with fluid flow - the natural science of fluids (liquids and gases) in motion. It has several sub disciplines itself, including aerodynamics (the study of air and other gases in motion) and hydrodynamics (the study of liquids in motion).

In many industrial applications of transient free convection flow problems, there occurs a heat source or a sink which is either a constant or temperature gradient or temperature dependent heat source. This heat source occurs in the form of a coil or a battery.

Therefore, in this section, contributions of earlier researchers in the flow field of Natural and mixed convective Heat and Mass transfer is discussed. A comprehensive survey of relevant papers may be found in the recent monograph by Nield and Bejan (2006). Most of the studies included there refer to bodies of relatively simple geometry such as flat plates, cylinders, and spheres. It gives a clear description of the work already done in this field and brings out the knowledge gap existing and where the geometry under consideration fits. Sparrow and Cess (1962) initially studied solutions of the steady flow and heat transfer of the stagnation point flow taking into account the constant volumetric heat generation. Foraboschi and Federico (1964) have assumed volumetric rate of heat generation of the type $Q = Q_o (T - T_o)$ when $T >= T_o$ and $Q = 0$ when $T < T_o$ in their study of steady state temperature profiles for laminar parabolic and piston flow in circular tubes. Neeraja (1993) has made a study of the fluid flow and heat transfer in a viscous incompressible fluid confined in an annulus bounded by two rigid cylinders. The flow is generated by periodic traveling waves imposed on the outer cylinder and the inner cylinder is maintained at constant temperature. The limiting case of fully developed natural convection in porous annuli is solved analytically for steady and transient case by Shaarawi, *et al*. (1990). Philip (1982) has obtained analytical solutions for the annular porous media valid for low modified Reynolds number. Taking G/R much less than 1, the coupled equations governing the flow, heat and mass transfer have been solved by regular perturbation method. Anghel *et al*. (2000) have examined the composite Soret and Dufour effects on free convective heat and mass transfer in a Darcian porous medium with Soret and Dufour effects. Gokhale and Behnaz-Farman (2007) analyzed transient free convection flow of an incompressible fluid past an isothermal plate with temperature gradient dependent heat sources. Implicit finite difference scheme which is unconditionally stable has been used to solve the governing partial differential equations of the flow. Transient temperature and velocity profiles are plotted to show the effect of heat source. Muthukumara, *et al*. (2007) has

analyzed the radiation effect on moving vertical plate with variable temperature and mass diffusion. Sreevani (2003) has analyzed the Soret effect on convective heat and mass transfer flow of a viscous fluid in a cylindrical annulus with heat generating sources. Sivaiah (2004) has discussed the convective heat and mass transfer flow in a circular duct with Soret effect. Srenivas Reddy (2006) has discussed the Soret effect on mixed convective heat and mass transfer through a porous cylindrical annulus. Again, Sallam (2009) has analyzed thermal-diffusion and diffusion-thermo effects on mixed convection heat and mass transfer in a porous medium. Heat and mass transfer have been solved by regular perturbation method.
Recently, Prasad (2006) analyzed the convective heat and mass transfer through a porous cylindrical annulus in the presence of heat generating source under radial magnetic field. Assuming the Eckert number E_c much less than 1 the governing equations have been solved by regular perturbation method. The flow through a porous medium in a porous cylindrical annulus does not provide the physical interpretation of mixed convective heat and mass flow and the study did not take care of cross-diffusion effect. Therefore in this research we investigate the problem of the combined influence of cross-diffusion effect using the finite difference method expressions taking the constant heat source effect into account.

2.0 Formulation of the problem

Consider unsteady, incompressible, viscous, electrically conducting fluid flow through a porous medium in a circular cylindrical annulus with cross-diffusion effects. Let the inner and outer radius be denoted by $r_i = a$ and $r_o = a + s$ respectively. The flow temperature and concentration in the fluid are assumed to be fully developed. Both the fluid and porous region have constant physical properties and the flow is a mixed convection flow taking place under thermal and molecular buoyancies and uniform axial pressure gradient. The annulus is stationary and the induced magnetic field is neglected by assuming a very small magnetic Reynold's number. For this reason, the uniform radial magnetic field $\boldsymbol{B_o}$ is considered as the total magnetic field acting on the whole system in the positive r direction as shown in the Figure 1. The Flow configuration and coordinate system is shown below

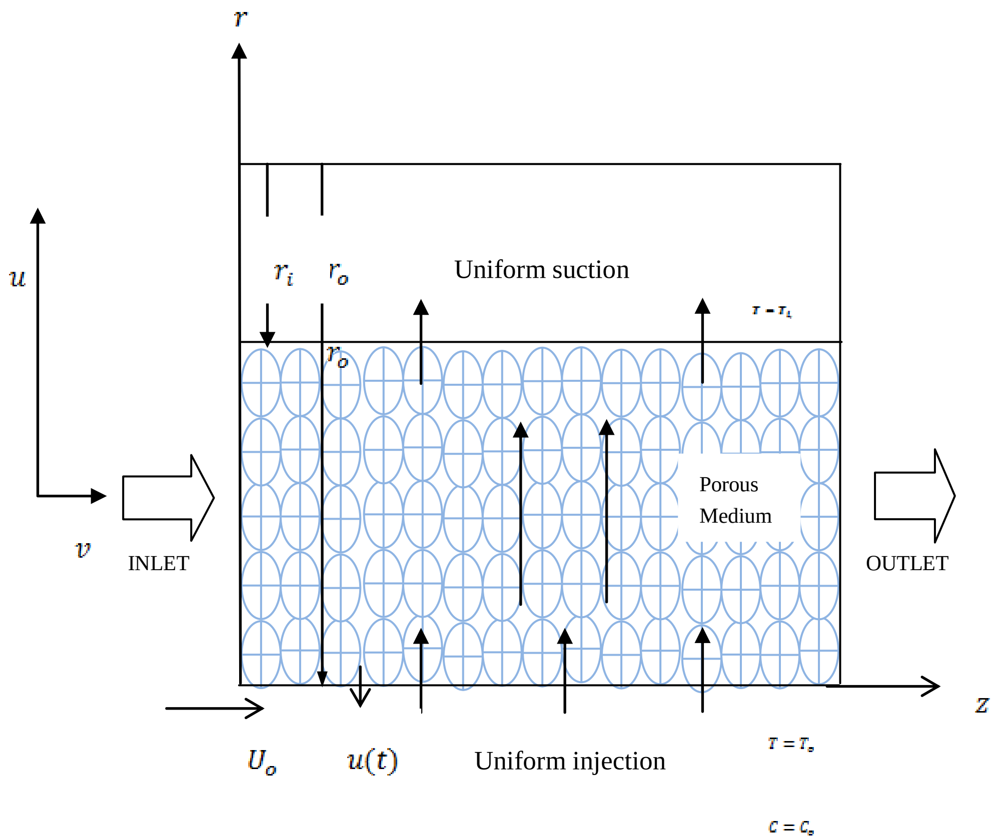

Figure 1: Flow configuration and coordinate system through horizontal porous annulus.

From the geometry of the problem, all the quantities are independent of the axial coordinate *z* except the pressure gradient $\frac{dP}{dz}$, which is assumed constant.

