Selection method by fuzzy set theory and preference matrix

Singh Prachi (Corresponding author)
Govt.V.Y.T.P.G.Auto.College, Durg(C.G.) India
E-mail: prachibksingh@gmail.com

Pathak H.K.
School of Studiess,Pt.R.S. University
Raipur(C.G.)India

Abstract

In fuzzy decision making problems, fuzzy ranking is one of the most preferred aeras. The aim of this paper to develop a new ranking method which is reliable and doesnot need tremendous arithmetic calculations. Also it can be used for all type of fuzzy numbers which are represented as crisp form or in linguistic form. Fuzzy multi criteria decision making commonly employs methods such as ordering method,Fuzzy Analytic Hierarchy Process [FAHP], Fuzzy Technique for Order Preference by Similarity to Ideal Solution [FTOPSIS]and hybrid method. The FAHP commonly uses triangular fuzzy numbers and trapezoidal fuzzy numbers while the FTOPSIS method identifies the best alternative as the one that is nearest to the positive ideal solution and farthest to the negative ideal solution. Although both these methods have been widely used, they have their drawbacks. The accuracy of these methods decreases as the number of alternative increases i.e. the more complex the problem, less the accuracy and all the methods have many computations. In order to overcome this problem, we propose a method which is a combination of method of Blin and Whinston(1973) and method of Shimura(1973). This way the advantages of both the methods may be utilized to arrive at a decision that involves vague data. In this paper, we use the concept of preference matrix to find the membership grades and calculate the ranking.

Keywords: Fuzzy set, preference matrix, multi person decision making, multi criteria decision making(MCDM), relativity function matrix.

1. Introduction

Making decision is undoubtedly one of the most fundamental activities of human being. We all are faced in our daily life with varieties of alternatives actions available to us and we have to decide which of the available action to take. In classical decision theory can be characterized by a set of decision alternatives, a set of nature, a relation assigning to each pair of a decision and state a result and finally the utility function which orders the results according to their desirability. In this way a decision is to be made under conditions of certainty . The alternative that leads to the outcome yielding the highest utility is chosen i.e. the decision making problem becomes an optimization problem of maximizing the expected utility. When probabilities of the outcomes are not known, or may not even be relevant and outcome for each action are characterized only approximately i.e. decisions are made under uncertainties, this is the prime domain for fuzzy-decision-making .

1.1 Literature review

The research work done on fuzzy multi-criteria decision making(FMCDM) analysis has been one of the fastest growing areas in the field of decision making and operation research in real world scenario since a couple of decades. In 1970 Bellman and Zadeh initially suggested a fuzzy model of decision making in which relevant goals and constrains are expressed in terms of fuzzy sets. The consequent decision is determined by an appropriate aggregation of these fuzzy sets. It is a problem of individual decision making. When the decision is made by more than one users it is called multiperson-decision making . If in spite of multiple-person there exists multicriteria, the procedure is called multicriteria decision making. In multicriteria decision making it is assumed that the number of criteria need to be finite.

A fuzzy model group decision was proposed by Blin and Whinston(1973) and Blin(1974). Here, each member(criterion), totally or partially orders a set X of decision-alternatives, then a social choice function S is defined which produces the most acceptable overall group preference ordering denoted by:

$S: X \times X \rightarrow [0, 1]$ where $S(x_{\_i}, x_{\_j})$ indicates the degree of group preference of alternative $x_{\_i}$ over x_j. By a simple method of computation

$$S(x_{\_i}, x_{\_j}) = N(x_{\_i}, x_{\_j}) / n$$

Where $N(x_{\_i}, x_{\_j})$ = total number of popularity of x_i over $x_{\_j}$ by the total number of decision makers n.

