Application of Branch and Bound Technique for n×3 Flow Shop Scheduling, In Which Processing Time Associated With Their Respective Probabilities

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Abstract:
The paper deals branch and bound technique to solve a 3 stage flow-shop scheduling problem in which probabilities are associated with their processing time. Our objective is to obtain an optimal sequence of jobs to minimizing the total elapsed time. The working of the algorithm has been illustrated by numerical example.

Keywords: Flow-Shop, Branch and Bound, Scheduling, Make Span, Total Elapsed Time.

1. Introduction:

Many applied and experimental situations, which generally arise in manufacturing concern to get an optimal schedule of jobs in set of machines, diverted the attention of researchers and engineers. In flow-shop scheduling, the objective is to obtain a sequence of jobs which when processed in a fixed order of machines, will optimize some well defined criteria. Various researchers have done a lot of work in this direction. Johnson[1], first of all gave a method to minimise the makespan for n-job, two-machine scheduling problems. The work was further extended by Ignall and Scharge [3], Cambell[7], Maggu and Dass [17], Heydari [21], Yoshida and Hitomi [20], Lomnicki [4], Palmer [2], Bestwick and Hastings [6], Nawaz et al. [9] , Sarin and Lefoka [13] , Koulamas [16], Dannenbring [8] , etc. by considering various parameters.Yoshida and Hitomi [20] considered two stage flow shop problem to minimize the makespan whenever set up times are separated from processing time. The basic concept of equivalent job for a job block has been introduced by Maggu and Dass [17]. Singh T.P. and Gupta Deepak [18] studied the optimal two stage production schedule in which processing time and set up time both were associated with probabilities including job block criteria. Heydari [21] dealt with a flow shop scheduling problem where n jobs are processed in two disjoint job blocks in a string consists of one job block in which order of jobs is fixed and other job block in which order of jobs is arbitrary.

Lomnicki [4] introduced the concept of flow shop scheduling with the help of branch and bound method. Further the work was developed by Ignall and Scharge[3], Chandrasekharan [22] , Brown and Lomnicki [5] , with the branch and bound technique to the machine scheduling problem by introducing different parameters. In practical situations processing times are not always deterministic so we have associated probabilities with their processing times of all the jobs on all the three machines. This paper combines the study made by Lomnicki[4], Singh T.P. and Gupta Deepak [18] and hence the problem discussed here is wider and has significant use of theoretical results in process industries.

2. Assumptions:

1. No passing is allowed.
2. Each operation once started must performed till completion.
3. A job is entity, i.e. no job may be processed by more than one machine at a time.

3. Notations:

We are given n jobs to be processed on three stage flowshop scheduling problem and we have used the following notations:

A_i : Processing time for job i on machine A
B_i : Processing time for job i on machine B
C_i : Processing time for job i on machine C
p_{i1} : Expected processing time for job i on machine A
p_{i2} : Expected processing time for job i on machine B
p_{i3} : Expected processing time for job i on machine C
C_{ij} : Completion time for job i on machines A, B and C.
S_0 : Optimal sequence
J_r : Partial schedule of r scheduled jobs.
J_r' : The set of remaining (n-r) free jobs.

4. Mathematical Development:

Consider n jobs say i=1, 2, 3 … n are processed on three machines A, B & C in the order ABC. A job i (i=1,2,3…n) has processing time A_i , B_i & C_i on each machine respectively, assuming their respective probabilities p_i , q_i & r_i such that $0\leq p_i \leq 1$, $\Sigma p_i = 1$, $0 \leq q_i \leq 1$, $\Sigma q_i = 1$, $0\leq r_i \leq 1$, $\Sigma r_i = 1$. The mathematical model of the problem in matrix form can be stated as :

Jobs	Machine A		Machine B		Machine C	
i	A_i	p_i	B_i	q_i	C_i	r_i
1 2 3 4 --- --- n	A_1 A_2 A_3 A_4 --- --- A_n	p_1 p_2 p_3 p_4 --- --- p_n	B_1 B_2 B_3 B_4 --- --- B_n	q_1 q_2 q_3 q_4 --- --- q_n	C_1 C_2 C_3 C_4 --- --- C_n	r_1 r_2 r_3 r_4 --- --- r_n

Tableau – 1

Our objective is to obtain the optimal schedule of all jobs which minimize the total elapsed time, using branch and bound technique.

1. Algorithm:

Step1: Calculate expected processing time p_{i1}, p_{i2} & p_{i3} on machines A, B & C respectively as follows:

$p_{i1} = A_i * p_i$, $p_{i2} = B_i * q_i$ and $p_{i3} = C_i * r_i$

Step2: Calculate

(i) $g_1 = t(J_r,1) + \sum_{i \in J'_r} p_{i1} + \min_{i \in J'_r}(p_{i2} + p_{i3})$

(ii) $g_2 = t(J_r,2) + \sum_{i \in j'_r} p_{i2} + \min_{i \in J'_r}(p_{i3})$

(iii) $g_3 = t(J_r,3) + \sum_{i \in j'_r} p_{i3}$

Step 3: Calculate

$g = \max [g_1, g_2, g_3]$

We evaluate g first for the n classes of permutations, i.e. for these starting with 1, 2, 3………n respectively, having labelled the appropriate vertices of the scheduling tree by these values.

