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Application of Fuzzy-Parametric Linear Programming Problem

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Abstract-- In this paper, we have created a link between the fuzzy linear programming and parametric linear programming by a new procedure for solving fuzzy-parametric linear programming problems, where the matrix coefficients are uncertain values ($\widetilde{a_{ij}}$) and the changes in the coefficients of objective function (c_j). Then find parametric functions for the optimal basis and alternative basis. The value of critical points which determine the beginning of the alternative basis will be approximations to the value of the critical point that determine the ends of the optimal basis. We use the ready program Win(QSB) with real data in formulation that helps to improve the computational performance.

In this study, Practical application of the General Company for electrical industries, development of certain products for the purpose of competition and increase profits. In this case the labour environment become fuzzy, then we used fuzzy linear programming from other hand the company expects wide occur change in the prices of raw materials, then we use parametric linear programming.

Keywords-- Fuzzy linear programming; parametric linear programming; fuzzy decisive set method; fuzzy-parametric linear programming.

1. Introduction

In 1955 the roots of parametric linear programming problem studied by satty T. L. and Gass S. I. [7] when the coefficients of objective function are changed. In 1982 Larry Jenkins [5] published: A method is developed for carrying out parametric analysis on a mixed integer linear program as either objective function coefficients or right hand side values of the constraints are varied continuously. And the root of fuzzy linear programming proposed by Bellman and Zadeh [1] that a fuzzy decision is defined as the fuzzy set of alternatives resulting from the intersection of the goal/objective and constraints. Tanaka [8] adopted this concept to problems of mathematical programming. Negoita [6] formulated the fuzzy linear programming problem with fuzzy coefficients.

In our paper proposed to make link between fuzzy linear programming and parametric linear programming in compound formulation.

The paper is outlined as follows: Concepts of fuzzy/parametric L.P. problems and studied the link in section 2. In section 3 we examined the application of the fuzzy-parametric linear programming problems with real data of G.C.E.I. change in coefficients of objective function. Finally, we writ conclusions in section 4.

2. Concepts of fuzzy and parametric L.P.

2.1 Linear Programming Problems [4]:

Let x_j be (n) decision variables and (m) constraints, then the linear programming problems can defined as follows:

$$\text{Max } Z = \sum_{j=1}^{n} c_j\, x_j \;\Big|\; \sum_{j=1}^{n} a_{ij}\, x_j \le b_i, \qquad i = 1, 2, \dots, m, x_j \ge 0. \qquad \dots (1)$$

2.2 Linear Programming Problems with fuzzy matrix coefficients [2]:

If any matrix coefficient of linear programming problems be uncertain value, then, the form written as following:

$$\text{Max } Z = \sum_{j=1}^{n} c_j x_j \mid \sum_{j=1}^{n} \widetilde{a_{ij}} x_j \le b_i, \quad i = 1, 2, \dots, m, x_j \ge 0. \qquad \dots(2)$$

Assumption 1: By using fuzzy logic which defined by L.A. Zadeh [9] the membership function of fuzzy matrix coefficients are presented as:

$$\mu_{\widetilde{a_{ij}}}(x) = \begin{cases} 1 & , \ x \le a_{ij} \\ (a_{ij} + d_{ij} - x)/\, d_{ij} & , a_{ij} < x < a_{ij} + d_{ij} \\ 0 & , \ a_{ij} + d_{ij} \le x \end{cases} \quad \dots(3)$$

Where: $x \in R$

Therefore, the cost for every product is fuzzy, and the lower limit ($\ell Cost_j$) and upper limit ($uCost_j$) then, the cost calculated as follows:

$$\ell Cost_j = \sum_{i=1}^{m} a_{ij} \cdot q_i \qquad \dots(4)$$

$$uCost_j = \sum_{i=1}^{m} (a_{ij} + d_{ij}) \cdot q_i \qquad \dots(5)$$

Where, q_i is the price of raw materials.