Then the final nonfuzzy group preference can be determined by converting S into its resolution form

$$S = \bigcup_{(\alpha \in [0, 1])} \alpha\, {}^{\alpha}S$$

Yet another method was proposed by Shimura(1973), in which all given decision-alternatives are ordered on the basis of their pair wise comparisons. In this method $f(x_i, x_j)$ denotes the attractiveness grade given by an individual to $x_{\_i}$ with respect to $x_{\_j}$ alternative. This value then converted to relative preference grades $F(x_i, x_j)$ expressed as:

$$F(x_i, x_j) = f(x_i, x_j) / \max[f(x_i, x_j), f(x_j, x_i)] = \min[1, f(x_i, x_j) / f(x_j, x_i)]$$

For each $x_{\_i}$ in X, the overall relative preference grades $p(x_i)$ can now be calculated for x_i with respect to all the rest alternatives and denoted by $p(x_i)$:

$$p(x_i) = \min_{(xj \text{ in } X)} F(x_i, x_j)$$

The preference ordering of alternatives in X is then induced by the numerical ordering of these grades $p(x_i)$.

Fuzzy MCDM analysis has been one of the fastest growing areas in decision making and operations research during the last two decades. The Fuzzy Analytic Hierarchy Process [FAHP], first proposed by Van Laarhoven and Pedrycz(1983)was an extension of the method given by Saaty's (1980), in crisp form. This treatment dealt with fuzzy ratios of criteria components. These components are usually fuzzified the decision either triangular or trapezoidal membership functions. The FAHP was found easier to understand and effectively handle both qualitative and quantitative data in the multi-attribute decision making problems. In this method a comparison matrix for each criterion of each alternative with others is obtained. The geometric mean of each row is taken as weight factor and at last a fuzzy utility function aggregates all the matrix to give the final ordering. Technique for Order Performance by Similarity to Ideal Solution [TOPSIS] was first proposed by Hwang and Yoon(1981)}. In this method a FPIS (A^+) and a FNIS(A^-) for each row is defined and distances(d^+ and d^-) from these (A^+ and A^-) of each alternative from each row is calculated. Finally the rank preference ordering is given by descending order of closeness coefficient

$$CC_i = d^- / (d^+ + d^-)$$

i.e. the best decision-alternative would be the one that is nearest to the Positive Ideal Solution [PIS] and farthest from Negative Ideal Solution [NIS]. Both FAHP and FTOPSIS have been used individually to choose among with multiple criteria in various decision making processes. FTOPSIS method aims to define a positive or negative global solution, while FAHP method aims to arrive as consistent solution.

Although both these methods FAHP(Fuzzy Analytic Hierarchy Process) and FTOPSIS(Fuzzy Technique for Order Preference by Similarity to Ideal Solution) have been widely used however one did comment upon the drawbacks of the above mentioned techniques with an observation that the accuracy of these methods decreases as the number of alternative increases i.e. the more complex the problem less the accuracy. This weak performance may create potential problems for decision makers when complex real time situations are analyzed. In order to overcome this problems Sreeda.K.N and Sattanathan

R(2009)proposed a hybrid method that integrates both methods may be used when complex problems are encountered. This way the advantages of both the methods may be utilized to arrive at a decision that involves vague data.

In Karadogan A.et.el(2008) illustrated their logistics behind fuzzy multiple attribute decision making by (m × m) weight matrix where m represents decision criteria. This matrix is then used to compute m number of eigenvalues and eigenvectors to act as exponential portions over the decision membership-values so as to optimize the problems of choosing the most preferred alternative.

In due course of computing eigen-values and eigen-vectors, one point that struck was, in what order these computed eigen-vectors are chosen for m such participating criteria, as m numbers of-values are obtained by simplifying over mth degree polynomial in absolute scale. Moreover, treatment of fuzzy rating values for decision alternatives by an assumed exponent(randomly chosen eigen-vector from the computed m! times eigen vector.) opens the doorway towards exhaustive computation taking different combinations of exponents for optimizing the decision making procedure. Karadogan A.et.el(2008) explained their method with following example. This is an example of multi-criteria decision making for selection of best underground mining method out of five decision alternatives with different eighteen criteria.