Since the flow is fully developed, continuity equation takes the form

$$\frac{\partial v}{\partial r} = 0 \tag{2.1}$$

Considering a uniform injection of a second material from below and uniform suction to the top with velocity v_o then from $\frac{\partial v}{\partial r} = 0$, we have

$$v = v_o \tag{2.2}$$

The velocity vector of the fluid is

$$\vec{\boldsymbol{q}} = \vec{\boldsymbol{q}}(r,t) = u\boldsymbol{i} + v_o(r,t)\boldsymbol{j} \tag{2.3}$$

We consider a slow speed fluid flow such that the buoyancy force resulting from temperature and concentration differences in the flow field are comparable with the inertia and viscous forces. In the presence of heat transfer, let the density vary with temperature and also vary with concentration difference in the presence of mass transfer. The Boussinesque approximation is invoked so that the density variation is confined to the thermal and molecular buoyancy forces.

Since $\vec{q} = \vec{q}(r,t)$, the viscous term $\mu\nabla^2$q takes the form $\frac{\partial^2 u}{\partial r^2}$, and the momentum equation takes the dimensional form

$$\rho\left(\frac{\partial u'}{\partial t'} + v_o'\frac{\partial u'}{\partial r'}\right) = -\frac{dP'}{dz} + \mu\left(\frac{\partial^2 u'}{\partial r'^2} + \frac{1}{r'}\frac{\partial u'}{\partial r'}\right) - \frac{\mu}{k}u' \tag{2.4}$$

By taking into account the effect of viscous dissipation and constant heat source, the energy equation takes the form

$$\rho C_p\left(\frac{\partial T'}{\partial t'} + v_o'\frac{\partial T'}{\partial r'}\right) = K\left(\frac{\partial^2 T'}{\partial r'^2} + \frac{1}{r'}\frac{\partial T'}{\partial r'}\right) + \mu\left(\frac{\partial v}{\partial r}\right)^2 + \frac{D_m K_t}{C_s}\left(\frac{\partial^2 C'}{\partial r'^2} + \frac{1}{r'}\frac{\partial C'}{\partial r'}\right) + \frac{Q}{r} \tag{2.5}$$

The equation of concentration can be written as

$$\frac{\partial C'}{\partial t'} + v_o'\frac{\partial C'}{\partial r'} = D\left(\frac{\partial^2 C'}{\partial r'^2} + \frac{1}{r'}\frac{\partial C'}{\partial r'}\right) + \frac{D_m K_t}{C_s C_p}\left(\frac{\partial^2 T'}{\partial r'^2} + \frac{1}{r'}\frac{\partial T'}{\partial r'}\right) \tag{2.6}$$

2.1 Definition of Mesh

We want to use uniform mesh to represent a function of two variables $f(r,t)$ where r and t is the radial and distance along the annulus. Consider a rz-plane which is divided into uniform rectangular cells of width Δr and height Δt as shown in Fig. 2. Consider a reference point (i,j) where i and j represent r and t respectively. Using the notation $i \pm 1$ for $r \pm \Delta r$ and $j \pm 1$ for $t \pm \Delta t$ we can define the adjacent points to r and t, the points that are i and j units from the reference point will have coordinates $(i\Delta r, j\Delta t)$. The mesh can be defined as below

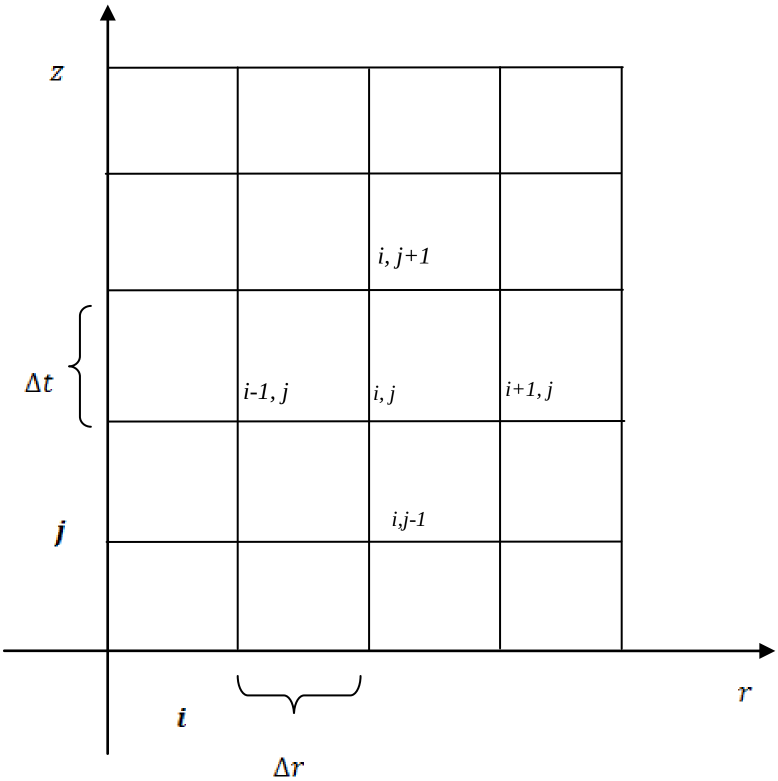

Figure 2: Mesh representing the fluid flow.

2.2 The Finite Difference Technique

The finite-difference equations are arrived at by the setting up of the finite-difference expressions for the numerical solution of Equation (2.5), (2.6) and (2.7) in section 2.0 for horizontal annulus. To implement a finite-difference solution, we divide the x-axis along the radius into discrete grid points, as shown in the Fig.3. The first grid point labeled point 1 is assumed to be at the surface of the inner cylinder $(r = 1)$. The points are evenly distributed along the $x-$ axis, with Δr denoting the spacing between the grid points. The last point namely, that at the surface of the outer cylinder$(r = 2)$, is denoted by n. Therefore we have a total number of n grid points distributed along the axis. Point i is simply an arbitrary grid point, with points $i - 1$ and $i + 1$ as the adjacent points. Since in the time marching approach, we know the flow-field at point r and we use the difference equations to solve explicitly for the variables at point $r + i\Delta r$.

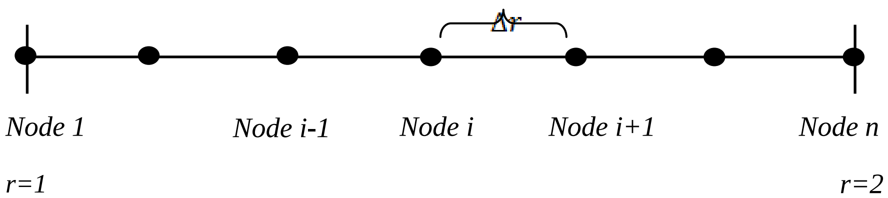

Figure 3: Grid point distribution across the annulus.

We are interested in replacing a partial derivative with a suitable algebraic difference quotient, i.e., a finite difference. Most common finite difference representations of derivatives are based on Taylor's series expansions. For example, referring to Fig.2. if $u_{i,j}$ denotes the r component of velocity at point (i, j), then the velocity $u_{i+1,j}$ at point $(i + 1, j)$ can be expressed in terms of a Taylor series expanded about point (i, j), as follows

$$u_{i+1,j} = u_{i,j} + \left(\frac{\partial u}{\partial r}\right)_{i,j} \Delta r + \left(\frac{\partial^2 u}{\partial r^2}\right)_{i,j} \frac{(\Delta r)^2}{2} + \left(\frac{\partial^3 u}{\partial r^3}\right)_{i,j} \frac{(\Delta r)^3}{6} + \ldots \tag{2.7}$$

Equation (2.8) is mathematically an exact expression for $u_{i+1,j}$ if

i. The number of terms is infinite and the series converges and /or

ii. $\Delta r \to 0$.