From formulas (4) and (5) the costs change with respect to parameter λ as follows:

$$Cost_j = \sum_{i=1}^{m} (a_{ij} + \lambda d_{ij}) \cdot q_i, 0 \le \lambda \le 1 \qquad \dots(6)$$

Therefore, the coefficients of objective function (c_j) should be fuzzy too, where the lower bound (min c_j) and the upper bound (max c_j). Then we can write the problem as follows:

$$\text{Max.} Z = \sum_{j=1}^{n} \tilde{c}_j x_j \mid \sum_{j=1}^{n} \tilde{a}_{ij} x_j \le b_i, \quad i = 1,2, \dots, m, \ x_j \ge 0. \qquad \dots(7)$$

To solve the problem (7) by using fuzzy decisive-set method which require to partitioned the problem to four sub problems as follows:

$$Z_1 = \max \sum_{j=1}^{n} \min. c_j x_j \mid \sum_{j=1}^{n} (a_{ij} + d_{ij}) x_j \le b_i, \quad i = 1,2, \dots, m \qquad \dots(8)$$

$$Z_2 = \max \sum_{j=1}^{n} \min. c_j x_j \mid \sum_{j=1}^{n} a_{ij} x_j \le b_i, \quad i = 1,2, \dots, m \qquad \dots(9)$$

$$Z_3 = \max \sum_{j=1}^{n} \max. c_j x_j \mid \sum_{j=1}^{n} a_{ij} x_j \le b_i, \quad i = 1,2, \dots, m \qquad \dots(10)$$

$$Z_4 = \max \sum_{j=1}^{n} \max. c_j x_j \mid \sum_{j=1}^{n} (a_{ij} + d_{ij}) x_j \le b_i, \quad i = 1,2, \dots, m \qquad \dots(11)$$

Noting, logically that the value of Z_4 is not take it, Then we ignore its results.

$Z\ell = \min\{Z_1, Z_2, Z_3\}$, $Zu = \max\{Z_1, Z_2, Z_3\}$ …(12)

Then, the membership function of the objective function become as:

$$\mu_G(\tilde{C}X) = \begin{cases} 0 & , \quad \tilde{C}X \le Z\ell \\ \dfrac{(\tilde{C}X - Z\ell)}{Zu - Z\ell} & , \quad Z\ell < \tilde{C}X < Zu \\ 1 & , \quad Zu \le \tilde{C}X \end{cases} \quad \ldots(13)$$

And, the membership of constraints become as:

$$\mu_{ci}(x) = \begin{cases} 0 & , \quad b_i \le \sum_{j=1}^{n} a_{ij}\, x_j \\ \left(b_i - \sum_{j=1}^{n} a_{ij}\, x_j\right) / \sum_{j=1}^{n} d_{ij}\, x_j & , \quad \sum_{j=1}^{n} a_{ij}\, x_j < b_i < \sum_{j=1}^{n} (a_{ij} + d_{ij}) x_j \\ 1 & , \quad \sum_{j=1}^{n} (a_{ij} + d_{ij}) x_j \le b_i \end{cases} \quad \ldots(14)$$

The costs are increasing when λ increasing and the objective function coefficients are decreasing when λ increasing, then the membership of objective function coefficients are as follows:

$$\mu_{cj}(x) = \begin{cases} 0 & , x \le \min c_j \\ (\max c_j - x)/(\max c_j - \min c_j) & , \min c_j < x < \max c_j \\ 1 & , \max c_j \le x \end{cases} \quad \ldots(15)$$

$$\begin{aligned} & \text{Max. } \lambda \\ & \mu_G(X) \ge \lambda, \\ & \mu_{ci}(X) \ge \lambda, \ i = 1, 2, \ldots, m, \\ & x_j \ge 0, 0 \le \lambda \le 1. \end{aligned} \quad \ldots(16)$$

From (13, 14, 15) the problem (16) become as follows:

$$\begin{aligned} & \text{Max. } \lambda \\ & \sum_{j=1}^{n} [\max c_j - \lambda(\max c_j - \min c_j) x_j \ge \lambda(Zu - Z\ell) + Z\ell, \\ & \sum_{j=1}^{n} (a_{ij} + \lambda d_{ij}) x_j \le b_i, i = 1,2, \ldots, m, \\ & x_j \ge 0, 0 \le \lambda \le 1. \end{aligned} \quad \ldots(17)$$

Notice that, the objective function and the constraints in problem (17) containing the cross product terms λX are not convex. Therefore the solution of this problem requires the special approach adopted for solving general non-convex optimization problems.