Example (Karadogan A.et.el(2008))**:** The decision maker was asked to define the membership degree of each criterion i.e. conferred with subject experts and membership degrees are given to each criterion in single value between [0,1], not in triangle or trapezoidal fuzzy numbers.The reason behind that is the fractional exponential of these numbers is not possible. The grading given by decision maker is as follows:

C_1 =.80/ A_1+.75 A_2+.95/ A_3+.90/ A_4+.85 A_5

C_2 =.75/ A_1+.80/ A_2+.88/ A_3+.85/ A_4+.82/ A_5

C_3 =.70/ A_1+.65/ A_2+.87/ A_3+.85/ A_4+.92/ A_5

C_4 =.70/ A_1+.75/ A_2+.90/ A_3+.80/ A_4+.65/ A_5

C_5 =.55/ A_1+.60/ A_2+.70/ A_3+.75/ A_4+.85/ A_5

C_6 =.50/ A_1+.55/ A_2+.65/ A_3+.75/ A_4+.85/ A_5

C_7 =.70/ A_1+.65/ A_2+.85/ A_3+.75/ A_4+.90/ A_5

C_8 =.40/ A_1+.50/ A_2+.70/ A_3+.80/ A_4+1.00/ A_5

C_9 =.65/ A_1+.75/ A_2+.85/ A_3+.60/ A_4+.95/ A_5

C_{10}=.60/ A_1+.55/ A_2+.85/ A_3+.65/ A_4+.80/ A_5

C_{11} =.80/A_1+.75/ A_2+.90/ A_3+.65/ A_4+.95/ A_5

C_{12} =.78/ A_1+.70/ A_2+.90/ A_3+.75/ A_4+.65 / A_5

C_{13} =.50/ A_1+.72/ A_2+.80/ A_3+.60/ A_4+.85/ A_5

C_{14} =r.85/ A_1+.45/ A_2+.75/ A_3+.60/ A_4+.50/ A_5

C_{15} =.60/ A_1+.55/ A_2+.80/ A_3}+.70/ A_4+.95/ A_5

C_{16} =.60/ A_1+.55/ A_2+.80/ A_3+.65/ A_4+.90/ A_5

C_{17} =.75/ A_1+.70/ A_2+.82/ A_3+.80/ A_4+.90/ A_5

C_{18} =.65/ A_1+.70/ A_2+.80/ A_3+.75/ A_4}+.60/ A_5

Weight is obtained from eigen vector of relative preference matrix. Now they took exponent of weight to their respective criteria. Finally ``max of min'' gives the ordering of alternatives. Also in all these methods one has to define fuzzy numbers (single, triangular or trapezoidal), which is not an easier task.

Fuzzy ranking methods are another methods for solving MCDM problems. Ordering of fuzzy quantities is based on extracting various features from fuzzy sets. These features may be a center of gravity, an area under the membership function, or various intersection points between fuzzy sets. In their review, Bortolan and Degani(1985) find that for simple cases, most fuzzy set ranking methods produce consistent ranking. Difficult cases however, produce different rankings for different methods. This means that if membership functions overlap (or intersect) for some values of x, or if the supports of fuzzy numbers differ even slightly, different methods will most likely produce different rankings. From Bortolan-Degani(1985) and Wang-Kerre(2001a), (2001b) papers, the following methods were considered in our study: Balwin and Guild(1979), Campos Ibanes and Munoz(1989), Chang and Lee(1994), Chen(1985), Chen and Klien(1997), Fortemps and Roubens(1996), Kim and Park(1990), Liou and Wang(1992), Peneva and Popchev(1998). In these methods it is assumed that, the ranking methods must be able to rank several fuzzy sets of various shapes (triangular and trapezoidal) which are non-normal and non-convex. There must exist a rational numeric preference relation or linguistic interpretation that conveys which alternatives are most favored. But the method of Chen and Klien(1997) gives limited control to the decision maker in specifying his/her preferences, method of Peneva and Popchev(1998) is rejected because it requires fuzzy quantities to be triangular, method of Kim and Park(1990) is extremely similar to Chen(1985) method, both are based on finding intersections of minimizing/maximizing sets with fuzzy numbers in question. The only difference between the two methods is in the specification of risk preferences – Chen's method does it by varying exponents of the maximizing and minimizing sets, while Kim and Park's method emphasizes intersections of minimizing/maximizing sets with fuzzy numbers differently, although if all alternatives are relatively close together, Chen method can give reasonable results but it is deemed that this method is illogical also this method uses only two degrees of membership i.e.an objection can be raised that not enough fuzzy information is used in the ranking. Method of Balwin and Guild(1979)} can give reasonable results only when fuzzy sets overlap.