From there we pursue the finite – difference representations of derivatives. Solving Eq. (2.7) for $(\partial u/\partial r)_{i,j}$, we obtain

$$\underbrace{\left(\frac{\partial u}{\partial r}\right)_{i,j} = \frac{(u_{i+1,j} - u_{i,j})}{\Delta r}}_{\text{Finite-difference representation}} \underbrace{- \left(\frac{\partial^2 u}{\partial r^2}\right)_{i,j} \frac{(\Delta r)^2}{2} - \left(\frac{\partial^3 u}{\partial r^3}\right)_{i,j} \frac{(\Delta r)^3}{6} + \ldots}_{\text{Truncation error}} \tag{2.8}$$

In Eq. (2.8) the actual partial derivative evaluated at point (i, j) is given on the left side. The first term on the right side, namely $(u_{i+1,j} - u_{i,j})/\Delta r$, is a finite difference representation of the partial derivative. The remaining terms on the right side constitute the truncation error. That is, if we wish to approximate the partial derivative with the above algebraic finite-difference quotient,

$$\left(\frac{\partial u}{\partial r}\right)_{i,j} \approx \frac{(u_{i+1,j} - u_{i,j})}{\Delta r} \tag{2.9}$$

Then the truncation error in Eq. (2.8) tells us what is being neglected in this approximation. In Eq. (2.9), the lowest-order term in the truncation error involves Δr to the first power; hence, the finite-difference expression in Eq. (2.8) is called *first-order-accurate*. We can more formally write Eq. (2.9) as

$$\left(\frac{\partial u}{\partial r}\right)_{i,j} = \frac{(u_{i+1,j} - u_{i,j})}{\Delta r} + 0(\Delta r) \tag{2.10}$$

In Eq. (2.10) the symbol $0(\Delta r)$ is a formal mathematical notation which represents “terms of order Δr.” Equation (2.10) is a more precise notation than Eq. (2.9) which involves the “approximately equal” notation. Also referring to Fig.2.3, note that the finite-difference expression in Eq. (2.10) uses information to the *right* of the grid point(i, j); that is, it uses $u_{i+1,j}$ as well as $u_{i,j}$. No information to the left of (i, j) is used. As a result, the finite difference in Eq. (2.10) is called a forward difference. For this reason, we now identify the first-order-accurate difference representation for the derivative $(\partial u/\partial r)_{i,j}$ expressed by Eq. (2.10) as a first-order-forward difference, repeated below

$$\left(\frac{\partial u}{\partial r}\right)_{i,j} = \frac{(u_{i+1,j} - u_{i,j})}{\Delta r} + 0(\Delta r) \tag{2.11}$$

Let us now write a Taylor series expansion for $u_{i-1,j}$, expanded about $u_{i,j}$.

$$u_{i-1,j} = u_{i,j} + \left(\frac{\partial u}{\partial r}\right)_{i,j} (-\Delta r) + \left(\frac{\partial^2 u}{\partial r^2}\right)_{i,j} \frac{(-\Delta r)^2}{2} + \left(\frac{\partial^3 u}{\partial r^3}\right)_{i,j} \frac{(-\Delta r)^3}{6} + \ldots$$

or $$u_{i-1,j} = u_{i,j} - \left(\frac{\partial u}{\partial r}\right)_{i,j} \Delta r + \left(\frac{\partial^2 u}{\partial r^2}\right)_{i,j} \frac{(\Delta r)^2}{2} - \left(\frac{\partial^3 u}{\partial r^3}\right)_{i,j} \frac{(\Delta r)^3}{6} + \ldots \tag{2.12}$$

Solving for $(\partial u/\partial r)_{i,j}$, we obtain

$$\left(\frac{\partial u}{\partial r}\right)_{i,j} = \frac{(u_{i,j} - u_{i-1,j}\ )}{\Delta r} + 0(\Delta r) \tag{2.13}$$

The information used in forming the finite-difference quotient in Eq. (2.13) comes from the *left* of the grid point(i,j); that is, it uses $u_{i-1,j}$ as well as $u_{i,j}$. No information to the right of (i,j) is used. As a result, the finite difference in Eq. (2.13) is called a rearward (or backward) difference.

2.3 Initial and boundary conditions

In the present problem, we know that the temperature (θ) and the concentration (C) increases gradually from the inner wall to outer wall while the velocity vector (q) increases as the flow expand towards the center of the annulus. Hence, we choose initial conditions that qualitatively behave in the same fashion.
The initial conditions and boundary conditions relevant to the fluid flow configuration are respectively

$$\left.\begin{aligned} q(a,0) = q(a+s,0) = 0 \\ \theta(a,0) = \theta(a+s,0) = 0 \\ C(a,0) = C(a+s,0) = 0 \end{aligned}\right\} t = 0 \tag{2.14}$$

and

$$\left.\begin{aligned} q(a,t) = 0 \\ q(a+s,t) = 0 \\ \theta(a,t) = 0 \\ \theta(a+s,t) = 1 \\ C(a,t) = 0 \\ C(a+s,t) = 1 \end{aligned}\right\} t > 0 \tag{2.15}$$

where v is the axial velocity in the porous region, θ & C are the temperature and concentrations of the fluid, μ Dynamic viscosity, K is the permeability of porous medium, D is the molecular diffusivity, D_m is the coefficient of mass diffusivity, T_m is the mean fluid temperature, K_t is the thermal diffusion, C_s is the concentration susceptibility, C_p is the specific heat.

2.4 Non-dimensionalization process

Using the general scaling variables in equation, the boundary conditions can be non-dimensionalized. That is, we use the general scaling variables and non-dimensional parameters quoted above to normalize the boundary layer equations governing the fluid flow under consideration and make the solution bounded, for example non-dimensional velocity such that it varies from 0 to 1.

The relevant corresponding boundary conditions in non-dimensional form are

$$u = 0, \quad \theta = 0, \quad C = 0 \ \ at \ \ r = 1$$

$$u = 0, \quad \theta = 1, \quad C = 1 \quad at \ r = 1 + s \tag{2.16}$$

The terms in the momentum equation (2.4) can be non-dimensionalized as follows
Dividing both side of equation by ρ, we have

$$\frac{\partial u'}{\partial t'} + v_o'\frac{\partial u'}{\partial r'} = -\frac{1}{\rho}\frac{dP'}{dz'} + \frac{\mu}{\rho}\left(\frac{\partial^2 u'}{\partial r'^2} + \frac{1}{r'}\frac{\partial u'}{\partial r'}\right) - \frac{\mu}{\rho k}u'$$

Non-dimensionalising term by term as follows

$$\frac{\partial u'}{\partial t'} = \frac{\partial(uU_o)}{\partial(tv/U_o^2)} = \frac{U_o^3}{v}\frac{\partial u}{\partial t}$$

$$\frac{\partial u'}{\partial r'} = \frac{\partial(uU_o)}{\partial(rv/U_o)} = \frac{U_o^2}{v}\frac{\partial u}{\partial r}$$

$$v_o'\frac{\partial u'}{\partial r'} = S\frac{U_o^3}{v}\frac{\partial u}{\partial r}$$

$$\frac{dP'}{dz'} = \frac{d(\rho P U_o^2)}{d(zv/U_o)} = \rho\frac{U_o^3}{v}\frac{dP}{dz}$$

$$\frac{1}{\rho}\frac{dP'}{dz'} = \frac{U_o^3}{v}\frac{dP}{dz}$$

$$\frac{\partial^2 u'}{\partial r'^2} = \frac{\partial}{\partial r'}\left(\frac{\partial u'}{\partial r'}\right) = \frac{\partial}{\partial(rv/U_0)}\left(\frac{U_o^2}{v}\frac{\partial u}{\partial r}\right) = \frac{U_o^3}{v^2}\frac{\partial^2 u}{\partial r^2}$$