Step 4: Now explore the vertex with lowest label. Evaluate g for the (n-1) subclasses starting with this vertex and again concentrate on the lowest label vertex. Continuing this way, until we reach at the end of the tree represented by two single permutations, for which we evaluate the total work duration. Thus we get the optimal schedule of the jobs..

6. Numerical example:

Consider 6 jobs 3 machine flow shop problem. processing time of the jobs on each machine is given. Our objective is to find optimal sequence of jobs to find the minimum elapsed time.

Job	Machine A		Machine B		Machine C	
i	A_i	p_i	B_i	q_i	C_i	r_i
1	5	0.2	8	0.2	20	0.2
2	6	0.2	30	0.2	6	0.2
3	30	0.3	4	0.2	5	0.1
4	2	0.1	5	0.1	3	0.1
5	3	0.1	10	0.2	4	0.2
6	4	0.1	1	0.1	4	0.2

Tableau – 2

Solution:

Step1: Define expected processing time p_{i1}, p_{i2} & p_{i3} on machine A , B & C respectively as shown in the tableau – 3

Step2 & Step 3: Calculate

(i) $g_1 = t(J_r,1) + \sum_{i \in J'_r} p_{i1} + \min_{i \in J'_r}(p_{i2} + p_{i3})$

(ii) $g_2 = t(J_r,2) + \sum_{i \in j'_r} p_{i2} + \min_{i \in J'_r}(p_{i3})$

(iii) $g_3 = t(J_r,3) + \sum_{i \in j'_r} p_{i3}$

For J_1 = (1).Then $J'(1)$ = {2,3,4,5,6}, we get

$g_1 = 55$, $g_2 = 66$ & $g_3 = 55$

$g = \max(g_1, g_2, g_3) = \max(55,66,55) = 66$

similarly, we have

LB(2)= max(55,67,78) = 78

LB(3)= max(55,91,76) = 91

LB(4)= max(55,64,49) = 64

LB(5)= max(55,64,55) = 64

LB(6)= max(58,65,47) = 65

Step4: Now branch from J_1 = (4). Take J_2 =(41)

Then J'_2={2,3,5,6} and LB(41)= max(55,64,54) = 64

Proceeding in this way, we obtain lower bound values on the completion time on machine C as shown in the tableau- 4

Therefore, the minimum completion time on machine C is 67. Hence optimal sequence is S_0 :4-5-1-6-2-3 and total elapsed time:67

In-Out table for the given problem is in tableau-5

7. Remarks:

The study may further be extended by considering various parameters such as transportation time, break down interval, mean weightage time etc.

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Tables and Figures:

Table 3: The expected processing times for machine A, B and C are as follows:

Job i	Machine A p_{i1}	Machine B p_{i2}	Machine C p_{i3}
1	5	8	20
2	6	30	6
3	30	4	5
4	2	5	3
5	3	10	4
6	4	1	4

Table 4: lower bounds for respective jobs are as follows:

Node Jr	LB (Jr)
(1)	66
(2)	78
(3)	91
(4)	64
(5)	64
(6)	65
(41)	64
(42)	77
(43)	89
(45)	62
(46)	63
(451)	59
(452)	76
(453)	83
(456)	59
(4512)	61
(4513)	79
(4516)	59
(45162)	61
(45163)	86

Tableau 5: In-Out table for the given problem is as follows:

Job i	Machine A In-out	Machine B In-out	Machine C In-out
4	0-2	2-7	7-10
5	2-5	7-17	17-21
1	5-10	17-25	25-45
6	10-14	25-26	45-49
2	14-20	26-56	56-62
3	20-50	56-60	62-67=T

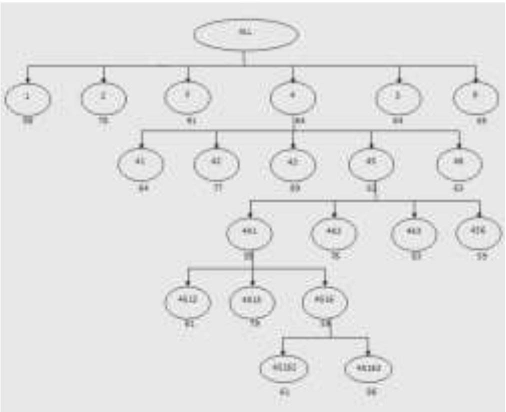

Figure-1 Branches for the optimal sequence