To avoid the above set of exhaustive computations and other explained complications, the authors come up with a simpler and logically sound technique to resolve the problems. The proposed method is a combination of Shimura's(1973) relative preference grades and Blin and Whinston's(1973) min-max composition.

1.1.2 Proposed methodology

Here we give some basic definitions.

Definition Fuzzy set: Fuzzy set A is a function which takes a set X to a unit interval [0,1]. Where A(x) denotes the membership grade, a real number in [0, 1] of an individual x in X due to function A.

Definition Alternative grading matrix: A matrix is constructed so that each entry a_{ij} represents a number by which an element S_i of ith row comes more times than an element S_j of jth column.

Table-1						
Alternative grading matrix						
Criteria	Rank					
	1	2	3	4	-	-
-						
C_n						
-						

Definition Relativity function: The above mentioned pair-wise membership values of the fuzzy preference matrix featuring the classification accuracy, were further used to measure relative membership values of classifying alternatives A_i over A_j in the form of relativity function as denoted in the expression below

$f(A_i / A_j) = f_{A_i}(A_j) / \max [f_{A_i}(A_j), f_{A_j}(A_i)]$, where $f(A_i / A_j)$ is the relativity function for choosing alternative A_i over alternative A_j.

1.1.3 The steps of computation

The proposed method makes of use the ranking method based on combination of Blin's(1974) method and Shimura's(1973) method that utilizes relative pair-wise preference function and overall group preference ordering. Every decision maker is asked to rank all the alternatives with respect to each criteria.

Step-1: In this method each alternative is ranked by their given numerical value or by linguistic preference.

Remark: By doing so it is seen that if we give the weight to all criterion then according to FTOPSIS method we have to multiply by this weight to the respective criteria values or according to the Karadogan A.et.el (2008) by exponent the weight to the respective criteria values, which cannot be applied for triangular or trapezoidal fuzzy numbers as the weights are fractional. It is observed that the ranking does not change after weighing i.e. there is no need of weighing of the criteria.

Step-2: Now construct the alternative grading matrix, so that each row shows that how many times an alternative is better than other alternatives.

Step-3: In the next step construct relativity function matrix and obtain minimum of each row. Finally ``Max of Min" gives the ranking of the alternatives.

Table-2

Alternative preference matrix

	A_1	A_2	A_3	A_4	A_5	--
A_1	-	(A_1, A_2)	(A_1, A_3)	(A_1, A_4)	(A_1, A_5)	-

Table-3 Overall alternative preference matrix

	A_1	A_2	--	--	Aj	--	Min of row
A_1	1	$f(A_1/A_2)$	--	--	$f(A_1/A_j)$	--	--
--	--	1	--	--	--	--	--
A_i	--	--	--	--	$f(A_i/A_j)$	--	--

Step-4: If ranking of any two alternatives is found same then the min-max computation method can be extended by ignoring the columns that contribute to this common minimum. Hence the alternatives can be re-ranked by computing their respective minimums over rest of the columns. This process of re-ranking continues in successive diminishing columns till all the alternatives are discretely ranked.

1.1.4 Explanation of method with examples

To explain the proposed method the previous explained example is taken because the result can be compared. The example of\ Karadogan A.et.el (2008) is a multi-criteria decision making In this all the decision alternatives are given membership grades in [0,1] for each criterion.

Example-1

Step-1: Rank all the alternatives by their given membership grades as in the Table-1A.

Step-2: Then construct the preference table, Table-2A . This fuzzy preference matrix has been obtained by analyzing statistical counts upon alternative preferences commulatively.

Step-3 and Step-4: The minimum of each row is find in the last column of above relativity function matrix which give the ranking of all alternatives. Hence by the Table-3A the ranking by proposed method methods is $A_5 > A_3 > A_4 > A_1 > A_2$. Which is same as the ranking calculated in the referred method.