$$\frac{\mu}{\rho}\frac{\partial^2 u'}{\partial r'^2} = \frac{v\rho}{\rho}\frac{U_o^3}{v^2}\frac{\partial^2 u}{\partial r^2} = \frac{U_o^3}{v}\frac{\partial^2 u}{\partial r^2}$$

$$\frac{1}{r'}\frac{\partial u'}{\partial r'} = \frac{1}{rv/U_0}\left(\frac{U_o^2}{v}\frac{\partial u}{\partial r}\right) = \frac{U_o^3}{v^2}\frac{1}{r}\frac{\partial u}{\partial r}$$

$$\frac{\mu}{\rho}\frac{1}{r'}\frac{\partial u'}{\partial r'} = \frac{v\rho}{\rho}\frac{U_o^3}{v^2}\frac{1}{r}\frac{\partial u}{\partial r} = \frac{U_o^3}{v}\frac{1}{r}\frac{\partial u}{\partial r}$$

$$\frac{\mu}{k}u' = \frac{\mu}{k}(uU_o) = \frac{\mu U_o}{k}u$$

$$\frac{\mu U_o}{\rho k}u = \frac{vU_o}{k}u$$

Substituting back the above non-dimensionalised terms we have

$$\frac{U_o^3}{v}\frac{\partial u}{\partial t} + S\frac{U_o^3}{v}\frac{\partial u}{\partial r} = -\frac{U_o^3}{v}\frac{dP}{dz} + \frac{U_o^3}{v}\frac{\partial^2 u}{\partial r^2} + \frac{U_o^3}{v}\frac{1}{r}\frac{\partial u}{\partial r} - \frac{vU_o}{k}u \tag{2.17}$$

Dividing each of the terms given above by $\frac{U_o^3}{v}$, the momentum equation (2.17) becomes

$$\frac{\partial u}{\partial t} + S\frac{\partial u}{\partial r} = -\frac{dP}{dz} + \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{U_o^2}{k}u \tag{2.18}$$

Combining, rearranging the terms and on introducing the non-dimensional parameters D^{-1} equation (2.18) takes the form

$$\frac{\partial u}{\partial t} + S\frac{\partial u}{\partial r} = -\frac{dP}{dz} + \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - D^{-1}u \tag{2.19}$$

Considering each of the terms in the equation of energy (2.5), can be non-dimensionalized as follows
Dividing both side of equation by ρC_p, we have

$$\frac{\partial T'}{\partial t'} + v_o'\frac{\partial T'}{\partial r'} = \frac{K}{\rho C_p}\left(\frac{\partial^2 T'}{\partial r'^2} + \frac{1}{r'}\frac{\partial T'}{\partial r'}\right) + \frac{\mu}{\rho C_p}\left(\frac{\partial v'}{\partial r'}\right)^2 + \frac{D_m K_t}{\rho C_p C_s}\left(\frac{\partial^2 C'}{\partial r'^2} + \frac{1}{r'}\frac{\partial C'}{\partial r'}\right) + \frac{Q}{\rho C_p r} \tag{2.20}$$

Therefore we have the transformations

$$\frac{\partial T'}{\partial t'} = \frac{\partial (T_o' - T_i')\theta}{\partial (tv/U_o^2)} = \frac{U_o^2 (T_o' - T_i')}{v}\frac{\partial \theta}{\partial t}$$

$$\frac{\partial T'}{\partial r'} = \frac{\partial (T_o' - T_i')\theta}{\partial (rv/U_o)} = \frac{U_o (T_o' - T_i')}{v}\frac{\partial \theta}{\partial r}$$

$$v_o'\frac{\partial T'}{\partial r'} = \frac{U_o^2 (T_o' - T_i')}{v} S\frac{\partial \theta}{\partial r}$$

$$\frac{\partial^2 T'}{\partial r'^2} = \frac{\partial}{\partial r'}\left(\frac{\partial T'}{\partial r'}\right) = \frac{\partial}{\partial (rv/U_0)}\left(\frac{U_o (T_o' - T_i')}{v}\frac{\partial \theta}{\partial r}\right) = \frac{U_o^2}{v^2}(T_o' - T_i')\frac{\partial^2 \theta}{\partial r^2}$$

$$\frac{K}{\rho C_p}\frac{\partial^2 T'}{\partial r'^2} = \frac{K}{\rho C_p}\frac{U_o^2}{v^2}(T_o' - T_i')\frac{\partial^2 \theta}{\partial r^2}$$

$$\frac{1}{r'}\frac{\partial T'}{\partial r'} = \frac{1}{rv/U_0}\frac{U_o (T_o' - T_i')}{v}\frac{\partial \theta}{\partial r} = \frac{U_o^2}{v^2}(T_o' - T_i')\frac{1}{r}\frac{\partial \theta}{\partial r}$$

$$\frac{K}{\rho C_p}\frac{1}{r'}\frac{\partial T'}{\partial r'} = \frac{K}{\rho C_p}\frac{U_o^2}{v^2}(T_o' - T_i')\frac{1}{r}\frac{\partial \theta}{\partial r}$$

$$\frac{\partial v'}{\partial r'} = \frac{\partial (vU_o)}{\partial (rv/U_o)} = \frac{U_o^2}{v}\frac{\partial v}{\partial r}$$

$$\frac{\mu}{\rho C_p}\left(\frac{\partial v'}{\partial r'}\right)^2 = \frac{v\rho}{\rho C_p}\left(\frac{U_o^2}{v}\frac{\partial v}{\partial r}\right)^2 = \frac{U_o^4}{vC_p}\left(\frac{\partial v}{\partial r}\right)^2$$

$$\frac{\partial C'}{\partial r'} = \frac{\partial (C_o' - C_i')\theta}{\partial (rv/U_o)} = \frac{U_o (C_o' - C_i')}{v}\frac{\partial C}{\partial r}$$

$$\frac{1}{r'}\frac{\partial C'}{\partial r'} = \frac{1}{rv/U_0}\frac{U_o (C_o' - C_i')}{v}\frac{\partial C}{\partial r} = \frac{U_o^2}{v^2}(C_o' - C_i')\frac{1}{r}\frac{\partial C}{\partial r}$$

$$\frac{D_mK_t}{\rho C_pC_s}\frac{1}{r'}\frac{\partial T'}{\partial r'} = \frac{D_mK_t}{\rho C_pC_sC_p}\frac{U_o^2}{v^2}(T_o' - T_i')\frac{1}{r}\frac{\partial\theta}{\partial r}$$

$$\frac{\partial^2 C'}{\partial r'^2} = \frac{\partial}{\partial r'}\left(\frac{\partial C'}{\partial r'}\right) = \frac{\partial}{\partial(rv/U_0)}\left(\frac{U_o(C_o' - C_i')}{v}\frac{\partial C}{\partial r}\right) = \frac{U_o^2}{v^2}(C_o' - C_i')\frac{\partial^2 C}{\partial r^2}$$

$$\frac{D_mK_t}{\rho C_pC_s}\frac{\partial^2 C'}{\partial r'^2} = \frac{D_mK_t}{\rho C_pC_s}\frac{U_o^2}{v^2}(C_o' - C_i')\frac{\partial^2 C}{\partial r^2}$$