2.3. Parametric Linear Programming Changes in C [3, 4]**:**

Parametric linear programming investigates the effect of predetermined continuous variations in the objective function coefficients on the optimum solution.
Let X = (x_1, x_2, …, x_n) and define the parametric linear programming changes in C as:

$$\text{Max } Z = \left\{(C + C't)X \,\middle|\, \sum_{j=1}^{n} P_j x_j \le b,\ \ X \ge 0\right\} \qquad \dots (18)$$

Where: C' : Variation profit vector $C' = (c'_1, c'_2, \dots, c'_n)$.
Let X_{Bi}, B_i, $C_{Bi}(t)$ be the elements that define the optimal solution associated with critical value t_i. Starting at $t_0 = 0$ with B_0 as its optimal basis. Next, the critical value t_{i+1} where (i = 0, 1, 2, …) and its optimal basis, if one exists, are determined. The changes in C can affect only the optimality of the problem, the current solution $X_{Bi} = B_i^{-1}b$ will remain optimal for some $t \ge t_i$ so long the reduced cost, $Z_j(t)$- $c_j(t)$, satisfies the following optimality condition:

$$Z_j(t) - C_j(t) = C_{Bi}(t)B_i^{-1}P_j - C_j(t) \ge 0, \qquad \text{for all j nonbasis} \quad \dots (19)$$

The value of t_{i+1} equals the largest t in [t_i, t_{i+1}] that satisfies all the optimality conditions.
To find the optimal solution at any value of the parameter t, we use the functions of t in each basis.

$$Z(t) = Z + t\,Z' \qquad \dots (20)$$

Were Z= CX, Z′= C′ X

2.4. Fuzzy-Parametric Linear Programming Problems:

The link between fuzzy linear programming where the coefficients matrix are uncertain and parametric linear programming with changes in coefficients of objective function are define as follows:

$$\text{Max } Z = \{(\tilde{C} + \tilde{C}'t)X \mid \sum_{j=1}^{n} \tilde{P}_j x_j \le b, X \ge 0\} \qquad \dots (21)$$

The general idea/steps of solving fuzzy-parametric linear programming problems is to start with the optimal solution at **t** = 0. Next, solving the fuzzy linear programming as in (17) to obtain λ^*. Then, the optimality conditions become as follows:

$$Z_j^*(t) - C_j^*(t) = C_{Bi}^*(t)(B_i^*)^{-1}P_j^* - C_j^*(t) \ge 0, \qquad \text{for all j nonbasic} \quad \dots (22)$$

And, $Z^*(t) = Z^* + t\,(Z^*)'$ …(23)
where $Z^* = C^*X^*$, $(Z^*)' = (C^*)'X^*$

3. Application

3.1 Description of Data Research

The General Company for Electrical Industries produces five commodities which are as follows:
1- Water pump.
2- Electric Fan.
3- Motor $^1/_4$HP.
4- Motor $^1/_3$HP.
5- Motor $^1/_2$HP.

The following tables are shows the real data of the company:

Table (1): profits and changes in profit

Productions	X_1 Water pump	X_2 Electric Fan	X_3 Motor $^1/_4$HP	X_4 Motor $^1/_3$HP	X_5 Motor $^1/_2$HP
Selling price	11000	32000	43000	45000	47000
Max C_j	1199	723	1419	-1919	- 4288
Min C_j	787	87	347	-1919	- 4288
$(Max.C_j)'$	2947.26	7855.02	11130.06	9079.94	7796.88
$(Min.C_j)'$	2642.38	7384.38	10336.78	9079.94	7796.88

3.2 Fuzzy-Parametric problem

We write the fuzzy-parametric linear programming problem with real data of the GCEI as follows:

$$\text{Max } Z = (\widetilde{787}+\widetilde{2642.38}\,\mathbf{t})\, x_1 + (\widetilde{87}+\widetilde{7384.38}\mathbf{t})\, x_2 + (\widetilde{347}+ \widetilde{106336.78}\ \mathbf{t})\, x_3 + (-1919 + 9079.94\ \mathbf{t})\, x_4 + (-4288 + 7796.88\ \mathbf{t})\, x_5$$

subject to:

$\widetilde{1.346}x_1$	$+\widetilde{3.900}x_2$	$+\widetilde{4.960}x_3$	$+7.488x_4$	$+7.488\, x_5$	$\leq$	75000
$\widetilde{0.072}x_1$		$+\widetilde{0.332}x_3$	$+0.466x_4$	$+0.466\, x_5$	$\leq$	21000
	$1.400\, x_2$	$+0.71\, x_3$	$+0.71\, x_4$	$+0.71\, x_5$	$\leq$	14000
$\widetilde{0.129}x_1$					$\leq$	4520
		$\widetilde{0.139}\, x_3$	$+ 0.12\, x_4$	$+0.162\, x_5$	$\leq$	15000
		$\widetilde{0.812}\, x_3$			$\leq$	14574
				$1.015x_5$	$\leq$	12673
	$\widetilde{0.198}\, x_2$				$\leq$	8580
	$\widetilde{0.182}\, x_2$				$\leq$	5635
			$0.800\, x_4$		$\leq$	14000
$\widetilde{0.006}x_1$					$\leq$	4000
$0.043x_1$					$\leq$	7500
		$\widetilde{0.371}\, x_3$	$+0.391x_4$	$+0.391\, x_5$	$\leq$	15000
	$0.297\, x_2$				$\leq$	2500
		$0.147\, x_3$	$+0.147\, x_4$	$+0.147\, x_5$	$\leq$	6000
		$\widetilde{0.970}\, x_3$	$+1.01\, x_4$	$+1.010\, x_5$	$\leq$	14000
		$\widetilde{0.749}\, x_3$	$+0.824x_4$	$+0.824\, x_5$	$\leq$	14000
$\widetilde{0.588}\, x_1$					$\leq$	14000
	$0.925\, x_2$				$\leq$	73000
	$\widetilde{1.003}\, x_2$				$\leq$	15000
$0.3591\, x_1$	$+.3879\, x_2$	$+.7409\, x_3$	$+.7409x_4$	$+ .7409\, x_5$	$\leq$	100000