Table-1A

Alternative grading matrix

Criteria	Rank				
	1	2	3	4	5
C_1	A_3	A_4	A_5	A_1	A_2
C_2	A_3	A_4	A_5	A_2	A_1
$C_3 = C_7$	A_5	A_3	A_4	A_1	A_2
C_4	A_3	A_4	A_2	A_5	A_1
$C_5 = C_6 = C_8$	A_5	A_4	A_3	A_2	A_1
C_9	A_5	A_3	A_2	A_1	A_4
C_{10}	A_3	A_5	A_4	A_1	A_2
C_{11}	A_5	A_3	A_1	A_2	A_4
C_{12}	A_3	A_1	A_4	A_2	A_5
C_{13}	A_5	A_3	A_2	A_4	A_1
C_{14}	A_1	A_3	A_4	A_5	A_2
$C_{15} = C_{16} = C_{17}$	A_5	A_3	A_4	A_1	A_2
C_{18}	A_3	A_4	A_2	A_1	A_5

Table-2A

Alternative preference matrix

	A_1	A_2	A_3	A_4	A_5
A_1	-	10	1	5	3
A_2	8	-	0	4	3
A_3	17	18	-	16	7
A_4	13	14	2	-	6
A_5	15	15	11	12	-

Table-3A

Relativity function matrix

	A_1	A_2	A_3	A_4	A_5	Min of the row
A_1	1	1	1/17	5/13	1/5	1/17
A_2	4/5	1	0	2/7	1/5	0
A_2	1	1	1	1	7/11	7/11
A_2	1	1	1/8	1	1/2	1/8
A_5	1	1	1	1	1	1

Example-2 : This example of Sreeda et el (2009) is taken here for the shake when at least two alternatives have same ranking. Assume there are three alternatives (A_1, A_2, A_3) with two aspects ($F_{1,}$, F_2) and five criteria (C_{11}, C_{12}, C_{13}, C_{14}, $C_{15.}$). The hierarchy is as shown in Table-4A.

Table-4A

Relationship among aspects, criteria and alternatives

Aspect	Criteria	Alternatives		
		A_1	A_2	A_3
F_1	C_{11}	(2/3, 5/6, 1)	(1/3,1/2, 2//3)	(2/3, 5/6, 1)
	C_{12}	(2/3, 5/6, 1)	(2/3, 5/6, 1)	(1/6,1/3,1/2)
	C_{13}	(2/3, 5/6, 1)	(1/3,1/2, 2/3)	(1/3,1/2, 2/3)
F_2	C_{21}	(1/2,2/3, 5/6)	(2/3, 5/6, 1)	(1/6,1/3,1/2)
	C_{22}	(1/2,2/3, /5/6)	(1/2,2/3, 5/6)	(1/3,1/2, 2/3)

By this table we construct the ranking as follows

$A_1 = A_3 > A_2$, $A_1 = A_2 > A_3$, $A_1 > A_2 = A_3$, $A_1 = A_2 > A_3$, $A_1 = A_2 > A_{3.}$

Then construct the preference table

Table-5A Alternative preference matrix				
	A_1	A_2	A_3	Min of the row
A_1	-	2	4	2
A_2	0	-	3	0
A_3	0	1	-	0

Here by Table-5A it is clear that $A_1 > A_2 = A_3$ i.e. no ranking for A_2 and A_3.

Step-4: To rank them compare A_2 and A_3 by deleting their minimum value 0 by rows. It is observed that by doing so $A_2 > A_3$ i.e. the final ordering is $A_2 > A_2 > A_3$. Which is same as given by Sreeda et el (2009).

1.1.5 Conclusion:

The proposed optimization process of fuzzy ranking type of decision making justifies with a note that the methodology works fine for the variety of criteria either expressed in mathematically crisp form or in linguistic form. Also this method needs no exhaustive computations, whether it be computation of m-dimensional eigen vector or it be calculation of m n manipulated fuzzy ratings traced upon weights for large sizes of m number of decision criteria and n number of decision alternatives. In proposed methodology the computation cost is minimum and there is no use of weight which is a lengthy process to find out.