Substituting back the above non-dimensionalised terms we have

$$\frac{U_o^2(T_o'-T_i')}{v}\frac{\partial\theta}{\partial t} + \frac{U_o^2(T_o'-T_i')}{v}S\frac{\partial\theta}{\partial r} = \frac{K}{\rho C_p}\frac{U_o^2}{v^2}(T_o' - T_i')\left(\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r}\right) + \frac{U_o^4}{vC_p}\left(\frac{\partial v}{\partial r}\right)^2 + \frac{D_mK_t}{\rho C_pC_s}\frac{U_o^2}{v^2}(C_o' - C_i')\left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r}\right) + \frac{Q}{\rho C_p r} \qquad (2.21)$$

If each of the terms given above is multiplied by $\frac{v}{U_o^2(T_o{}'-T_i{}')}$ we will have the energy equation in the form

$$\frac{\partial\theta}{\partial t} + S\frac{\partial\theta}{\partial r} = \frac{K}{v\rho C_p}\left(\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r}\right) + \frac{U_o^2}{C_p(T_i - T_o)}\left(\frac{\partial v}{\partial r}\right)^2 + \frac{D_mK_t}{\mu C_pC_s}\left(\frac{C_o'-C_i'}{T_o'-T_i'}\right)\left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r}\right) + \frac{Q}{r\rho C_p\left(T_o' - T_i'\right)} \qquad (2.22)$$

Combining, rearranging the terms and on introducing non-dimensional parameters P_r, E_c, D_u and α equation (2.22) reduces to

$$\frac{\partial\theta}{\partial t} + S\frac{\partial\theta}{\partial r} = \frac{1}{P_r}\left(\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r}\right) + E_c\left(\frac{\partial v}{\partial r}\right)^2 + D_u\left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r}\right) + \frac{\alpha}{r} \qquad (2.23)$$

The equation (2.6) of concentration can be transformed as follows

$$\frac{\partial C'}{\partial t'} + v_o'\frac{\partial C'}{\partial r'} = D\left(\frac{\partial^2 C'}{\partial r'^2} + \frac{1}{r'}\frac{\partial C'}{\partial r'}\right) + \frac{D_mK_t}{C_sC_p}\left(\frac{\partial^2 T'}{\partial r'^2} + \frac{1}{r'}\frac{\partial T'}{\partial r'}\right)$$

$$\frac{\partial C'}{\partial t'} = \frac{\partial(C_o' - C_i')C}{\partial(tv/U_o^2)} = \frac{U_o^2(C_o' - C_i')}{v}\frac{\partial C}{\partial t}$$

$$\frac{\partial C'}{\partial r'} = \frac{\partial(C_o' - C_i')\theta}{\partial(rv/U_o)} = \frac{U_o(T_o' - T_i')}{v}\frac{\partial C}{\partial r}$$

$$v_o'\frac{\partial T'}{\partial r'} = \frac{U_o^2(C_o' - C_i')}{v}S\frac{\partial C}{\partial r}$$

$$\frac{\partial^2 C'}{\partial r'^2} = \frac{\partial}{\partial r'}\left(\frac{\partial C'}{\partial r'}\right) = \frac{\partial}{\partial(rv/U_0)}\left(\frac{U_o(C_o' - C_i')}{v}\frac{\partial C}{\partial r}\right) = \frac{U_o^2}{v^2}(C_o' - C_i')\frac{\partial^2 C}{\partial r^2}$$

$$D\frac{\partial^2 C'}{\partial r'^2} = D\frac{U_o^2}{v^2}(C_o' - C_i')\frac{\partial^2 C}{\partial r^2}$$

$$\frac{1}{r'}\frac{\partial C'}{\partial r'} = \frac{1}{rv/U_0}\frac{U_o(C'_o - C'_i)}{v}\frac{\partial C}{\partial r} = \frac{U_o^2}{v^2}(C'_o - C'_i)\frac{1}{r}\frac{\partial C}{\partial r}$$

$$D\frac{1}{r'}\frac{\partial C'}{\partial r'} = D\frac{U_o^2}{v^2}(C'_o - C'_i)\frac{1}{r}\frac{\partial C}{\partial r}$$

$$\frac{\partial T'}{\partial r'} = \frac{\partial(T'_o - T'_i)\theta}{\partial(rv/U_o)} = \frac{U_o(T'_o - T'_i)}{v}\frac{\partial\theta}{\partial r}$$

$$\frac{1}{r'}\frac{\partial T'}{\partial r'} = \frac{1}{rv/U_0}\frac{U_o(T'_o - T'_i)}{v}\frac{\partial\theta}{\partial r} = \frac{U_o^2}{v^2}(T'_o - T'_i)\frac{1}{r}\frac{\partial\theta}{\partial r}$$

$$\frac{D_m K_t}{C_s C_p}\frac{1}{r'}\frac{\partial T'}{\partial r'} = \frac{D_m K_t}{C_s C_p}\frac{U_o^2}{v^2}(T'_o - T'_i)\frac{1}{r}\frac{\partial\theta}{\partial r}$$

$$\frac{\partial^2 T'}{\partial r'^2} = \frac{\partial}{\partial r'}\left(\frac{\partial T'}{\partial r'}\right) = \frac{\partial}{\partial(rv/U_0)}\left(\frac{U_o(T'_o - T'_i)}{v}\frac{\partial\theta}{\partial r}\right) = \frac{U_o^2}{v^2}(T'_o - T'_i)\frac{\partial^2\theta}{\partial r^2}$$

$$\frac{D_m K_t}{C_s C_p}\frac{\partial^2 C'}{\partial r'^2} = \frac{D_m K_t}{C_s C_p}\frac{U_o^2}{v^2}(T'_o - T'_i)\frac{\partial^2\theta}{\partial r^2}$$

Substituting back the above non-dimensionalised terms we have

$$\frac{U_o^2(C'_o - C'_i)}{v}\frac{\partial C}{\partial t} + \frac{U_o^2(C'_o - C'_i)}{v}S\frac{\partial C}{\partial r} = D\frac{U_o^2}{v^2}(C'_o - C'_i)\left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r}\right) + \frac{D_m K_t}{C_s C_p}\frac{U_o^2}{v^2}(T'_o - T'_i)\left(\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r}\right) \quad (2.24)$$

If each of the terms given above is multiplied by $\frac{v}{U_o^2(C'_o - C'_i)}$ we will have the energy equation in the form

$$\frac{\partial C}{\partial t} + S\frac{\partial C}{\partial r} = \frac{D}{v}\left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r}\right) + \frac{D_m K_t}{vC_s C_p}\left(\frac{T'_o - T'_i}{C'_o - C'_i}\right)\left(\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r}\right) \quad (2.25)$$

Using the non-dimensional parameters S_c and S_r equation (2.25) becomes

$$\frac{\partial C}{\partial t} + S\frac{\partial C}{\partial r} = \frac{1}{S_c}\left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r}\right) + S_r\left(\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r}\right) \quad (2.26)$$

Equations (2.19), (2.23) and (2.26) respectively give the final set of conservation of momentum, energy and concentration equations in non-dimensional form for horizontal annulus with constant heat source effect.