$x_j \geq 0$, j= 1, 2, 3, 4, 5, and x_j integer.

3.3 Fuzzy Problem

The first step to solve the fuzzy-parametric linear programming problem is put t = 0 it will turn the problem into a fuzzy linear programming problem, then we will the problem as follows:

Max $Z = \widetilde{787}x_1 + \widetilde{87}x_2 + \widetilde{347}x_3 -1919x_4 - 4288x_5$ subject to the same constraints in the previous problem.
Now, retail the current problem in to four subproblems and solve Z_1, Z_2, Z_3, by simplex method then the results is as follows:

Z_1=21369830, Z_2= 21743030, Z_3= 40835810.

Zu= 40835810, Zℓ= 21369830.

To solve the fuzzy problem using (17) we start with λ= 1 the solution will be empty, and then search for the best value of λ in [0, 1] symbolized by $\boldsymbol{\lambda}^*$, so that the solution be non- empty. After twenty two iteration we obtain $\boldsymbol{\lambda}^*$= 0.496446973. And we stop if $|\lambda_{22} - \lambda_{21}| = 2.39\times10^{-7} < 3\times10^{-7}$.

$\lambda_0 = 1$	e.
$\lambda_1 = 0.5$	e.
$\lambda_2 = 0.25$	n.e.
$\lambda_3 = 0.375$	n.e.
$\lambda_4 = 0.4375$	n.e.
$\lambda_5 = 0.46875$	n.e.
$\lambda_6 = 0.484375$	n.e.
$\lambda_7 = 0.4921875$	n.e.
$\lambda_8 = 0.49609375$	n.e.
$\lambda_9 = 0.498046875$	e.
λ_{10}= 0.497070312	e.
λ_{11}= 0.496582231	e.
λ_{12}= 0.49633799	n.e.
λ_{13}= 0.49646011	e.
λ_{14}= 0.49639905	n.e.
λ_{15}= 0.49642958	n.e.
λ_{16}= 0.496444825	n.e.
λ_{17}= 0.496452467	e.
λ_{18}= 0.496448646	e.
λ_{19}= 0.496446735	n.e.
λ_{20}= 0.49644769	e.
λ_{21}= 0.496447212	e.
λ_{22}= 0.496446973	n.e.

The optimal solution and decision variables in the optimal basis are as follows:

$Z_0^* = 31033670$, and X_1^*= 23729, X_3^*= 8384, $X_2^* = X_4^* = X_5^* = 0$

By optimality condition we obtain the first interval [0, 3.226830279] and by continue application the parametric analysis we obtain the second interval [3.226830286, ∞] for alternative basis.

The optimal solution and decision variables in the alternative basis are as follows:

$Z_1^* = 15158530$, and X_1^*= 2405, X_3^*= 14396, $X_2^* = X_4^* = X_5^* = 0$.

Table (2): Summary Solution of Real Data

t	x_1^*	x_2^*	x_3^*	x_4^*	x_5^*	$Z_i^*(t)$
$0 \le t \le 3.226830279$	23729	0	8384	0	0	31033670 + 156365446.3**t**
$3.226830286 \le t < \infty$	2405	0	14396	0	0	15158318.7 + 161284590.2**t**

4. Conclusions

1) The fuzzy matrix coefficients will cause a fuzzy in the coefficients of objective function.
2) The value of critical point that present the ends of the first interval will be approximate to the value of critical point that present beginning the second interval.
3) The best production in the optimal basis is X_1 then X_3 , while the alternative basis prefer the production X_3 first and then X_1.

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