References

Baldwin J.F. and Guild N.C.F. (1979), Comparison of fuzzy sets on the same decision space, Fuzzy Sets and Systems 2 ,213-231.

Buckley J.J(1985b), Fuzzy Hierarchical Analysis, Fuzzy Sets and Systems, 17(3), 233-247.

Blin J.M.(1974), Fuzzy relations in group decision theory, J.of Cyberneics4(2) 12-22.

Blin J.M.and Whinston A.B . (1973) , Fuzzy sets and social choice, J.of Cybernetics3(4) 28-36.

Bellmen R.E. and Zadeh L.A. (1970), Decision making in a fuzzy environment, Management sciences,17(4) 141-164.

Bortolan G., and Degani R. (1985), A review of some methods for ranking fuzzy subsets, Fuzzy Sets and Systems 15 1-19.

Campos Ibanes L.M., and Gonzales Munoz A. (1989), A subjective approach for ranking fuzzy subsets, Fuzzy Sets and Systems 29 145-153.

Chang P.T., and Lee E.S. (1994), Ranking of fuzzy sets based on the concept of existence, Computers and Mathematics with Applications 27(9) 1-21.

Chen S.H. (1985), Ranking fuzzy numbers with maximizing set and minimizing set, Fuzzy Sets and Systems 17113-129.

Chen C.B. and Klein C.M. (1997), A simple approach to ranking a group of aggregated fuzzy utilities, IEEE Transactions on Systems, Man and Cybernetics – Part B: Cybernetics 27(1) 26-35.

Fortemps P., and Roubens M. (1996), Ranking and deffuzification methods based on area compensation,

Fuzzy Sets and Systems 82, 319-330.

Hwang C.L.and Yoon K.(1981), Multiple Attributes Decision Making Methods and Applications, Springer, Berlin Heidelberg.

Laarhoven P.J.M.and Pedrycz W. (1983) , A Fuzzy Extension of Saaty's Priority Theory, Fuzzy Sets and Systems, 11(3), 229-241.

Liou T.S. and Wang M.J. (1992), Ranking fuzzy numbers with integral value, Fuzzy Sets and Systems 50 247-255.

Karadogan.A, Kahriman A. and Ozer U. (2008), Application of fuzzy set theory in the selection of underground mining method, The J.of the Southern African Ins.of Min.and Met. 108, 73-79.

Kim K. and Park K.S. (1990), Ranking fuzzy numbers with index of optimism, Fuzzy Sets and Systems 35, 143-150.

Peneva V. and Popchev I. (1998), Comparison of clusters from fuzzy numbers, Fuzzy Sets and Systems 97, 75-81.

Saaty T L .(1980),The Analytical Hierarchy Process, Mc Graw Hill, New York.

Shimura M. (1973), Fuzzy sets concept in rank-ordering objects, J.of Math.Anal.and Applications, 43(3), 717-733.

Sreeda.K.N and R.Sattanathan (2009), A Novel Hybrid Fuzzy Multicriteria Decision Making Method by Integrating Fuzzy AHP and Fuzzy TOPSIS, Int.J.of Algo., Comp. and Math. Vol.2(3) 47-55.

Shu-Hsien Liao and Kuo-Chung Lu, Evaluating anti-armor weapon using ranking fuzzy numbers, Tam.Ox.J.of Math.Sci.,16(2)(2000)241-257.

Triantaphyllou E (2000), Multi–criteria Decision making methods: A comparative study, Kluwer Academic Publishers, London.

Wang X. and Kere E.E. (2001a) , Reasonable properties of fuzzy quantities (I), Fuzzy Sets and Systems 118, 375-385.

Wang X., and Kere E.E. (2001b), Reasonable properties of fuzzy quantities (II), Fuzzy Sets and Systems 118 , 385-405.

Irfan Ertugrul and Nilsen Karakasoglu (2008), Comparison of fuzzy AHP and fuzzy TOPSIS methods for facility location selection, Int J Adv Manuf Technol , 39:783–795.

Zadeh L.A. (1965), Fuzzy Sets, Inform. Control~ 8, 338-353.