3.0 Methodology

3.1 Numerical Solution of the problem by Finite Difference expressions

We seek a solution of the system of equations (2.19), (2.23) and (2.26) together with the non-dimensional form of initial and boundary conditions (2.16). The system of equations is nonlinear and we apply the numerical approximation method of finite differences in the solution as described in section 2.2. We use the forward differences as approximations to the derivatives. The finite difference form of the momentum equations (2.19) the energy conservation equation (2.23) and concentration equation (2.26) which governs the fluid flow is given as

$$\frac{u_{i,j+1}-u_{i,j}}{\Delta t} = -\frac{dP}{dz} + \frac{u_{i-1,j}-2u_{i,j}+u_{i+1,j}}{(\Delta r)^2} + \left(\frac{1}{r_{i,j}} - S\right)\frac{u_{i+1,j}-u_{i,j}}{\Delta r} - D^{-1}u_{i,j} \tag{3.1}$$

$$\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta t} = \frac{1}{P_r}\left[\frac{\theta_{i-1,j}-2\theta_{i,j}+\theta_{i+1,j}}{(\Delta r)^2} + \frac{1}{r_{i,j}}\frac{\theta_{i+1,j}-\theta_{i,j}}{\Delta r}\right] + E_c\left(\frac{u_{i+1,j}-u_{i,j}}{\Delta r}\right)^2 + D_u\left[\frac{C_{i-1,j}-2C_{i,j}+C_{i+1,j}}{(\Delta r)^2} + \frac{1}{r_{i,j}}\frac{C_{i+1,j}-C_{i,j}}{\Delta r}\right] - S\frac{\theta_{i+1,j}-\theta_{i,j}}{\Delta r} + \frac{\alpha}{r_{i,j}} \tag{3.2}$$

and finally

$$\frac{C_{i,j+1}-C_{i,j}}{\Delta t} = \frac{1}{S_c}\left(\frac{C_{i-1,j}-2C_{i,j}+C_{i+1,j}}{(\Delta r)^2} + \frac{1}{r_{i,j}}\frac{C_{i+1,j}-C_{i,j}}{\Delta r}\right) + S_r\left(\frac{\theta_{i-1,j}-2\theta_{i,j}+\theta_{i+1,j}}{(\Delta r)^2} + \frac{1}{r_{i,j}}\frac{\theta_{i+1,j}-\theta_{i,j}}{\Delta r}\right) - S\frac{C_{i+1,j}-C_{i,j}}{\Delta r} \tag{3.3}$$

The finite difference form of the initial conditions and the boundary conditions (2.17) is given below

$$\left.\begin{aligned} u(1,0) = u(2,0) = 0 \\ \theta(1,0) = \theta(2,0) = 0 \\ C(1,0) = C(2,0) = 0 \end{aligned}\right\} j = 0 \tag{3.4}$$

and

$$\left.\begin{aligned} u(1,j) = 0 \\ u(2,j) = 0 \\ \theta(1,j) = 0 \\ \theta(2,j) = 1 \\ C(1,j) = 0 \\ C(2,j) = 1 \end{aligned}\right\} j > 0 \tag{3.5}$$

In this case unit vectors i and j represent r and t respectively. Rearranging each of these equations enables us to compute consecutive terms of the velocity u, the temperature θ, and concentration C using the initial values and boundary conditions given in the equations (3.4) and (3.5) respectively. Rearrangement of the equations (3.1), (3.2) and (3.3) will reduce to

$$u_{i,j+1} = -\Delta t\frac{dP}{dz} + \frac{\Delta t}{(\Delta r)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) + \frac{\Delta t}{\Delta r}\left(\frac{1}{r_{i,j}} - S\right)\left(u_{i+1,j} - u_{i,j}\right) - [\Delta t D^{-1} + 1]u_{i,j} \tag{3.6}$$

$$\theta_{i,j+1} = \frac{\Delta t}{(\Delta r)^2}\frac{1}{P_r}\left[\left(\theta_{i-1,j} - 2\theta_{i,j} + \theta_{i+1,j}\right) + \frac{\Delta r}{r_{i,j}}\left(\theta_{i+1,j} - \theta_{i,j}\right)\right] + \frac{\Delta t}{(\Delta r)^2}E_c\left(u_{i+1,j} - u_{i,j}\right)^2 + \frac{\Delta t}{(\Delta r)^2}D_u\left[\left(C_{i-1,j} - 2C_{i,j} + C_{i+1,j}\right) + \frac{\Delta r}{r_{i,j}}\left(C_{i+1,j} - C_{i,j}\right)\right] + \frac{\Delta t\alpha}{r} - S\frac{\Delta t}{\Delta r}\left(\theta_{i+1,j} - \theta_{i,j}\right) + \theta_{i,j} \tag{3.7}$$

and

$$C_{i,j+1} = \frac{\Delta t}{(\Delta r)^2 S_c}\left[\left(C_{i-1,j} - 2C_{i,j} + C_{i+1,j}\right) + \frac{\Delta r}{r_{i,j}}\left(C_{i+1,j} - C_{i,j}\right)\right] + \frac{\Delta t}{(\Delta r)^2} S_r \left[\left(\theta_{i-1,j} - 2\theta_{i,j} + \theta_{i+1,j}\right) + \frac{\Delta r}{r_{i,j}}\left(\theta_{i+1,j} - \theta_{i,j}\right)\right] - S\frac{\Delta t}{\Delta r}\left(C_{i+1,j} - C_{i,j}\right) + C_{i,j} \quad (3.8)$$

3.2 Results and discussions

In this section, discussion of the numerical results of the study and their interpretation are presented for the effects of constant heat source on unsteady mixed convective, viscous conducting fluid flow through an horizontal porous annulus. Since the present study involve a large number of non-dimensional parameters $\frac{dP}{dz}, D^{-1}, P_r, E_c, D_u, S_c, S$ and S_r, Computations for the radial velocity u, temperature θ and concentration C were made for $P_r = 0.71$ corresponding to air, Suction $S = 1$ and pressure gradient to be $\frac{dP}{dz} = 2.0$. The parameters that were varied included the Non-Darcy parameter, Eckert number, Dufour number, Schmidt Number, and Soret number. In concert with previous related studies, the Dufour and Soret numbers are chosen in such a way that their product is constant.

Also, the size ($\Delta r = 0.125$) of the annulus segment is fixed at 0.125; however, its location (L) is varied from 1.125 to 1.875.

These values of the parameters were varied one at a time and input into R computer program. Computations were done using the simultaneous model equations (3.6) to (3.8), the initial conditions (3.4) and the boundary conditions (3.5) and the curves plotted for each case. The results for the velocity profiles are represented in the figures labeled Fig. 3.1.Temperature and concentration profiles are represented in the figures labeled Fig. 3.2 to Fig. 3.5. The vertical axis for the Fig. 3.1 to Fig. 3.5 represents the distance from the inner pipe with $r = 1$ to outer pipe with $r = 2$. The numerical results of velocity, temperature and concentration distributions are presented as follows

3.2.1 Velocity profiles

We discuss how each of the parameters affects the velocity profiles (u) of the fluid flow as represented by the graphs in Figure 3.1.

The velocity variation with Non-Darcy parameter when the parameters $M = 2$, $S = 1$ and $\frac{dp}{dz} = 2$ are held constant .

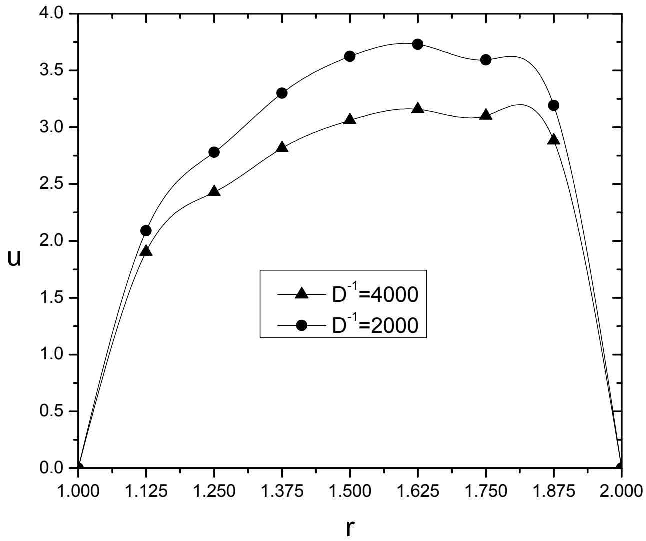

Figure 3.1 Velocity variations with Non-Darcy parameter

It is observed from Fig.3.1 that the velocity of the fluid decreases with increase in the value of the non-Darcy parameter. This is because the lesser the non-Darcy parameter, the larger the size of the pores inside the medium due to which drag force decreases and hence the velocity increases.

A decrease in a stream-wise velocity component, *u*, can result in a decrease in the amount of heat transferred from the walls to the fluid. Similarly, a decrease in the transverse velocity component means that the amount of fresh fluid which is extended from the low temperature region outside the boundary layer and directed towards the annuli walls is reduced thus decreasing the amount of heat transfer. The two effects are in the same direction reinforcing each other. Thus, increase in the non-Darcy parameter implies that the porous medium is offering more resistance to the fluid flow and this result in reduction in the velocity profiles.

3.2.2 Temperature profiles

The effects of various parameters on the temperature profile of the fluid flow were considered as discussed below with reference to Fig. 3.2 and Fig. 3.3

The variation of temperature profile with Dufour parameter when the parameters $P_r = 0.71$, $\alpha = 2$, $E_c = 0.01$, and $S = 1$are held constant.

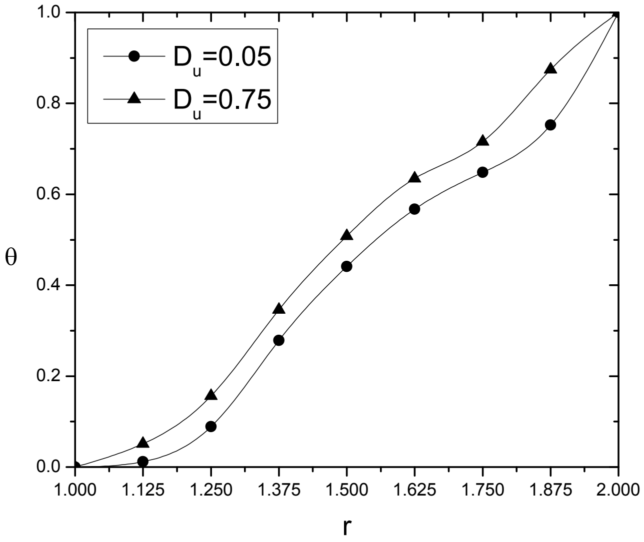

Figure.3.2 Temperature variations with Dufour parameter

Figure 3.2 depicts that the diffusion thermal effects greatly affects the fluid temperature. As the values of the Dufour parameter increase, the fluid temperature also increases. That is not surprising realizing the fact that the thermal boundary becomes thicker for larger Dufour number. Therefore, with an increase in the Dufour number the rate of thermal diffusion rises. This scenario is valid for horizontal case where the dimensionless wall temperature is unity for all parameter values.
From the figure it can be seen that the heat transfer rates are higher for aiding flows than for the corresponding buoyancy in the opposing flows.
The variation of temperature profile with Eckert parameter when the parameters $P_r = 0.71$, $D_u = 0.5$, $\alpha = 2,$ and $S = 1$ are held constant.

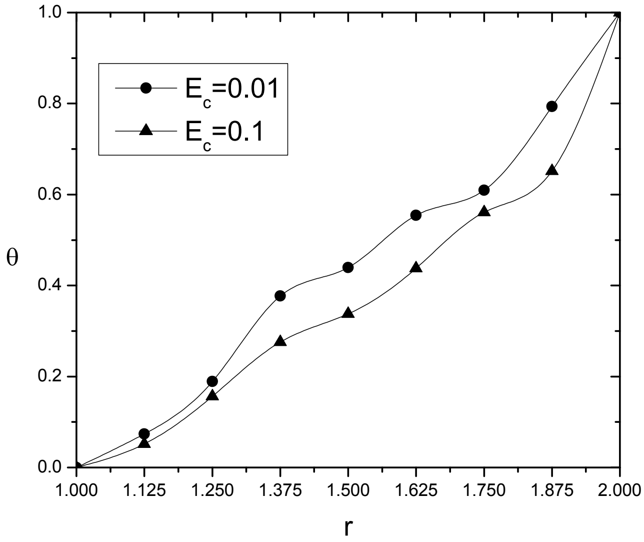

Figure3.3 Temperature variations with Eckert parameter

Figure 3.3 shows the effect of Eckert number on the flow field. We find that an increase in the Eckert number has the decreasing effect on the Temperature profile.
From these figures it is noteworthy that the thermal boundary layer thickness decreases with decreasing values of Eckert parameter. A reduction in the value of the Eckert number $(E_c = 0.01)$ leads to a decrease in the temperature near the inner wall of the annulus. This is because an increase in the Eckert number leads to a decrease in the thermal energy and cosequently a decrease in the temperature profiles.
Hence the Dufour parameter enhanced thermal diffusion while an increase in the Eckert parameter slowed down the rate of internal diffusion within the boundary.

3.2.3 Concentration profiles

The variation of concentration profile with Soret parameter when the parameters $S_c = 1.3$, and $S = 1$ are held is constant

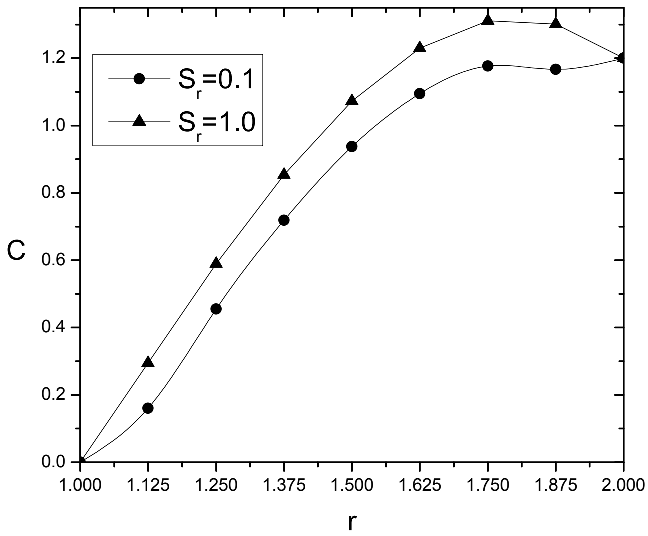

Figure.3.4 Concentration variations with Soret parameter

Figure 3.4 shows the influence of the Soret parameter on the concentration profiles. It can be seen that the concentration increases with increasing values of Soret Number. From this figure we observe that the concentration profiles increase significantly with increase of the Soret number values.
On the other hand an increase in the Soret effect reduces the temperature within the thermal boundary layer leading to an increase in the temperature gradient at the wall and an increase in heat transfer rate at the wall.
The variation of concentration profiles with the Temperature gradient shows that the actual concentration enhances with increase in the temperature gradient this is because the thermal boundary layer becomes thicker for larger the Temperature gradient.

The variation of concentration profile with Schmidt parameter when the parameters $S_r = 0.5$, and $S = 1$ are held constant.

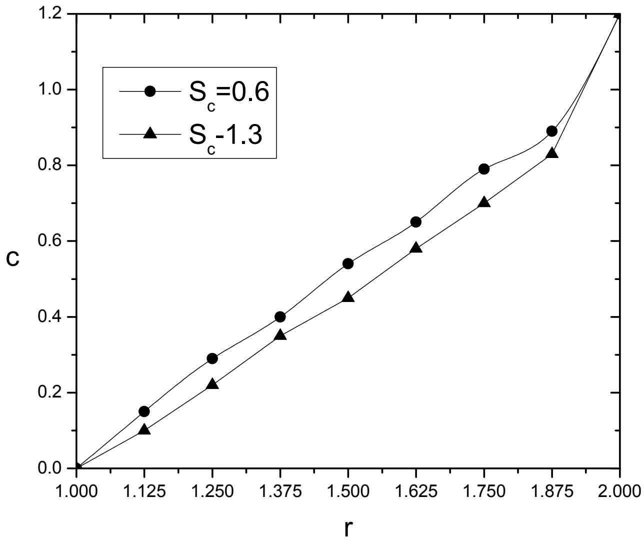

Figure.3.5 Concentration variations with Schmidt number

Fig.3.5 illustrates the effect of Schmidt number on the concentration field. It is noticed that an increase in the value of the Schmidt number causes a decrease in the concentration. The mass diffusion parameter is inversely proportional to the concentration and therefore its increase results in a decrease in the concentration profiles. Furthermore, it is interesting to note that the concentration profile falls rapidly for water vapor $(Sc = 0.6)$. Physically this is true because of the fact that the water vapors can be used for maintaining normal concentration field. The more the molecular diffusivity is, the smaller the concentration in the flow field.

4.0 Conclusions and Recommendations

In this section, a conclusion is given with reference to the results obtained in the previous sections. Recommendations to further areas of research are also given.

4.1 Conclusions

The approximate analytical solutions corresponding to the present study analyses the unsteady mixed convection of an electrically conducting incompressible viscous fluid flow through a cylindrical annulus are obtained using finite difference technique. The cross-diffusion effects are also considered in the presence of constant heat source and required expressions of momentum; energy and concentration profiles are evaluated. The accuracy of the obtained solutions is checked through imposed conditions and graphs. The research has gradually come up with the fluid flow model by beginning with a simple model of the fluid flow and the building on it. Using the finite difference technique and the general scaling variables, the governing equations are transformed into a set of partial differential equations, where numerical solution has been presented for a wide range of parameters.

Furthermore, some well-known established results from the literature are obtained as limiting cases from the present approximate solutions. Numerical results for the velocity field, temperature and concentration field are graphically displayed.

Finally, discussion on the effect of each of these parameters on the velocity, temperature and concentration profiles is explained in details.

Non-Darcy parameter: With respect to variation of velocity with Non-Darcy parameter we found that the lesser the permeability of porous medium, the larger the magnitude of velocity; and for further lowering of the permeability, the larger the magnitude of velocity in the entire flow region.

Soret number: It is observed that decreasing values of Soret number leads to reduction in the concentration distribution in the flow field. In other words, it can be seen that the concentration increases with increasing values of Soret parameter.

Dufour number: It is observed that the variation of temperature profile with Dufour parameter shows that the actual temperature enhances gradually with increase with Dufour parameter.

Eckert number: An increase in the Eckert number causes an increase in the temperature of the fluid next to the walls. Thus, it may be used to reduce the rate of cooling. For the horizontal case, fluid temperature near the wall is predicted to exceed wall temperature inferring that the direction of heat transfer is reversed from the fluid to the wall.

The results have shown that the fluid velocity, temperature, and concentration profiles are appreciably influenced by the Soret and Dufour effects; they also play a significant role and should not be neglected. We therefore conclude that cross-diffusion effects have to be considered in the fluid, heat, and mass transfer. We also showed that the magnetic field and viscous dissipation parameters have greater effects on the fluid velocity, temperature, and concentration boundary layer thickness. It is also noted that the finite difference method is valid even for systems of highly nonlinear differential equations. Furthermore, it has great potential for being used in many other related studies involving complicated nonlinear problems in science and engineering, especially in the field of fluid mechanics, which is rich in nonlinear phenomena.

The following main results are concluded from this study.

1. It was found that the effect of increasing the Non-Darcy parameter decelerates/suppress the fluid velocity/motion while enhancing the temperature and concentration profiles. It was also observed that the velocity decreases if Dufour parameter & Eckert parameter increases.
2. Temperature increases with increasing Dufour parameter and decreases with increase in Eckert parameter.
3. The effect of Soret number is that it reduces the temperature and enhances the velocity and the concentration profiles. Dufour number had an opposite effects on the temperature and concentration distributions. An increase in viscous dissipation parameter enhances temperature and reduces the concentration distributions.

4.2 Recommendations

Clearly, since the present study provides approximate solutions and can be used as bench mark by numerical analysts; the research work provides a basis for further investigation while including the following considerations.

- Study more complex phenomenon and geometrical configurations. For example rectangular and spherical coordinate systems.
- Strong magnetic field whereby the system is not stationary and inclined at an angle
- Varying heat sources and fluid viscosity.

REFERENCES

Anghel, M., Takhar, H.S. and Pop, I. (2000): *Dofour, Soret effects on free convection boundary layer over a vertical surface embedded in a porous medium*. Studia universities- Bolyai, Mathematica, **45**, 11-21.

Antonio Barletle (1999) *Combined forced and free convection with viscous dissipation in a vertical duct. Int. J. Heat and mass transfer*, **42**, 2243-2253.

Gokhale, M.Y. Behnaz-Farnam (2007): "*Transient free convection flow on an isothermal plate with constant heat sources*'', International review of Pure and Applied Mathematics, **3**, 129-136.

Incropera, F.P. & Dewitt, D.P. (1985): *Fundamentals of heat and mass transfer*. John wiley and sons, Inc.

Muthukumara, et al. (2007): "*Study of MHD and Radiation effects on moving vertical plate with variable temperature and mass diffusion*", Int. Rev. Pure Appl. Math., **3(1):** 95- 103.

Neeraja, G: (1993): *Ph.D thesis*, S.P. Mahila University, Tirupathi, India

Nield and Bejan(2006) *Monograph of Comprehensive survey of relevant papers*

Prasad, P.M.V: (2006): *Hydro magnetic convective heat and mass transfer through a porous medium in channel/pipes. Ph. D thesis*, S.K. University, Anantapur, India.

Nield, D.A. and Bejan, A., (2006): "*Convection in porous media*", 3rd edition, New-York, Springer-Verlag.

Philip. J. R: (1982): *Axisymmetric free convection at small Rayleigh numbers in porous cavities, Int. J. Heat and Mass transfer*, **25**, 1689-1699.

Sallam,N. (2009): *Thermal-Diffusion and Diffusion-Thermo Effects on Mixed Convection Heat and Mass transfer in a Porous Medium*. J.Porous Media.
Sreevani, M. (2003)*: Mixed convection heat and mass transfer through a porous medium in channels with dissipative effects*, Ph.D thesis S, K.University, Anantpur, India.

Shaarawi,El.M.A.I and Al-Nimr. M.A: (1990): *Fully developed laminar natural convection in open ended vertical concentric annuli, Int. J. Heat and mass transfer,* 1873-1884

Sivaiah (2004): *Thermo-diffusion effect on convective heat and mass transfer flow through a porous medium in ducts*.Ph.D thesis, S.K.University.Anantapur.

Sparrow, E.M., and Cess, R.D.(1962): *Trans,ASME SERE*, March

Sreenivas Reddy, B: (2006): *Thermo-diffusion effect on convective heat and mass transfer through a porous medium, Ph. D thesis*, S.K. University, Anantapur, India.

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